A Community Compromised Approach to Combinatorial Coalition Problem

Laor Boongasame, Veera Boonjing, and Ho-Fung Leung

***Abstract*—**Buyer coalition with a combination of items is a group of buyers joining together to purchase a combination of items with a larger discount. The primary aim of existing buyer coalition with a combination of items research is to generate a large total discount. However, the aim is hard to achieve because this research is based on the assumption that each buyer completely knows other buyers' information or at least one buyer knows other buyers' information in a coalition by exchange of information. These assumption contrast with the real world environment where buyers join a coalition with incomplete information, i.e., they concerned only with their expected discounts. Therefore, this paper proposes a new buyer community coalition formation with a combination of items scheme, called the *Community Compromised Combinatorial Coalition* scheme, under such an environment of incomplete information. In order to generate a larger total discount, after buyers who want to join a coalition propose their minimum required saving, a coalition structure that gives a maximum total retail prices is formed. Then, the total discount division of the coalition is divided among buyers in the coalition depending on their minimum required saving and is a Pareto optimal. In mathematical analysis, we compare concepts of this scheme with concepts of the existing buyer coalition scheme. Our mathematical analysis results show that the total discount of the coalition in this scheme is larger than that in the existing buyer coalition scheme.

***Keywords*—**Group decision and negotiations, group buying, game theory, combinatorial coalition formation, Pareto optimality.

I. Introduction

THERE are several existing buyer coalition researches [1], [2], [4], [7], [8], [10], [11], [12], [15], and 18]. Some existing buyer coalition researches [2], [8], [11], [15], and [18] form a buyer coalition with only one type of item. However, if a seller offers discount price in each transaction based on a total retail price of the items, each buyer in a coalition may want to purchase a combination of items. Therefore, forming buyer coalition with a combination of items or the Combinatorial Coalition Formation Scheme (*CCF*) [10] is proposed. Such forming can enlarge the total retail price in each transaction and gives a larger total discount.

The *CCF* scheme has applied the concepts from the core theory in coalition formation theories (e.g., the Shapley value [14], the core [6], the bargain set [3], the kernel [5], the nucleolus [13], the coarse core [17], the private core [19], and the fine core [16]). These theories are based on the assumption that each player in a coalition completely knows all other players' private information, i.e., each player in the coalition knows all other players' payoffs, or at least one player knows the other players' information in a coalition by exchange of information. However, the assumptions contrast with a buyer coalition forming in practical where buyers generally have private information and do not want to reveal their information to the other buyers in the coalition.

This paper proposes a new buyer community coalition with a combination of items scheme, called the *C*ommunity *C*ompromised *C*ombinatorial *C*oalition *(C4*) Scheme. The solution of this scheme is based on the assumption that each buyer does not know information of other buyers. By the assumption, this solution can attract buyers to form the coalition by giving at least the required minimum discount to them. Thus, this system or the honest coordinator can achieve the goal of this scheme, which is to generate a large total discount, by selecting a coalition structure that gives total retail price as large as possible. By this way, a total discount of the coalition is higher than that of the competitive coalition which is formed from other solutions (such as the core theory), but buyers with high reservation price in this scheme may pay for buyers with low reservation price more than they do in the other solutions. Since "buyers" in this paper means buyers in a community and they have the feeling that they belong to the community, it is not difficult to understand that individual buyers compromise their benefits so that the community can obtain the maximum benefits for all buyers.

There are three desired goals of this scheme: (1) to maximize the total discount; (2) to recognise and respect the individual buyers' required minimum discounts; and (3) to distribute the total discount among buyers in a coalition in a Pareto optimal manner. The stability in the total discount division of the coalition in this scheme is guaranteed in terms of Pareto optimality because it always exists (not be occasionally an empty set) unlike the core in game theory [2].

The correctness of properties of this scheme is proven by mathematical analysis. To guarantee the good results of the *C4* scheme to be presented in this research, it compares the results with those of the *CCF* scheme [10].

