Some complexiton type solutions of the (3+1)-dimensional Jimbo-Miwa equation

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Abstract—By means of the extended homoclinic test approach (shortly EHTA) with the aid of a symbolic computation system such as Maple, some complexiton type solutions for the (3+1)-dimensional Jimbo-Miwa equation are presented.

Keywords—Jimbo-Miwa equation, Painlevé analysis, Hirota's bilinear form, Computerized symbolic computation.

I. Introduction

THE Jimbo-Miwa equation is used to describe certain interesting (3+1)-dimensional waves in physics but not pass any of the conventional integrability tests. This equation is the second equation in the well known Painlevè hierarchy of integrable systems. The (3+1)-dimensional Jimbo-Miwa equation is

$$u_{xxxy} + 3\,u_y\,u_{xx} + 3\,u_x\,u_{xy} + 2\,u_{yt} - 3\,u_{xz} = 0, \qquad (1)$$

where $u : \mathbb{R}_x \times \mathbb{R}_y \times \mathbb{R}_z \times \mathbb{R}_t^+ \to \mathbb{R}$.
There are many efforts to solve equation (1). Tang and Liang [1] applied the multi-linear variable separation scheme to (1). [2] by applying the Painlevé test showed that (1) is not integrable and through the obtained truncated Painlevé expansions constructed two bilinear equations. Starting from these bilinear equations, one soliton, two soliton and dromin solutions are also obtained. [3] obtained a new class of cross kink-wave and periodic solitary-wave solution for (1) by using two-soliton methoid, bilinear method and transforming parameters into complex ones. [4] obtained some exact solutions of (1) by an extended rational expansion method and symbolic computation. [5] obtained two new types of exact periodic solitary-wave and kinky periodic-wave solutions to (1) by applying EHTA. [6] presented exact and explicit generalized solitary solutions for the equation by the Exp-function method. [7] derived multiple front solutions by employing Hirota's bilinear method for (1). [8] by using rational function transformations approached to exact solution of the equation. [9] obtained new exact solutions, including solitary wave solutions, periodic wave solutions and variable separations solutions of (1) by a kind of classic, efficient and well-developed method, the mapping approach. [10] presented the traveling wave solutions for the equation by the $(\frac{G'}{G})$-expansion method. [11] used the generalized three-wave method to obtain exact three-wave solutions including periodic cross-kink wave solutions, doubly periodic solitary wave solutions and breather type of two-solitary wave solutions for (1). [12] presented abundant new exact solutions for the Jimbo-Miwa equation (1) by using the generalized Riccati equation mapping method. In that work, authors presented twenty seven solutions for the equation. However, Kudryashov and Sinelshchikov [13] showed that eight from those twenty seven solutions are wrong and do not satisfy the equation. Also, the other nineteen exact solutions are not new and can be found from the well-known solution. One can find another schemes to solve (1) in Refs. [14], [15], [16]. In this paper we present some complexiton type solutions of the equation involve two kinds of transcendental functions. We use EHTA to obtain these solutions.

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II. Extended homoclinic test approach

The basic idea of this method applies the Painlevé analysis to make a transformation as

$$u = T(f) \qquad (2)$$

for some new and unknown function f. Then we use this transformation in a high dimensional nonlinear equation of the general form

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, \cdots) = 0, \qquad (3)$$

where $u = u(x, y, z, t)$ and F is a polynomial of u and its derivatives. By substituting (2) in (3), the first one converts into the Hirota's bilinear form, which it will solve by taking a special form for f and assuming that the obtained Hirota's bilinear form has solutions in EHTA, then we can specify the unknown function f, (for more details see [17]).

