# Real-time Multi-resolution Decomposition of Degrading Fault Signals using Entropy measure

E M Lalitha

***Abstract*—** Trend identification in gradually degrading faults poses a challenge due to embedded local large data variations in signals, which sometimes can interfere with the gradual trend. This paper describes combining Multi-resolution Signal Decomposition (MSD) property and denoising effect of wavelet transforms with entropy measure to eliminate local high frequency variations and retain the gradual pattern present in the data. Information content in wavelet coefficients at every level given by entropy value serves as a measure for roughness present in the signal, and is explored as a means to automatically optimize number of MSD levels. Such automation is critical in real-time applications, since further decompositions than required can lead to signal pattern distortion. Trending algorithm using basic statistical measures is then applied on the wavelet-approximated signal. The proposed methodology was tested for degrading faults in gas turbines. Encouraging results were obtained, demonstrating the utility of proposed novel approach for monitoring applications.

***Keywords*—-** entropy, multi-resolution decomposition, real-time applications, wavelets

## I. Introduction

Continuous fault monitoring is necessary to identify faults at an incipient stage to prevent failures. High frequency data variations within normal operating limits pose a challenge in identifying a gradual fault or trend present in the data! One such case is with the gas turbine engines. Figure 1 shows different signals that need to be monitored for the trend. It can be seen that though the data has large local variations, there is a gradual trend seen in the parameters that can lead to a fault. Fault-free region in this data is identified prior and any deviation of trend from the fault-free region trend is signaled as an alarm.

Various studies indicate trending using monotonic regression filters [1], syntactic pattern recognition approaches [2], and slope-based approaches [3]. A statistical trending algorithm that identifies mean-shift of a set of values when compared to baseline set is given in [4]. Wavelet transforms [5] have been used for denoising and localization of polynomial trends in noisy environments. Wavelet theory based Adaptive System for TRend Analysis (WASTRA) describes a non-linear adaptive algorithm for identification of trends from sensor data [6].

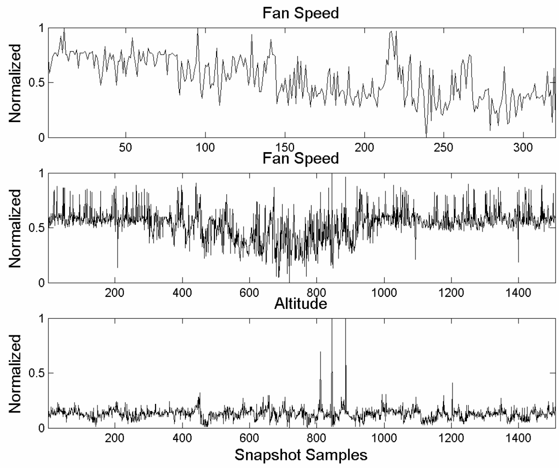


Figure 1: Parameters of a Gas turbine that required to be monitored

In most of the studies using wavelet transforms, it is seen that the extent of wavelet approximation depends on the application. Number of MSD levels plays an important role during signal approximation and experiments revealed that higher MSD levels than optimum can disturb the inherent signal patterns (Section 2.1). In real-time situations it is not possible to either use an optimal pre-selected MSD level or manually identify the best suitable one. Hence it is crucial to find this level automatically.

As MSD approximations smoothen the signal at every level, the author proposes a measure of roughness to indicate the optimum MSD levels. Denoising after certain levels of MSD, do not give any improvement, but rather, degrades the signal. Entropy is a good measure for information content present in the signal, which indirectly relates to roughness present in the signal, and hence this measure is explored to identify the optimum MSD levels required. Wavelet approximated signal thus obtained is then subjected to simple trending algorithm to identify the gradual signal trend.

Section 2 deals with the methodology adopted in this work, and Section 3 presents results. Conclusions are given in Section 4, with references in Section 5.

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## II. METHODOLOGY

Daubechies mother wavelet with 4 coefficients was used for approximation. A trade-off is sought between the quality of results and the complexity of convolution step execution during filtering operation of wavelet approximation.