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This paper is organized in five sections. Section II presents preliminaries and related works. Section III elaborates the *C*ommunity *C*ompromised *C*ombinatorial *C*oalition *S*cheme. Section IV discusses properties of the Community Compromised Combinatorial Coalition Scheme. Lastly, section V concludes the paper.

II. Preliminaries and Related works

In this section, we first present a buyer coalition with a combination of items formation problem in II.A. The *CCF* scheme [10] has applied the concept from the core in game theory, one of the most popular solution concepts. This is a reason that the *CCF* scheme is often cited. Consequently, we describe the concepts of the *CCF* scheme in sections II.B.

A. A buyer coalition with a combination of items problem

Some previous buyer coalition research works [2], [8], [11], [15], and [18] concentrate on forming a buyer coalition with only one type of item. Since a total discount of forming a buyer coalition with a combination of items is more than that in forming a buyer coalition with only one type of item [10], therefore, buyers willing to form a coalition with a combination of items. What are the solutions for a buyer coalition with a combination of items that allow buyers in the coalition get more discounts? Such a buyer coalition formation problem is called a *Buyer Coalition with a Combination of Items problem.*

1) A motivating example

An illustrative example of the buyer coalition with a combination of items problem is shown as follows. A book store of a university is required to provide books at discount prices for all students in the university. However, the book store often has a problem on stocks of the books. The problem makes the students pay for the books at higher prices. Therefore, the manager of the book store implements a new policy that students who want to purchase books with the book store have to place an order that includes titles, units, and reservation prices of the required books in advance before the manager of the book store orders the books from a publisher company. Suppose some students want to purchase two titles of books as follows: (i) Introduction to Statistics and (ii) Elements of English. The retail price of Introduction to Statistics, and Elements of English equals \$900, and \$1000 respectively. The publisher company's discount policy depending on a total retail price of each transaction for the books is shown in Table I. Different students generally place different reservation prices at different time different place. For example, Ying sends an e-mail to the manager at 3 PM that "Please, purchase two copies of Introduction to Statistics and a copy of Elements of English at most \$2650." The students' reservation prices are shown in Table II. Each reservation price of the students is lower than the total retail price of all items in his order after discount. For example, when Pong orders two copies of Introduction to Statistics and three copies of Elements of English, her reservation price is \$4450, which is less than \$(4800-300) = \$4500. After all students order the books at their reservation prices, the manager forms a buyer coalition.

2) Problem formulation

TABLE I
A PUBLISHER COMPANY'S DISCOUNT POLICY DEPENDING ON A TOTAL RETAIL PRICE OF EACH TRANSACTION.

A total retail price of each transaction P (\$)	Discount $D(P)$ (\$)
>= 3000	\$300
>= 7000	\$700
>= 8000	\$800

TABLE II
STUDENTS' RESERVATION PRICES OF THIS EXAMPLE

Students' name	Units of (i)	Units of (ii)	Total retail price P_k (\$)	Discount $D(P_k)$ (\$)	P_k after discount $P_{k\min}$ (\$)	R_k (\$)
Ying	2	1	2,800	0	2,800	2650
Pong	2	3	4,800	300	4,500	4450
Wang	1	-	900	0	900	790