III. Application

In this section, we investigate explicit formula of solutions of equation (1). To solve (1), we use the EHTA [17]. By this idea we obtain some analytic solutions for the problem. By using Painlevé analysis we set

$$u = 2(\ln f)_x \qquad (4)$$

where $f(x, y, z, t)$ is an unknown real function which will be determined. Substituting Eq. (4) into Eq. (1), we obtain the following Hirota's bilinear form

$$(D_x^3 D_y + 2\,D_t D_y - 3\,D_x D_z) f \cdot f = 0, \qquad (5)$$

where the D-operator, is defined by

$$D_x^m D_y^k D_z^p D_t^n f(x,y,z,t) \cdot g(x,y,z,t) =$$

$$(\tfrac{\partial}{\partial x_1} - \tfrac{\partial}{\partial x_2})^m (\tfrac{\partial}{\partial y_1} - \tfrac{\partial}{\partial y_2})^k (\tfrac{\partial}{\partial z_1} - \tfrac{\partial}{\partial z_2})^p (\tfrac{\partial}{\partial t_1} - \tfrac{\partial}{\partial t_2})^n$$

$$[f(x_1, y_1, z_1, t_1) g(x_2, y_2, z_2, t_2)],$$

where the right hand side is computed in

$$x_1 = x_2 = x,\ y_1 = y_2 = y,\ z_1 = z_2 = z,\ t_1 = t_2 = t.$$

Now we suppose the solution of Eq. (5) as

$$f(x,y,z,t) = e^{-\xi_1} + \delta_1 \cos(\xi_2) + \delta_2 e^{\xi_1} \tag{6}$$

where

$$\xi_i = a_i x + b_i y + c_i z + d_i t, \quad i = 1,2 \tag{7}$$

and $a_i, b_i, c_i, d_i,\ \delta_i$ are some constants to be determined later. Substituting Eq. (6) into Eq. (5), and equating all coefficients of $\sin(a_2\,x + b_2\,y + c_2\,z + d_2\,t)$ and $\cos(a_2\,x + b_2\,y + c_2\,z + d_2\,t)$ to zero, we get the following set of algebraic equations for a_i,b_i,c_i,d_i,δ_i, $(i = 1, 2)$

$$\begin{aligned}
&{a_1}^3 b_1 + b_2 {a_2}^3 - 3\,b_2 a_2 {a_1}^2 + 2\,d_1 b_1 - 2\,b_2 d_2 - \\
&\quad 3\,a_1 c_1 + 3\,c_2 a_2 - 3\,a_1 b_1 {a_2}^2 = 0,\\[6pt]
&3\,a_2 {a_1}^2 b_1 - {a_2}^3 b_1 + {a_1}^3 b_2 + 2\,d_2 b_1 + 2\,d_1 b_2 - \\
&\quad 3\,a_1 b_2 {a_2}^2 - 3\,a_2 c_1 - 3\,a_1 c_2 = 0,\\[6pt]
&4\,{\delta_1}^2 b_2 {a_2}^3 - 2\,{\delta_1}^2 b_2 d_2 + 3\,{\delta_1}^2 c_2 a_2 + 16\,{a_1}^3 b_1 \delta_2 + \\
&\quad 8\,d_1 b_1 \delta_2 - 12\,a_1 c_1 \delta_2 = 0.
\end{aligned} \tag{8}$$

Solving the system of equations (8) with the aid of Maple, yields the following cases:

Case 1:

$$\begin{aligned}
&a_1 = 0, c_1 = \frac{{b_1}^2{\delta_1}^2 b_2 {a_2}^2 + {b_1}^2 c_2 {\delta_1}^2 - 4\,{b_1}^2 \delta_2 c_2 + {\delta_1}^2 {a_2}^2 {b_2}^3}{b_2({\delta_1}^2 - 4\,\delta_2) b_1},\\
&d_1 = \frac{3}{2}\frac{{\delta_1}^2 {a_2}^3 b_2}{({\delta_1}^2 - 4\,\delta_2) b_1},\\
&d_2 = \frac{1}{2}\frac{a_2\left(-4\,\delta_2 b_2 {a_2}^2 + 4\,{\delta_1}^2 b_2 {a_2}^2 + 3\,{\delta_1}^2 c_2 - 12\,\delta_2 c_2\right)}{b_2({\delta_1}^2 - 4\,\delta_2)}
\end{aligned} \tag{9}$$

for some arbitrary real constants a_2, b_1, b_2, c_2, δ_1 and δ_2. Substitute Eqs. (9) into Eq. (4) with Eq. (6), we obtain the solution as

