Efficient Antenna Array Beamforming with Robustness against Random Steering Mismatch

Ju-Hong Lee, Ching-Wei Liao, Kun-Che Lee

***Abstract*—**This paper deals with the problem of using antenna sensors for adaptive beamforming in the presence of random steering mismatch. We present an efficient adaptive array beamformer with robustness to deal with the considered problem. The robustness of the proposed beamformer comes from the efficient designation of the steering vector. Using the received array data vector, we construct a novel correlation matrix associated with the received array data vector and a correlation matrix associated with signal sources. Then, the eigenvector associated with the largest eigenvalue of the constructed signal correlation matrix is designated as an appropriate estimate of the steering vector. Finally, the adaptive weight vector required for adaptive beamforming is obtained by using the estimated steering vector and the constructed correlation matrix of the array data vector. Simulation results confirm the effectiveness of the proposed method.

***Keywords*—**Adaptive beamforming, Antenna array, Linearly constrained minimum variance, Robustness, Steering vector.

I. Introduction

Adaptive beamforming using antenna array of sensors is useful in the process of adaptively detecting and preserving the presence of the desired signal, while suppressing the interference and the background noise. For conventional adaptive array beamforming, we require the priori information of either the impinging direction or the waveform of the desired signal to adapt the weights [1, 2]. The adaptive weights of an antenna array beamformer under a steered-beam constraint are calculated by minimizing the output power of the beamformer subject to the constraint that forces the beamformer to make a constant response in the steering direction. Hence, the performance of the beamformer is very sensitive to the accuracy of the steering operation. However, the true direction vector of the desired signal may not be exactly known in some applications, e.g., the application in land mobile-cellular radio systems. It has been shown in [2, 3, 4, 5, 6] that even a small mismatch between the true direction vector of the desired signal and the steering vector deteriorates the effectiveness of a steered-beam beamformer.

In the literature, research endeavor has been devoted to tackle the adaptive array beamforming problem due to random steering mismatch. Several robust methods were recently developed by imposing either a spherical or an ellipsoidal uncertainty constraint directly on the steering vector for the conventional linearly main-beam constrained minimum variance (LCMV) beamforming [7-10]. These methods belong to the class of diagonal loading (DL) techniques which has been widely utilized to provide robustness against spatial uncertainty. It has been shown that this kind of ad hoc techniques usually helps to reduce the array beamforming sidelobes. They are robust against array steering vector errors if the diagonal loading factor is properly selected. However, the selection of the optimal diagonal loading factor is not clear. Moreover, it is somewhat difficult to find the relationship between the loading value and the level of uncertainty constraint or the preset level of robustness. Unlike the conventional DL techniques suffering from the problem of choosing an appropriate loading factor, a fully data-dependent loading technique was presented by [11] to avoid the trade-off suffered by the conventional DL techniques. Instead of determining the optimal loading factor, this technique generates an appropriate loading matrix directly from the sample correlation matrix of the received array data vector. Simulation results using this technique to deal with the problem of random steering mismatch were also presented in [11]. Recently, a Bayesian approach was presented in [12] to deal with the situation where the steering vector is assumed to be a random vector under a prior distribution. However, this approach relies on the assumptions of a Bingham prior distribution for the steering vector and an inverse Wishart prior distribution for the interference covariance matrix. Moreover, a robust technique against the random steering mismatch was presented in [13]. This technique relies on a reconstruction the interference-plus-noise covariance matrix first, and then an optimization scheme to estimate the direction vector of the desired signal.

In this paper, we present an efficient adaptive array beamforming technique to deal with the problem due to the random steering mismatch. Based on the reconstruction of the correlation matrices associated with the data vector and the signal sources received by an antenna array, we first construct a correlation matrix associated with the array data vector. Then, an efficient procedure is proposed to find an appropriate estimate of the steering vector. Using the reconstructed correlation matrix and the estimated steering vector, we finally find the weight vector required for performing adaptive beamforming according to the linearly constrained minimum variance (LCMV) criteria [14]. The novelty of the proposed technique is that it does not require both of much prior information like [12] as well as complicated optimization procedure like [13]. Moreover, we observe from the simulation results that the proposed technique outperforms the existing

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techniques in additional to its easiness of implementation.