Let $G = \{g_1, g_2, ..., g_i\}$ be the collection of items, $B = \{b_1, b_2, ..., b_k\}$ be the set of buyers. There is only one seller S who can supply unlimited units of the items in G and its own price discount schedule is a non-monotonic ascending discount function $F: T \rightarrow D(T)$ where T is a total retail price in one transaction and $D(T)$ is the corresponding discount for T. There is a set of all possible retail prices per unit $P = \{p_1, p_2, ...p_i\}$ of each item g_i in G. Each buyer $b_k \in B$ places only one bid, $bid_k = \{q_{k1}, q_{k2}, ...q_{ki}; R_k\}$, where $q_{ki} > 0$ is the quantity of each item g_i that b_k requires, and R_k be the reservation price of buyer b_k, the maximum price which buyer b_k is willing to pay for $\{q_{k1}, q_{k2}, ...q_{ki}\}$. Different buyers generally have different reservation prices R_k and $R_k \le P_{k\min}$ where $P_k = \sum_{g_i \in bid_k} (q_{ki} \times p_i)$ is the total retail price of all items for each bid_k, and $P_{k\min} = P_k - D(P_k)$ is the total retail price of all items for each bid_k after discount, *called the discounted total retail price of items of buyer* b_k. Different reservation prices generally are proposed at different time different places. Denote by $T_C = \sum_{b_k \in C} P_k$ the total retail price of items of all buyers in coalition C where $C \subseteq B$ be a subset of buyers who can join together to purchase identical items with a larger discount in one transaction and $D(T_C)$ or $TD(C)$ the discount of total retail price of items in coalition C.

B. The combinatorial coalition formation (CCF) scheme

The *CCF* scheme [10] aims at generating the highest utility from a buyer community coalition. The coalition structure, called LVC_A, can be found using the following three steps. Let $v(C) = \sum_{b_k \in C} R_k - (T_C - D(T_C))$ is *the Utility of a coalition* C.

Step 1: Determine $AC = \{C \subseteq B : v(C) \geq 0\}$

Step 2: Determine $VC_A = \{C \in AC : v(C) \geq v(C''), \forall C'' \in AC\}$

Step 3: Determine $LVC_A = \{C \in VC_A : \sum_{b_k \in C} P_k \geq \sum_{b_k \in C''} P_k, \forall C'' \in VC_A\}$

Example 1

Consider the scenario in the motivating example again. The coalition structure LVC_A is found as follows. First, we find the set AC which is ({Ying, Pong}, {Ying, Wang}, {Ying, Pong, Wang}). The utilities of any coalitions in these students are shown in Table III. Then, we find the set VC_A which is ({Ying, Pong}). Finally, we find the set LVC_A which is ({Ying, Pong}).

TABLE III
THE UTILITIES OF ANY COALITIONS IN THESE STUDENTS IN THIS EXAMPLE

Coalitions	The utilities of coalitions
{Ying, Pong}	(7100-7600+700) = 200
{ Ying, Wang}	(3440-3700+300) = 40
{ Pong, Wang}	(5240-5700+300) = -160
{Ying, Pong, Wang}	(7890-8500+800) = 190

In summary, the total retail price in the coalition is \$7600, and the book store saves \$700 in total for the students in the university.

III. A COMMUNITY COMPROMISED COMBINATORIAL COALITION SCHEME

A. The community compromised combinatorial coalition (C4) scheme

1) Definition of terms

In order to describe the *C4* scheme, we first define some terms.

The reservation price of a buyer is the maximum price which a buyer is willing to pay for all units of all items in his order. Since a seller's discount policy is based on the total retail price of each transaction; therefore, if buyers form a coalition to purchase the items together on one transaction, they will get more discounts**.** *The discount* is the discount that the seller gives for the total retail price of each transaction. *The Utility of a coalition* represents the difference between the total reservation price of buyers in a coalition and the total retail price of items of the buyers in the coalition after discount.

Definition 1: Let $B = \{b_1, b_2, ..., b_k\}$ be the set of buyers, R_k be the reservation prices of buyer b_k, $T_C = \sum_{b_k \in C} P_k$ be the total retail price of items of all buyer b_k in C where $C \subseteq B$, and $D(T_C)$ be the discount of T_C. *The Utility of a coalition* C is defined as $v(C) = \sum_{b_k \in C} R_k - (T_C - D(T_C))$.