The methodology adopted is given in the flowchart in Figure 2. This is an iterative procedure, wherein the signal is subjected to wavelet decomposition and moving average at every level of MSD. Moving average proved to be useful in further smoothening the signal at every level. This output is then tested with a defined criterion to check if optimum level of MSD is reached. If the criterion is met, decompositions are automatically stopped and a trend algorithm that is developed identifies the trend in the signal, else, further decompositions are carried out. If the trends present in fault-free region and the test region are different, an alarm is signaled.

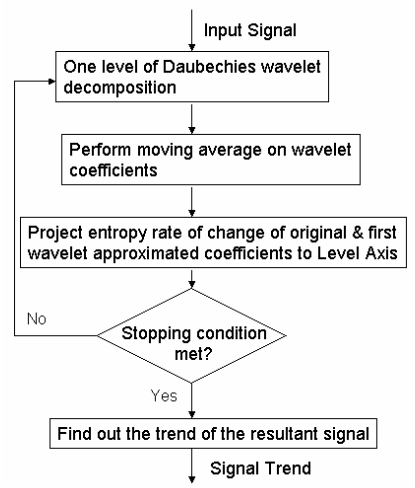


Figure 2: Method followed in this work.

### A. MSD and Wavelet Transform

Representation and analysis of digital data using wavelets has emerged as a very powerful tool in recent years. The choice of mother wavelet depends on the application and no general rule exists for the selection. The signal is decomposed at every level into approximate and detailed coefficients. This principle is shown in Figure 3.

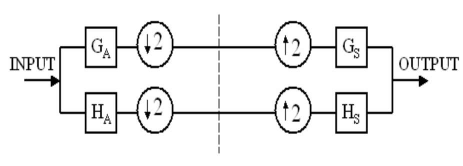


Figure 3: 1-level Analysis and Synthesis of a signal

The signal is passed through a set of low-pass and high-pass filters (Quadrature Mirror Filters) [7]. These filters are designed such that there is perfect reconstruction at the output end. $G_A$ and $H_A$ are analysis filters, which decompose the signal and $G_S$ and $H_S$ are synthesis filters that reconstruct the decomposed signal. Approximate coefficients at every level are further subjected to decompositions to higher levels. Reconstruction of the signal is loss-less when approximate and detailed coefficients at all levels are considered for reconstruction.

It was observed that the levels of decomposition play an important role that decides on the signal pattern. Figure 4 shows situations with adequate MSD levels (Figure 4(b)) and with one extra level of decomposition (Figure 4(c)). This experiment shows that the signal pattern is distorted if more MSD levels are performed than optimum.

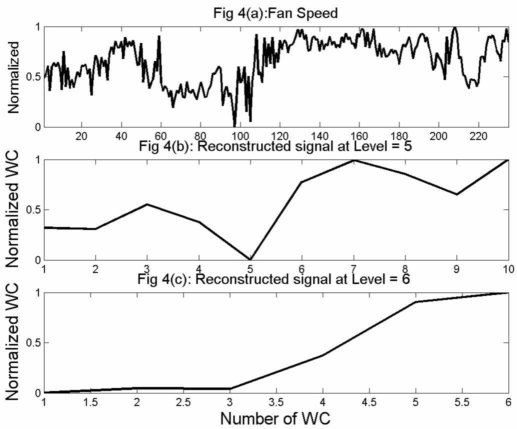


Figure 4: Wavelet approximation of Fan Speed at levels 5 and 6 respectively. WC – Wavelet Coefficients.

To address this issue, a stopping condition to arrive at an optimum MSD level using the first approximation of rate of change of entropy measure of wavelet coefficients is defined, which is discussed in Section 2.2.

### B. Entropy and Proposed Criterion

Claude Shannon introduced a revolutionary new probabilistic way of thinking about communication and simultaneously created the first truly mathematical theory of entropy, which employs probability and ergodic theory to study the statistical characteristics of data [8]-[9]. Entropy helps in describing irregularity, complexity or level of uncertainty of a signal [10].

