An Optimal Bayesian Maintenance Policy for a Partially Observable System Subject to Two Failure Modes

Akram Khaleghei and Viliam Makis

{akhalegh,Makis}@mie.utoronto.ca

Department of Mechanical and Industrial Engineering
University of Toronto

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Outline

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Background

- Most systems may fail in several ways due to wear, fatigue, shock, design deficiency, etc.
- Maintenance engineers seek to extend the techniques to include several failure modes to evaluate system vulnerability to random failures which is called competing risks analysis.
- Recently, due to the advances in data collection, analysis, and modeling, it became possible to develop effective condition based maintenance (CBM) programs.
The collected CM data carries only partial information about the hidden state of the equipment and the dimensionality of such data is typically very large, with lots of redundancy, noise, and substantial cross and auto correlation present. Our main objective is to utilize partial information for early fault detection of the operating equipment with two modes of failure. Modern competing risks analysis has been dominantly applied in biomedical and public health research and very few competing risk models have been developed for the technical applications (Zhanshan and Krings, 2008).
Model development

- We model the degradation process as a continuous time homogenous Markov chain $\{X_t\}$ with state space $\mathcal{X} = \{1, 2, 3\}$.
- Vector data $Y_{n\Delta}$ are collected at regular epoch $\Delta$ time unit apart and it is assumed that $Y_{n\Delta}|X_n = i \sim N_d(\mu_i, \Sigma_i)$ for $i = 1, 2$ (Yang and Makis, 2000).
- $N_1$ sudden failure, $N_2$ degradation failure and $M$ suspension histories are collected.
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Objective:

• Find \((\Pi^*, T^*, \Delta^*)\) minimizing the long run average cost per unit time.
The following cost and time components are considered:

- $C_S$: Sampling cost,
- $C_I$: Inspection cost which takes $T_I$ time units,
- $C_{PM}(h)$: Preventive maintenance cost which takes $T_P$ time units,
- $C_F$: Failure cost which takes $T_F$ time units,
- $C_{LP}$: Loss of production cost rate for performing PM, CM, or system inspection,
- $C_{OM}$: Cost rate in the warning state.
By renewal theory, the cost minimization problem is equivalent to finding an optimal control limit such that:

\[
g(\bar{\Pi}^*, \bar{T}^*, \Delta^*) = \inf_{\bar{(\Pi, T)}} \left( \frac{E(\bar{\Pi}, \bar{T})(CC)}{E(\bar{\Pi}, \bar{T})(CL)} \right)
\]

where \(CL\) and \(CC\) denote the cycle length and cycle cost, respectively and \((\bar{\Pi}, \bar{T})\) is a fixed control limit pair. We assume that a cycle is completed when the system is brought back to the healthy state and machine condition is as good as new.
• The semi-Markov decision process (SMDP) is developed to determine the optimal control limits.

• The SMDP is defined to be in state \((l, h)\) if the \(\Pi_h \in \left[\frac{l-1}{L}, \frac{l}{L}\right]\) and \(h\Delta\) represents the age of the system. We denote set \(K_1 = \{(l, h)| 1 \leq l \leq L, h \in \mathbb{N}^+\}\).

• If \(\Pi_h\) is above the control limit, and it is false alarm, the SMDP is defined to be in state \((0, h)\) otherwise the SMDP is defined to be in state \((PM, h)\). We denote set \(K_2 = \{(0, h), (PM, h)\}\).

• The machine starts working in a good condition \(K_3 = \{(0, 0)\}\).

• \(K = K_1 \cup K_2 \cup K_3\) is the state space for the SMDP.
The long-run expected average cost $g(\bar{\Pi}, \bar{T}, \Delta)$ can be obtained by solving the following system of linear equations,

\[
v(i,h) = C(i,h) - g(\bar{\Pi}, \bar{T}, \Delta)\tau(i,h) \\
+ \sum_{(i',h+1)\in K} P(i,h)(i',h+1) v(i',h+1) \quad \forall (i, h) \in K
\]

\[
v(s_1,s_2) = 0 \quad \exists (s_1, s_2) \in K
\]
Transition probabilities: Let $\xi = \min(\xi_1, \xi_2)$,

\[
P(i,h)(i',h+1) = \begin{cases} 
P(0,0) = 1 - R_1((h+1)\Delta|h\Delta).R_2(\Delta|\Pi_h) 
\end{cases}
\]