This paper is organized as follows. Section II presents the brief theory of adaptive array beamforming based on the LCMV criteria. In Section III, we present the problem formation and present an efficient robust technique based on two efficient steps. One is for steering vector estimation. The other is for the reconstruction of the correlation matrix related to array data vector. Section IV provides several simulation results for confirming the effectiveness of the proposed technique. Finally, we conclude the paper in Section V.

II. ADAPTIVE BEAMFORMING USING UNIFORM LINEAR ARRAY

Consider that there are J+1 far-field signal sources including a desired signal and J interferers impinging on an M-element uniform linear array (ULA). The data vector $x(t)$ received by the ULA can be expressed as follows:

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{s}(t)$ = [$s_d(t)$ $s_1(t)$ ….. $s_J(t)$]^T contains the complex waveforms $s_d(t)$ of the desired signal impinging on the ULA with direction angle θ_d off array broadside with power = σ_s^2 and $s_j(t)$ of the jth interferer with direction angle θ_j and power = σ_j^2 , j = 1, 2, …, J, respectively, $\boldsymbol{A}$ = [$\boldsymbol{a}(\theta_d)$ $\boldsymbol{a}(\theta_1)$ ….. $\boldsymbol{a}(\theta_J)$] contains the direction vectors $\boldsymbol{a}(\theta_d)$ of the desired signal and $\boldsymbol{a}(\theta_j)$ of the jth interferer, j = 1, 2, …., J, respectively, and $\boldsymbol{n}(t)$ = [$n_1(t)$ $n_2(t)$ ….. $n_M(t)$]^T represents the spatially white background noise vector. The superscript T denotes transpose operation. Let $\boldsymbol{s}(t)$ and $\boldsymbol{n}(t)$ be uncorrelated, the $M \times M$ ensemble correlation matrix of $\boldsymbol{x}(t)$ is given by

$$\boldsymbol{R}_{xx} = \mathrm{E}\{\boldsymbol{x}(t)\boldsymbol{x}(t)^{\mathrm{H}}\} = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^{\mathrm{H}} + \sigma_n^2\boldsymbol{I}_M\,, \tag{2}$$

where the superscript H denotes complex conjugate transpose, $\boldsymbol{R}_{ss}$ = E{$\boldsymbol{s}(t)\boldsymbol{s}(t)^{\mathrm{H}}$} is the signal correlation matrix, σ_n^2 is the noise power, and $\boldsymbol{I}_M$ is the identity matrix with size M×M. Let the ULA use a weight vector $\boldsymbol{w}$ = [w_1, w_2, ….. w_M]^T for processing the received data vector $\boldsymbol{x}(t)$ to produce the array output signal $y(t)$ = $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(t)$. According to the linearly constrained minimum variance (LCMV) criterion, the adaptive beamformer minimizes the power of the output signal $y(t)$ subject to some preset constraints. Assume that the first signal with direction vector $\boldsymbol{a}(\theta_d)$ is designated as the desired signal. The adaptive beamforming problem is shown as follows [14]:

$$\begin{aligned} &\textit{Minimize} \quad \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{xx}\boldsymbol{w} \\ &\textit{Subject to} \quad \boldsymbol{a}(\theta_d)^{\mathrm{H}}\boldsymbol{w} = 1\,. \end{aligned} \tag{3}$$

The optimal weight vector for the solution of the optimization problem (4) is given by

$$\boldsymbol{w} = \boldsymbol{R}_{xx}^{-1}\,\boldsymbol{a}(\theta_d)\,[\,\boldsymbol{a}(\theta_d)^{\mathrm{H}}\boldsymbol{R}_{xx}^{-1}\,\boldsymbol{a}(\theta_d)\,]^{-1}. \tag{4}$$

It has been shown in the literature [7-10] that using the weight vector given by (4) is very effective for adaptive array beamforming without any imperfections. However, it cannot mitigate the performance degradation due to even a small imperfection in real environments [7-10].