Mathematically, entropy is expressed as:

$$E = -\sum_i p_i \log_2 p_i \quad .....(1)$$

where, $i$ = number of states in the data and $p_i$ is the probability at each state.

Entropy in this work is utilized as a quantitative irregularity measure. Wavelet approximation at each level of decomposition smoothens the signal, which can be quantified by the irregularity measure.

Entropy of wavelet-approximated signal at first MSD level is calculated using Eqn (1). The rate of change of entropy between original and first level of decomposition is projected on to the MSD axis, which gives the number of optimum levels. This projection gives nearly the minimum of the entropy curve when considered as a polynomial, beyond which further iterations will degrade the signal. This concept is similar to Newton's method for polynomial solutions, where a tangent at a point projected on to the axis, when mapped to the polynomial function, gives approximate minimum function value [11].

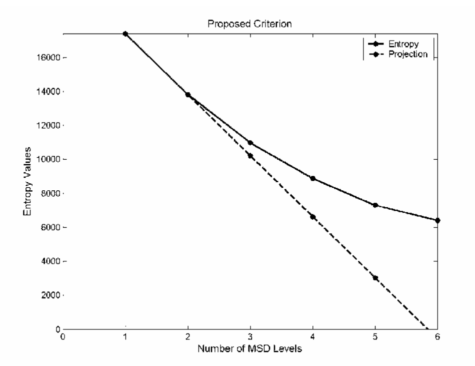


Figure 5(a): Proposed Criterion to automatically identify the optimum MSD levels.

Figure 5(a) gives the plot of entropy at different levels with the projection of rate of change of entropy after first level onto the number of MSD levels axis, which is the proposed stopping condition. In this case the number of decomposition levels that are identified is 5. Figure 5(b) shows wavelet-approximated signal after applying the proposed criterion. Entropy decreases after each level of decomposition, indicating the smoothness of the signal.

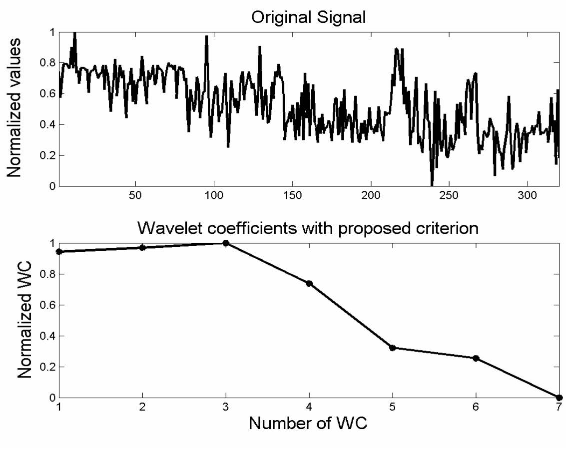


Figure 5(b): Wavelet Approximated signal using the proposed criteria. WC – wavelet coefficients.

## III. Results and Discussion

### A. Pattern Extraction

Figure 6 gives example-tested patterns along with the proposed criterion Value of MSD level identified to be the optimum is the projection of first rate of change of entropy of the wavelet coefficients. Experiments were performed on more than 40 data sets consisting of different patterns, and it was observed that consistently, the pattern extraction is accurate.

The advantage of this method is that one can go in for trending with the wavelet approximation coefficients or with