$$f(x,y,z,t) = e^{-\xi_1} + \delta_1 \cos(\xi_2) + \delta_2 e^{\xi_1}$$

and

$$u(x,y,z,t) = \frac{-2\,\delta_1 \sin(\xi_2)\,a_2}{e^{-\xi_1} + \delta_1 \cos(\xi_2) + \delta_2 e^{\xi_1}} \tag{10}$$

for

$$\xi_1 = b_1 y + c_1 z + d_1 t \quad , \quad \xi_2 = a_2 x + b_2 y + c_2 z + d_2 t.$$

If $\delta_2 > 0$, then we obtain the exact solution as

$$u(x,y,z,t) = \frac{-2\,\delta_1 \sin(\xi_2)\,a_2}{2\sqrt{\delta_2}\cosh(\xi_1 - \theta) + \delta_1 \cos(\xi_2)}$$

for

$$\theta = \frac{1}{2}\ \ln(\delta_2).$$

If $\delta_2 < 0$, then we obtain the following exact solution

$$u(x,y,z,t) = \frac{-2\delta_1 \sin(\xi_2)\,a_2}{2\sqrt{-\delta_2}\sinh(\xi_1 - \theta) + \delta_1 \cos(\xi_2)}$$

for

$$\theta = \frac{1}{2}\ \ln(-\delta_2).$$

Case 2:

$$\begin{aligned}
&a_2 = 0, b_2 = 0, c_1 = \frac{1}{3}\frac{b_1\left({a_1}^3 + 2\,d_1\right)}{a_1},\\
&c_2 = \frac{2}{3}\frac{d_2 b_1}{a_1}, \delta_2 = 0
\end{aligned} \tag{11}$$

for some arbitrary real constants a_1, b_1, d_1, d_2 and δ_1. Substitute Eqs. (11) into Eq. (4) with Eq. (6), we obtain the solution as follows

$$f(x,y,z,t) = e^{-\xi_1} + \delta_1 \cos(\xi_2)$$

and

$$u(x,y,z,t) = \frac{-2\,a_1 e^{-\xi_1}}{e^{-\xi_1} + \delta_1 \cos(\xi_2)} \tag{12}$$

for

$$\xi_1 = a_1 x + b_1 y + c_1 z + d_1 t \quad , \quad \xi_2 = c_2 z + d_2 t.$$

Case 3:

$$\begin{aligned}
&c_1 = \frac{1}{3}\frac{b_2\left({a_1}^3 + 2\,d_1 - 3\,a_1 {a_2}^2\right)}{a_2}, c_2 = \frac{1}{3}\frac{b_2\left(2\,d_2 + 3\,a_2 {a_1}^2 - {a_2}^3\right)}{a_2}\\
&\delta_2 = -\frac{1}{4}\frac{{\delta_1}^2 {a_2}^2}{{a_1}^2}, \quad b_1 = \frac{a_1 b_2}{a_2}
\end{aligned} \tag{13}$$

for some arbitrary real constants a_1, a_2, b_2, d_1, d_2 and δ_1. Substitute Eqs. (13) into Eq. (4) with Eq. (6), we obtain the solution as follows

$$f(x,y,z,t) = e^{-\xi_1} + \delta_1 \cos(\xi_2) + \delta_2 e^{\xi_1}$$

and

$$u(x,y,z,t) = \frac{2(-a_1 e^{-\xi_1} - \delta_1 \sin(\xi_2)\,a_2 + \delta_2 a_1 e^{\xi_1})}{e^{-\xi_1} + \delta_1 \cos(\xi_2) + \delta_2 e^{\xi_1}} \tag{14}$$

or

$$u(x,y,z,t) = \frac{2(2\,a_1\sqrt{\delta_2}\sinh(\xi_1 - \theta) - \delta_1 \sin(\xi_2)\,a_2)}{2\sqrt{\delta_2}\cosh(\xi_1 - \theta) + \delta_1 \cos(\xi_2)}$$