Example 2

Smith, Sam, and Simon is a group of buyers who join together to purchase books in one transaction. Smith is willing to pay for a copy of an identical Introduction to Statistics and a copy of an identical Elements of English at \$1800. Sam is willing to pay for a copy of Introduction to Statistics and two copies of Elements of English at \$2700. Simon is willing to pay for a copy of Introduction to Statistics at \$870. Suppose that the retail price of Introduction to Statistics equals \$1100 and the retail price of Elements of English equals \$900. If a total retail price of each transaction is more than \$5000, a discount at 10% will be given.

Let S be a coalition of Smith, Sam, and Simon who join together to purchase the books in one transaction. Therefore, *the Utility of Coalition* S is

$$v(S) = \sum_{b_k \in S} R_k - (T_S - D(T_S))$$
$$= 150$$

Different buyers generally have different reservation prices. *Required Minimum Saving of a buyer* represents the difference between the discounted total retail price of items of a buyer and the reservation price of the buyer.

Definition 2: Let $P_{k\min}$ be the discounted total retail price of items of buyer b_k, and R_k be the reservation prices of buyer b_k. *Required minimum saving of buyer* b_k is defined as $RD_k = P_{k\min} - R_k > 0$.

Actual discount of a buyer represents the final discount which a buyer gets from forming a coalition. Buyers in a coalition generally get different actual discounts depending on their reservation prices. *Final Price of a buyer* represents the difference between the discounted total price of items of a buyer and the actual discount of the buyer.

Definition 3: Let $P_{k\min}$ be the discounted total retail price of items of buyer b_k, and AD_k be the actual discount of buyer b_k. *Final Price of buyer* b_k is defined as $F_k = P_{k\min} - AD_k$.

A buyer can purchases any units of the items without forming with other buyers at his discounted total retail price of items which is higher than or equal to his reservation price. The buyer get the utility equals the difference between his reservation price and his discounted total retail price of items. However, the buyer gets more discounts and purchases the items at a price lower or equal to his reservation price if the buyer is a member of a coalition. The buyer gets the utility equals his actual discount.

Definition 4: Let $P_{k\min}$ be the discounted total retail price of items of buyer b_k, R_k be the reservation prices of buyer b_k, and AD_k be the actual discount of buyer b_k in the coalition C. *The Utility of buyer* b_k is defined as

$$v(\{k\}) = \begin{cases} AD_k & (b_k \in C) \\ (R_k - P_{k\min}) & (b_k \notin C) \end{cases}$$

2) Concepts of the C4 Scheme

The aim of a new buyer community coalition formation with a combination of items scheme, called the *Community Compromised Combinatorial Coalition (C4) Scheme*, is to generate a large total discount from a buyer community coalition with incomplete information. There are two main activities that are proposed to achieve the aim. First, we find a coalition structure that gives the maximum total retail price with the highest non-negative utility. Second, we divide the total discount of the coalition among buyers in the coalition depending on the required minimum saving RD_k of buyer b_k. Moreover, the total discount of the coalition is divided completely so that buyers in the coalition achieve the efficiency of the total discount division.

This scheme for forming a coalition C consists of two stages. In the first stage, we first find the set AC of all coalitions with non-negative. Then, we find the set VC_C of the coalitions with the maximum total retail price from AC. Finally, we find the set LVC_C of the coalitions with the highest utility from VC_C. In this way, we can find the set of coalition structures that give the maximum total retail price with the highest non-negative utility if such a coalition exists. Additionally, we describe the first stage in a formal way as follows. Let $B = \{b_1, b_2, ..., b_k\}$ be a set of buyers and $C \subseteq B$ be a subset of buyers who can join together to purchase identical items with a larger discount. Then

$$AC = \{C \subseteq B : v(C) \geq 0\},$$

$$VC_C = \{C \in AC : \sum_{b_k \in C} P_k \geq \sum_{b_k \in C''} P_k \ \forall C'' \in AC\},$$