\[
P(i,h)(i',h+1) = \begin{cases} 
P(0,0) = 1 - \frac{i - .5}{L} & \text{for } i \geq \bar{\Pi} \text{ and } h < \bar{T} 
\end{cases}
\]

\[
P(i,h)(0,0) = \begin{cases} 
P(0,0) = 1 \quad \quad \text{for } h \geq \bar{T} 
\end{cases}
\]
Expected sojourn times:

\[
\tau(i,h) = \Delta \sum_{i' \in K_1} P(i,h)(i',h+1) + \int_{0}^{\Delta} (T_F + t) \left[ -\frac{d}{dt} \left( R_1(h\Delta + t|\Pi_h) \right) \right] dt \quad \text{for } i > \bar{\Pi}, \ h < \bar{T}
\]

\[
\tau(PM,h) = T_{PM}
\]

\[
\tau(i,h) = T_I \quad \text{for } i \geq \bar{\Pi}, \ h < \bar{T}
\]

\[
\tau(i,h) = T_{PM} \quad \text{for } h \geq \bar{T}
\]
Expected costs:

\[
C_{(i,h)} = C_S \sum_{i' \in K_1} P(i,h)(i',h+1) + (C_{LP} \cdot T_F + C_F)P(i,h)(0,0)
\]
\[
+ C_{OM} \int_0^\Delta P(X_t = 1 | \xi > h\Delta, \Pi_h) dt \quad \text{for } i < \bar{\Pi}, h < \bar{T}
\]
\[
C_{(PM,h)} = C_{PM}(h) + C_{LP} \cdot T_{PM}
\]
\[
C_{(i,h)} = C_I + C_{LP} \cdot T_I \quad \text{for } i \geq \bar{\Pi}, h < \bar{T}
\]
\[
C_{(i,h)} = C_{PM}(h) + C_{LP} \cdot T_{PM} \quad \text{for } h \geq \bar{T}
\]
Illustration using simulated data

- Assume \( \lambda_0 = 0.15 \) and \( \lambda_1 = 0.3 \).
- \( Y_n \sim N_2(\mu_i, \Sigma_i) \) for \( i = 1, 2 \):
  
  \[
  \mu_0 = \begin{pmatrix} .2 \\ -1 \end{pmatrix} \quad \Sigma_0 = \begin{pmatrix} 1.5 & .5 \\ .5 & 1.5 \end{pmatrix}
  \]
  
  \[
  \mu_1 = \begin{pmatrix} .8 \\ -1 \end{pmatrix} \quad \Sigma_1 = \begin{pmatrix} 2.5 & 2.5 \\ 2.5 & 3 \end{pmatrix}
  \]

- Time to sudden failure follows Weibull distribution with scale parameter \( \lambda = 10 \) and shape parameter \( k = 2 \).
- If the age of a system exceeds a threshold \( \bar{T} \Delta \leq 10\Delta \), preventive maintenance activity is initiated.
• $T_I = 2$, $T_{PM} = 3$, $T_F = 10$, $C_I = $10 , $C_{PM}(h) = $(500 + 10h) , $C_F = $1500 and $C_{LP} = $20 per unit time, $C_{OM} = $2 per unit time.

• It has been found that when $L \geq 25$, the partition leads to a high degree of precision, so that $L = 25$.

Table : Comparisons with other method.

<table>
<thead>
<tr>
<th></th>
<th>MVBCH with age replacement</th>
<th>MVBCH with no age replacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\Pi}^*$</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>$\Delta^*$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$T^*$</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>$g(\Pi^<em>, T^</em>, \Delta^*)$</td>
<td>$47.21$</td>
<td>$58.95$</td>
</tr>
</tbody>
</table>
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Posterior probability

Sampling epoch

Stop for system inspection

$\Pi^*$
Conclusions and future research

- By introducing the maintenance policy proposed in this paper which is easy to implement, the maintenance cost will be reduced substantially and also the machine safety and reliability will be improved.
- In our future work, the new maintenance policy will be applied to real data.
- We hope that the results obtained in this paper will motivate future research in this area by allowing the state sojourn times to have more general distributions.
J Yang and V Makis,
Dynamic response of residual to external deviations in a controlled production process.

LP Chen, ZS Ye, and B Huang
Condition-based maintenance for systems under dependent competing failures,

Z Ma and A Krings,
Competing risks analysis of reliability, survivability, and prognostics and health management (PHM)
*IEEEAC*, 2008.