for

$$\theta = \frac{1}{2}\ \ln(\delta_2) \quad , \quad \delta_2 = \frac{1}{4}\frac{{\delta_1}^2 {a_2}^2}{{a_1}^2} > 0$$

and

$$\xi_1 = a_1 x + b_1 y + c_1 z + d_1 t \quad , \quad \xi_2 = a_2 x + b_2 y + c_2 z + d_2 t$$

and

$$u(x,y,z,t) = \frac{2(2\,a_1\sqrt{-\delta_2}\cosh(\xi_1 - \theta) - \delta_1 \sin(\xi_2)\,a_2)}{2\sqrt{-\delta_2}\sinh(\xi_1 - \theta) + \delta_1 \cos(\xi_2)}$$

for

$$\theta = \frac{1}{2}\ \ln(-\delta_2) \quad , \quad \delta_2 = -\frac{1}{4}\frac{{\delta_1}^2 {a_2}^2}{{a_1}^2} < 0.$$

Figures 1-3 show the plots of solutions for equation (1) for some special cases of the parameters' solutions in any Case.

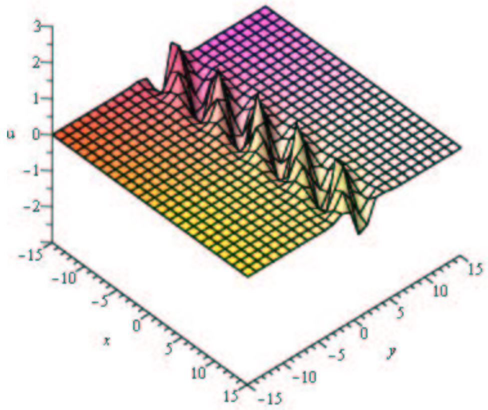

Fig. 1. Periodic solitary-wave solution for Case 1 for $a_2 = b_1 = b_2 = c_2 = \delta_1 = \delta_2 = 1, c_1 = \frac{1}{3}, d_1 = -0.5, d_2 = 1.5$ and $a_1 = t = 0$.

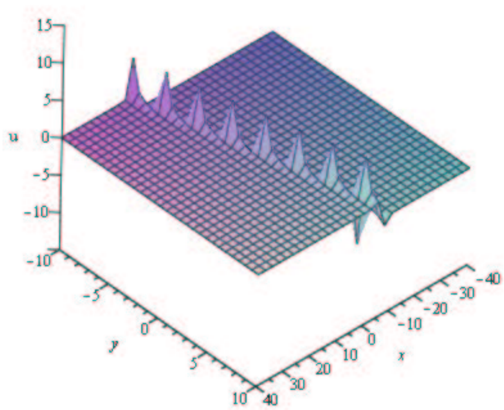

Fig. 2. Periodic solitary-wave solution for Case 2 for $a_1 = b_1 = c_1 = c_2 = d_1 = \delta_1 = 1, d_2 = 1.5$ and $t = 0$.

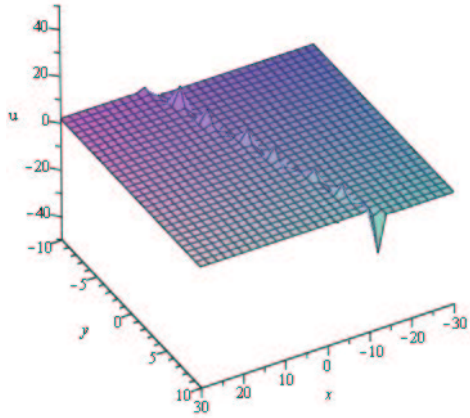

Fig. 3. Periodic solitary-wave solution for Case 3 for $a_1 = b_1 = d_1 = \delta_1 = 1, c_1 = -3, c_2 = -1, b_2 = 2, d_2 = -0.5, \delta_2 = -1$ and $t = 0$.

IV. Conclusions

In this paper, using the EHTA we obtained some explicit formulas of solutions for the (3+1)-dimensional Jimbo-Miwa equation. The presented solutions involve two kinds of transcendental functions, and so, they are complexiton type solutions but not traveling solutions. The result provide good supplements to the existing literature on related research. The solution procedure is very simple and straightforward and can be applied on another nonlinear equations. It must be noted that, all obtained solutions have checked in the Jimbo-Miwa equation. All solutions satisfy in the equation.