$$LVC_C = \{C \in VC_C : v(C) \geq v(C'') \forall C'' \in VC_C\}.$$

In the second stage, there are two cases in this stage. In the first case, if LVC_C is empty, VC_C and AC are also empty. Consequently, there is no a coalition structure that gives the maximum total retail price with the highest non-negative utility. In the second case, if $LVC_C \neq \varnothing$, let $C^* \in LVC_C$ be a coalition structure. The coalition C^* has two properties hold. First, the actual discounts of any buyers in C^* are more than or equal to their required minimum saving. Second, the sum of actual discounts of all buyers in the coalition C^* equals the total discount $TD(C^*) - \sum_{b_k \in C^*} D(P_k)$ of the coalition C^*. Additionally, we describe this stage in a formal way. Let AD_k be the actual discount of buyer b_k in C^* and RD_k be the required minimum saving of buyer b_k. The coalition C^* has the two properties as follows.

Property 1: If $LVC_C \neq \varnothing$ then we have $AD_k \geq RD_k \ \forall b_k \in C^*$ where $C^* \in LVC_C$.

Property 2: If $LVC_C \neq \varnothing$ then we have $\sum_{b_k \in C^*} AD_k = TD(C^*) - \sum_{b_k \in C^*} D(P_k)$ where $C^* \in LVC_C$.

B. The motivating example revisited

Consider the scenario in the motivating example again and we give additional information as follows. Each student in the university can place only one reservation price to the manager of the book store, not having any information about the reservation price given by the other students.

We describe forming a coalition by concepts of the *C4* scheme in section III.A.

In the first stage, the coalition structure is determined as follows. First, we find the set AC which is ({Ying, Pong}, { Ying, Wang}, {Ying, Pong, Wang}). Then, we find the set VC_C which is ({Ying, Pong, Wang}). Finally, we find the set LVC_C is ({Ying, Pong, Wang}).

In the second stage, since $LVC_C \neq \varnothing$, let $C^* \in LVC_C$ be a coalition structure. The coalition C^* has two properties holds. First, $AD_k \geq RD_k \ \forall b_k \in C^*$, i.e., $AD_{Somwang} \geq RD_{Somwang}$.

Second, $\sum_{b_k \in C^*} AD_k = TD(C^*) - \sum_{b_k \in C^*} D(P_k)$. We give an example of the required minimum saving, the actual discounts, and the final prices of buyers in the coalition as shown in Table 4.

From this example, we observe that (i) $AD_k \geq RD_k \ \forall b_k \in C^*$ (ii) forming the coalition C^* gives larger discounts for three persons and the book store saves $800 in total for the students. However, if some students form other coalitions to get the maximum actual discounts, it may cause a decrease in the total discount. For example, the coalition ({Ying, Pong}) in Example 1, it saves only $700 in total for the students and there are only two persons in the coalition and (iii) the total discount is divided among students completely.

IV. PROPERTIES OF THE COMMUNITY COMPROMISED COMBINATORIAL COALITION SCHEME

In this section, we show some properties of the *C4* scheme.

This scheme guarantees that the utilities of all buyers in a coalition are more than or equal to the utilities of buyers not in the coalition.

Proposition 1: (*Utilities of buyers in a coalition*) If $LVC_C \neq \varnothing$, let $C^* \in LVC_C$. $\forall b_g \in C^*$ and $\forall b_h \notin C^*$, we have $v(\{b_g\}) \geq v(\{b_h\})$.

Proof

Consider a buyer $b_h \notin C^*$ and a buyer $b_g \in C^*$. We have $v(\{b_h\}) = R_h - P_{h\min}$ and $v(\{b_g\}) = AD_g$ by Definition 4. Since $R_h \leq P_{h\min}$, $v(\{h\}) \leq 0$. On the other hand, since $AD_g \geq RD_g > 0$ by Property 1 and Definition 2, $v(\{b_g\}) = AD_g \geq 0$. Therefore, $v(\{b_g\}) \geq v(\{b_h\})$.□

Proposition 1 shows that this scheme motivates buyers to be members of the coalition. Additionally, this scheme