III. PROBLEM FORMULATION AND PROPOSED ROBUST TECHNIQUE

A. Problem Formulation

In practice, $\boldsymbol{R}_{xx}$ is unavailable and the knowledge of $\boldsymbol{a}(\theta_d)$ may be inaccurate. A sample matrix inversion (SMI) approach is commonly used to solve the constrained minimization problem of (3) by using a sample correlation matrix instead of the ensemble one. Under the actual steering vector $\boldsymbol{a}$, the solution of (3) can be expressed as

$$\boldsymbol{w}_{smi} = \hat{\boldsymbol{R}}_{xx}^{-1}\,\boldsymbol{a}\,[\boldsymbol{a}^{\mathrm{H}}\,\hat{\boldsymbol{R}}_{xx}^{-1}\,\boldsymbol{a}]^{-1}, \tag{5}$$

where $\boldsymbol{a} \neq \boldsymbol{a}(\theta_d)$ if steering vector error exists. The sample correlation matrix $\hat{\boldsymbol{R}}_{xx}$ is computed from the received data vector $\boldsymbol{x}(t)$ as follows:

$$\hat{\boldsymbol{R}}_{xx} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{x}(t_n)\boldsymbol{x}(t_n)^{\mathrm{H}}\,, \tag{6}$$

where N denotes the number of data snapshots used and t_n the nth time instant. Without steering vector error, $\boldsymbol{w}_{smi}$ converges to $\boldsymbol{w}$ given by (4) as N increases.

In the presence of random steering vector, we consider that the actual steering vector $\boldsymbol{a}$ is expressed by

$$\boldsymbol{a} = \boldsymbol{a}(\theta_d) + \sigma_e\boldsymbol{\Delta}\,, \tag{7}$$

where σ_e denotes a error index and $\boldsymbol{\Delta}$ a Gaussian random vector with zero mean vector and covariance matrix equal to the identity matrix $\boldsymbol{I}_M$. Since $\boldsymbol{a} \neq \boldsymbol{a}(\theta_d)$, using the weight vector $\boldsymbol{w}_{smi}$ given by (5) for adaptive beamforming, the adaptive antenna array will suppress the desired signal because the desired signal becomes a undesired signal or interferer.

B. Proposed Robust Technique

Due to the random steering vector error, the correlation matrix computed according to (6) is deteriorated and cannot provide the performance that we are looking for. To deal with this problem, we first construct an appropriate correlation matrix to replace one obtained by (6). In practice, the information regarding the number of interferers, as well as their actual steering vectors and powers, and the noise power is unavailable. The reconstruction of the correlation matrix of the received array data vector $\boldsymbol{x}(t)$ is not an easy task. Recently, a feasible approach was presented in [13]. Based on the concept of [13], we are able to easily reconstruct $\hat{\boldsymbol{R}}_{xx}$ as follows:

$$\hat{\boldsymbol{R}}'_{xx} = \int_{-90}^{90}\frac{\boldsymbol{a}(\theta)\boldsymbol{a}(\theta)^{\mathrm{H}}}{\boldsymbol{a}(\theta)\hat{\boldsymbol{R}}_{xx}^{-1}\boldsymbol{a}(\theta)^{\mathrm{H}}}d\theta\,, \tag{8}$$

where $\boldsymbol{a}(\theta)$ denotes the direction vector associated with the direction angle θ in degree under the ULA. $\hat{\boldsymbol{R}}_{xx}$ is computed from (6). According to our simulation experience, the integration over the entire angle range of [-90°, 90°] makes the reconstructed correlation matrix include all of the signal sources impinging on the ULA. This leads to eliminating the drawback of [13] due to possibly missing some interference signals during the reconstruction process. As a result, the adaptive array beamformer using the reconstructed data correlation matrix $\hat{\boldsymbol{R}}'_{\boldsymbol{xx}}$ is able to suppress the interference signals more effectively.

For the next step, we present a novel method to find an appropriate estimate of the actual direction vector for the desired signal. Only the data vector $\boldsymbol{x}(t)$ received by the M array sensors is required. To prevent the use of the existing estimation schemes which usually resort to employing complicated optimization algorithms, we propose an efficient manner to achieve this goal as follows. The basic idea is to take the similar procedure for obtaining (8). However, we only require the information regarding the signal sources contained in $\boldsymbol{x}(t)$ to perform the estimation. Hence, we reconstruct the signal correlation matrix $\hat{\boldsymbol{R}}'_{\boldsymbol{ss}}$ associated with $\boldsymbol{x}(t)$ as follows:

$$\hat{\boldsymbol{R}}'_{\boldsymbol{ss}} = \int_{\theta_p-\delta}^{\theta_p+\delta} \frac{\boldsymbol{a}(\theta)\boldsymbol{a}(\theta)^{\mathrm{H}}}{\boldsymbol{a}(\theta)\hat{\boldsymbol{E}}_{\boldsymbol{n}}\hat{\boldsymbol{E}}_{\boldsymbol{n}}^{\mathrm{H}}\boldsymbol{a}(\theta)^{\mathrm{H}}} d\theta , \tag{9}$$