References

[1] X.Y. Tang, Z.F. Liang, *Variable separation solutions for the (3+1)-dimensional Jimbo-Miwa equation*, Phys. Lett. A, 351 (2006) 398–402.

[2] G.-Q. Xu, *The soliton solutions, dromions of the Kadomtsev-Petviashvili and Jimbo-Miwa equations in (3+1)-dimensions*, Chaos, Solitons and Fractals, 30 (2006) 71–76.

[3] Z.D. Dai, Z. Li, Z.J. Liu, D.L. Li, *Exact cross kink-wave solutions and resonance for the Jimbo-Miwa equation*, Physica A, 384 (2007) 285–290.

[4] D. Wang, W. Sun, C. Kong, H.Q. Zhang, *New extended rational expansion method and exact solutions of Boussinesq equation and Jimbo-Miwa equations*, Appl. Math. Comput., 189 (2007) 878–886.

[5] Z.D. Dai, J. Liu, X.P. Zeng, Z.J. Liu, *Periodic kink-wave and kinky periodic-wave solutions for the Jimbo-Miwa equation*, Phys. Lett. A, 372 (2008) 5984–5986.

[6] T. Ozis, I. Aslan, *Exact and explicit solutions to the (3+1)-dimensional Jimbo-Miwa equation via the Exp-function method*, Phys. Lett. A, 372 (2008) 7011–7015.

[7] A.M. Wazwaz, *Multiple-soliton solutions for the Calogero-Bogoyavlenskii-Schiff, Jimbo-Miwa and YTSF equations*, Appl. Math. Comput., 203 (2008) 592–597.

[8] W.-X. Ma, J.-H. Lee, *A transformed rational function method and exact solutions to the 3+1 dimensional Jimbo-Miwa equation*, Chaos, Solitons and Fractals, 42 (2009) 1356–1363.

[9] S.-H. Ma, J.-P. Fang, C.-L. Zheng, *New exact solutions for the (3+1)-dimensional Jimbo-Miwa system*, Chaos, Solitons and Fractals, 40 (2009) 1352–1355.

[10] M. Song, Y. Geb, *Application of the $(\frac{G'}{G})$-expansion method to (3+1)-dimensional nonlinear evolution equations*, Comput. Math. Appl., 60 (2010) 1220–1227.

[11] Z. Li, Z.D. Dai, J. Liu, *Exact three-wave solutions for the (3+1)-dimensional Jimbo-Miwa equation*, Comput. Math. Appl., (in press), doi:10.1016/j.camwa.2010.08.070.

[12] Z. Li, Z.D. Dai, *Abundant new exact solutions for the (3+1)-dimensional Jimbo-Miwa equation*, J. Math. Anal. Appl., 361 (2010) 587–590.

[13] N.A. Kudryashov, D. I. Sinelshchikov, *A note on "Abundant new exact solutions for the (3+1)-dimensional imbo-Miwa equation"*, J. Math. Anal. Appl., 371 (2010) 393–396.

[14] W. Hong, K.S. Oh, *New solitonic solutions to a (3+1)-dimensional Jimbo-Miwa equation*, Comput. Math. Appl., 39(5-6) (2000) 29–31.

[15] D.L. Wang, H.W. Tam, W.M. Xue, *Soliton solutions to the Jimbo-Miwa equations and the Fordy-Gibbons-Jimbo-Miwa equation*, Phys. Lett. A, 262(4-5) (1999) 310–320.

[16] B. Tian, Y.T. Gao, H. Woopyo, *The solitonic features of a non integrable (3+1)-dimensional Jimbo-Miwa equation*, Comput. Math. Appl., 44(3-4) (2002) 525–528.

[17] Z. Zhao, Z.D. Dai, S. Han, *The EHTA for nonlinear evolution equations*, Appl. Math. Comput., (in press), doi:10.1016/j.amc.2010.09.069.