TABLE IV
AN EXAMPLE OF THE REQUIRED MINIMUM SAVING, THE ACTUAL DISCOUNTS, AND THE FINAL PRICES OF BUYERS IN THE COALITION OF THE MOTIVATING EXAMPLE REVISITED

Students, Name	$P_{k\,\min}$ ($)	R_k ($)	RD_k ($)	AD_k ($)	F_k ($)
Ying	2800	2650	150	150	2650
Pong	4500	4450	50	240	4260
Wang	900	790	110	110	790

guarantees that the actual discount division in the coalition is a Pareto optimal so that buyers in the coalition achieve the efficiency of the actual discount division in the coalition.

Proposition 2: (*Pareto Optimality in a coalition*) If $LVC_C \neq \varnothing$, let $C^* \in LVC_C$, and $(AD_k)_{k\in C^*}$ be the actual discounts of buyers in the coalition C^*. $(AD_k)_{k\in C^*}$ is Pareto optimal.

Proof

Suppose there exists $(AD'_k)_{k\in C^*}$ such that $AD'_i > AD_i$ for some i. Since $\sum_{b_k\in C^*} AD_k = TD(C^*) - \sum_{b_k\in C^*} D(P_k)$ by Property 2, $\sum_{b_k\in C^*} AD'_k > TD(C^*) - \sum_{b_k\in C^*} D(P_k)$. However, $C^* \in LVC_C$ and $TD(C^*) - \sum_{b_k\in C^*} D(P_k)$ is the largest discount of coalition C^*. Thus they are contradictions.□

Proposition 2 shows that no buyer can get a better bargain than the actual discount without making some other buyers worse off.

Proposition 3: (*Stability of the payoff division in a coalition*) If $LVC_C \neq \varnothing$, let $C^* \in LVC_C$. C^* might be, or might not be, in the core.

Proof

Let $D(T_C) = \$100, \$300, 10\%$ if the total retail price of each transaction is more $\$1000, \$3000, \$5000$; respectively.

We look at two cases.

Case 1: C^* is not in the core.

Let $B = \{b_1, b_2, b_3, b_4, b_5\}$ be a set of buyers, C^* be the coalition of buyers $\{b_1, b_2, b_3, b_4, b_5\}$, $R_1 = 2750, R_2 = 4500, R_3 = 870, R_4 = 2700, R_5 = 1800$ be the reservation prices of buyers b_1, b_2, b_3, b_4, b_5 in C^*; respectively, $P_1 = 2900, P_2 = 5100, P_3 = 900, P_4 = 3100, P_5 = 2000$ be the total retail price of all items for each bid_k in C^*; respectively. $C^* \in LVC_C$ because $v(C^*) = \sum_{b_k\in C^*} R_k - (T_{C^*} - D(T_{C^*})) = 20 > 0$ and C^* have the maximum total retail price $T_{C^*} = 14000$ with the largest coalition utility. If we remove buyer b_4, *and* b_5 from the coalition C^*, then we find that subset $S^* = C^* \setminus \{b_4, b_5\}$ has $v(S^*) = 110 > v(C^*)$. Therefore, C^* is not in the core.

Case 2: C^* is in the core.

Let $B = \{b_1, b_2, b_3\}$ be a set of buyers, C^* be the coalition of buyers $\{b_1, b_2, b_3\}$, $R_1 = 2750, R_2 = 4500, R_3 = 870$ be the reservation prices of buyers b_1, b_2, b_3 in C^*; respectively, $P_1 = 2900, P_2 = 5100, P_3 = 900$ be the total retail price of all items for each bid_k in C^*; respectively. $C^* \in LVC_C$ because $v(C^*) = \sum_{b_k\in C^*} R_k - (T_{C^*} - D(T_{C^*})) = 110 > 0$ and C^* have the maximum total retail price $T_{C^*} = 8900$ with the largest coalition utility. Since $v(C^* \setminus \{b_1\}), v(C^* \setminus \{b_2\}), v(C^* \setminus \{b_3\})$ and $v(C^* \setminus \{b_1, b_2\}), v(C^* \setminus \{b_1, b_3\}), v(C^* \setminus \{b_2, b_3\}) < v(C^*)$; therefore, C^* is in the core.□

Proposition 3 shows that "a coalition structure based the *Community Compromised Combinatorial Coalition Scheme* might be, or might not be, in the core".