where θ_p denotes the presumed direction angle of the desired signal and δ the tolerance range decided by experiment for making $\theta_d \in [\theta_p - \delta, \theta_p + \delta]$. Moreover, $\hat{\boldsymbol{E}}_{\boldsymbol{N}}$ represents the basis matrix spanning the noise subspace associated with $\boldsymbol{x}(t)$. To obtain $\hat{\boldsymbol{E}}_{\boldsymbol{N}}$, we can first perform the eigenvalue decomposition (EVD) of $\hat{\boldsymbol{R}}_{\boldsymbol{xx}}$ and then utilize the well-know and very reliable information theoretic approach presented by [15] to estimate the number of signal sources. The noise subspace $\hat{\boldsymbol{E}}_{\boldsymbol{N}}$ can be easily obtained based on the estimated signal number according to [16]. After obtaining $\hat{\boldsymbol{R}}'_{\boldsymbol{ss}}$, we again perform its EVD and have

$$\hat{\boldsymbol{R}}'_{\boldsymbol{ss}} = \sum_{i=1}^{M} \lambda_i \boldsymbol{e}_i \boldsymbol{e}_i^{\mathrm{H}} , \tag{10}$$

where λ_i denotes the ith eigenvalue and $\boldsymbol{e}_i$ the eigenvector associated with λ_i . Let $\boldsymbol{e}_1$ be the eigenvector associated with the maximum eigenvalue. Then, $\boldsymbol{e}_1$ can be the appropriate estimate of the direction vector of the desired signal. Accordingly, the resultant weight vector required for achieving adaptive beamforming against the considered random steering mismatch is given by

$$\boldsymbol{w}_r = \hat{\boldsymbol{R}}'^{-1}_{\boldsymbol{xx}} \boldsymbol{e}_1 \left(\boldsymbol{e}_1^{\mathrm{H}} \hat{\boldsymbol{R}}'^{-1}_{\boldsymbol{xx}} \boldsymbol{e}_1 \right)^{-1} . \tag{11}$$

As regards the implementation of the proposed technique, the required computational complexity is dominated by the integrations shown by (8) and (9), and the EVD of (10). Clearly, the proposed technique requires less computational complexity than the existing robust techniques like [11-13].

IV. Computer Simulation Results

In this section, we present an example for illustration and comparison. For the simulations, we use a uniform linear array with omnidirectional array elements and M = 10. The inter-element spacing is set to half of the desired signal wavelength. Assume that all of the signal sources are binary phase-shift keying (BPSK) signals with rectangular pulse. The desired signal source impinges on the array from direction angle $\theta_d = 0^{\circ}$ off array broadside, while two interference signal sources impinge on the array from the direction angles $\theta_1 = -30^{\circ}$ and $\theta_2 = 30^{\circ}$ off array broadside, respectively. Moreover, the signal-to-noise power ratio (SNR) of the desired signal is set to 10 dB and the interference-to-noise power ratio (INR) is set to 20 dB, respectively. All the simulations are obtained by averaging 100 Monte-Carlo runs for illustration and comparison. Simulation results of using the proposed method, the fully data-dependent loading technique of [11], the robust technique of [13], and the conventional LCMV method of [14] are presented. In addition, the theoretically optimal performance without error (termed the ideal case) of using LCMV of [14] is also provided for confirming the effectiveness of the proposed technique. When using the proposed technique and the robust technique of [13], we set the angle range $[\theta_p - \delta, \theta_p + \delta]$ to $[-5^{\mathrm{o}}, +5^{\mathrm{o}}]$.

Figure 1 depicts the resulting array beam patterns of utilizing the above methods for comparison. We note from the figure that the LCMV of [11] cannot cure the problem of random steering vector mismatch. The proposed technique outperforms the other two existing robust techniques. Figure 2 plots the resulting array output SINR versus the number of the data samples used for simulations. Again, we observe from this figure that the LCMV of [11] fails to deal with the problem of random steering vector mismatch. The performance of the proposed technique is significantly better than the other two existing robust techniques. Finally, Figure 3 shows the array output SINR versus the variance of the random steering mismatch. This figure shows that the proposed technique possesses the advantages of better robustness against random steering mismatch over the existing robust techniques.