Proposition 4: (*Total retail price in a coalition*) Let

$$AC = \{C \subseteq B : v(C) \geq 0\},$$
$$VC_A = \{C \in AC : v(C) \geq v(C'')\ \forall C'' \in AC\},$$
$$LVC_A = \{C \in VC_A : \textstyle\sum_{b_k\in C} P_k \geq \sum_{b_k\in C''} P_k\ \ \forall C'' \in VC_A\},$$
$$VC_C = \{C \in AC : \textstyle\sum_{b_k\in C} P_k \geq \sum_{b_k\in C''} P_k\ \ \forall C'' \in AC\},$$
$$LVC_C = \{C \in VC_C : v(C) \geq v(C'')\forall C'' \in VC_C\},$$

$C^{**} \in LVC_A$ is a coalition found by using the *CCF* scheme, and $C^* \in LVC_C$ is a coalition found by using the *C4* scheme. We have $\sum_{b_k\in C^*} P_k \geq \sum_{b_k\in C^{**}} P_k$ or $|T_{C^*}| \geq |T_{C^{**}}|$.

Proof

Let $C^* \in LVC_C$ and $C^{**} \in LVC_A$. Therefore $C^* \in VC_C$ and $C^{**} \in VC_A$. Since VC_C contains only all the largest coalitions in AC, and elements of VC_A are all chosen from AC as well, we have $\sum_{b_k\in C^*} P_k \geq \sum_{b_k\in C^{**}} P_k$ or $|T_{C^*}| \geq |T_{C^{**}}|$.□

Proposition 4 shows that this scheme gives a higher total retail price of each transaction than that in the *CCF* scheme. Finally, the primary aim of this scheme is to generate a large total discount. This scheme guarantees that a total discount of a coalition in the *C4* scheme is larger than or equal to that in the *CCF* scheme.

Proposition 5: (*Total discount of a coalition*) Let $C^{**} \in LVC_A$ be a coalition found by using the *CCF* scheme, and $C^* \in LVC_C$ be a coalition found by using the *C4* scheme. We have $TD(C^*) \geq TD(C^{**})$ or $D(T_{C^*}) \geq D(T_{C^{**}})$.

Proof

Since $|T_{C^*}| \geq |T_{C^{**}}|$ by Proposition 4, and the fact that $D(\bullet)$ is a ascending function**,** therefore, $TD(C^*) \geq TD(C^{**})$ or $D(T_{C^*}) \geq D(T_{C^{**}})$.□

Proposition 5 shows that there is a larger total discount to divide for all buyers in the coalition.

V. Conclusion

This paper proposes concepts of a new buyer coalition with a combination of items scheme, called Community Compromised Combinatorial Coalition (*C4*) scheme, which is based on the assumption that each buyer only has incomplete information of other buyers. Although buyers of a coalition in the *C4* scheme do not get the maximum discounts over any other coalitions like the solution of the core, the buyers are willing to join the coalition because they get discounts at their required minimum saving or higher. The solution of the *C4* scheme consists of two approaches. First, a coalition structure that gives the maximum total retail price is formed. Second, the total discount division of the coalition is divided among buyers in the coalition depending on buyers' required minimum saving. The total discount division of the coalition is the Pareto optimal so that buyers in the coalition achieve the efficiency of the total discount division. The total discount of the coalition in the *C4* scheme is larger than or equal to that in the *CCF* scheme. The *C4* scheme guarantees that all buyers in the coalition can purchase any units of the items at their reservation prices or lower. The above approaches can be proved by mathematical analysis

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