V. Conclusion

This paper has presented an efficient technique for dealing with the problem of random steering mismatch in adaptive array beamforming. Only the information of the data vector received by an antenna array is required for developing the proposed technique. Based on the sample correlation matrix associated with the array data vector, we reconstruct two correlation matrices and find an appropriate estimate for the steering vector. Then, the reconstructed correlation matrix of the array data vector and the estimated steering vector are utilized for adaptive beamforming. Simulation results have shown the efficiency of the proposed technique in terms of better robustness and less computational complexity than the existing robust techniques.

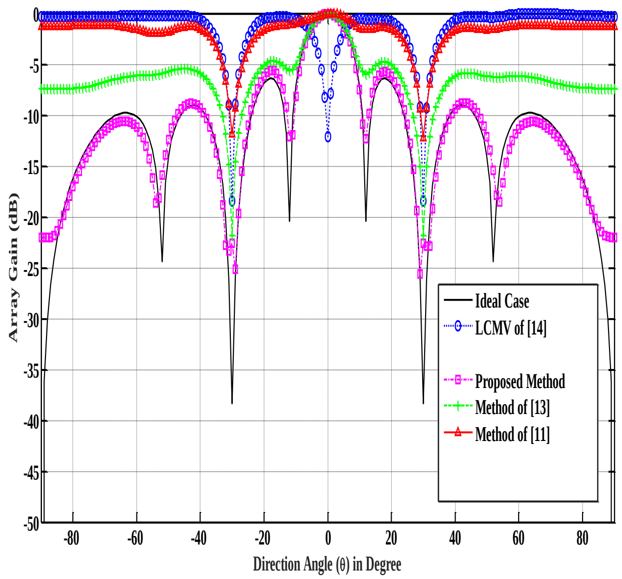

Fig. 1 The array beam pattern with $\sigma_e^2 = 1$ and 100 data samples.

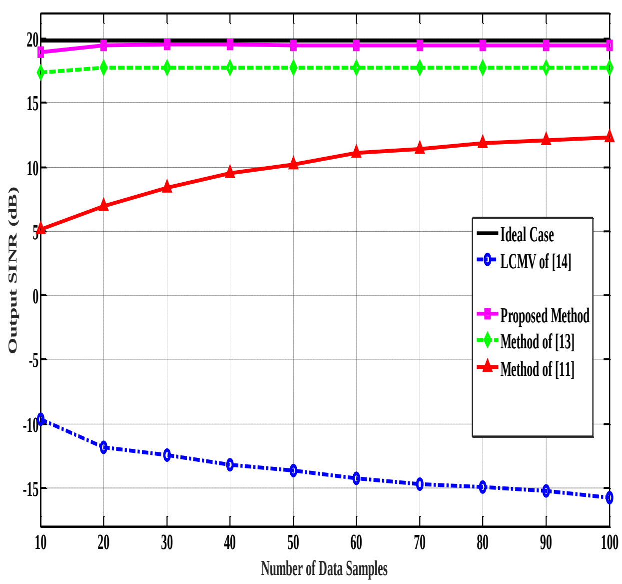

Fig. 2 The array output SINR versus number of data samples with $\sigma_e^2 = 1$

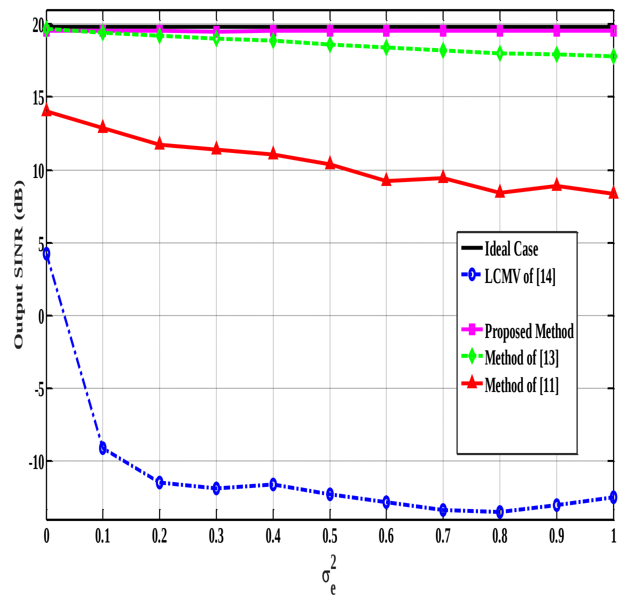

Fig. 3 The array output SINR versus σ_e^2 with 100 data samples.

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