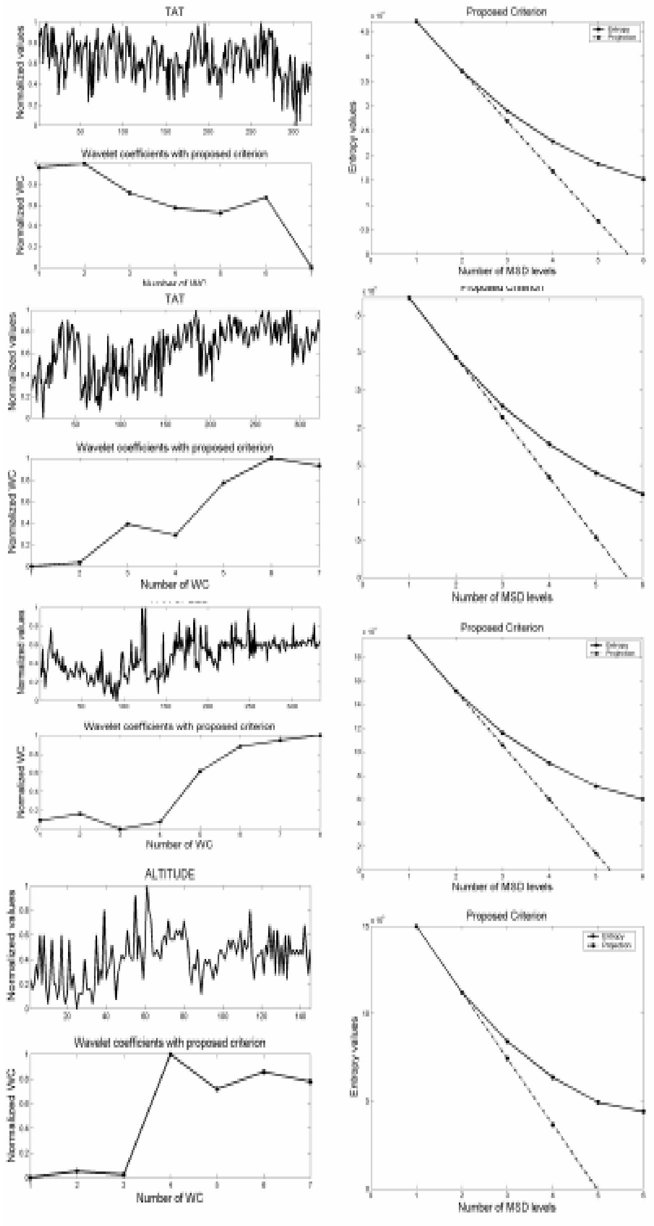


Figure 6: Patterns extracted using wavelet approximations at MSD levels identified by defined stopping criterion along

with their optimum levels indicated by the condition.

the reconstructed signal at the level prescribed by the stopping condition. It is beneficial to perform trending on the reduced wavelet coefficients in terms of speed and memory requirements, but depends on the application.

### *B. Trend Identification*

Due to the methodology of extracting underlying pattern present in the signal, it is seen that a simple trending algorithm is sufficient to identify the gradual trend. In this work basic slope based approach is utilized for this purpose. The trends that are seen in this context are increasing trend, decreasing trend and consistent trend. Work is in progress to fine-tune the present algorithm and also develop methods for trends that explode, diminish, frequency change etc. The flowchart of developed method is given in Figure 7. Individual trend is captured between two successive wavelet coefficients, which contribute to the overall trend in the signal.

## III. Conclusions

Advantages of using wavelet decomposition in identifying gradual degradation trend in the data is demonstrated highlighting the importance of automated selection of optimal MSD levels for real time applications. A quantitative measure using entropy is proposed that can help in finding the most suitable decomposition level, which can vary with different signals. Wavelet approximated signal using this stopping condition is tested for trend identification by a simple trend

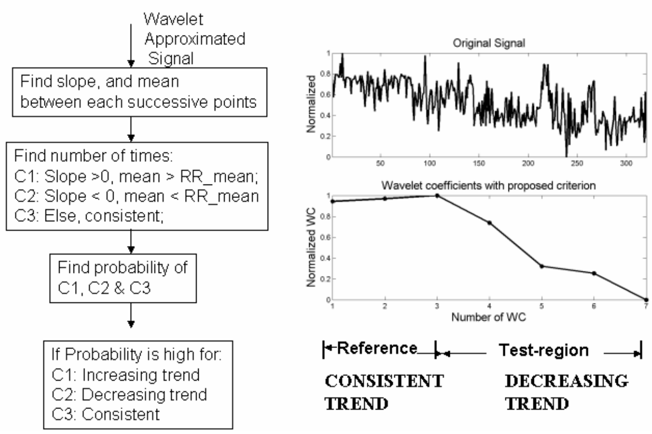


Figure 7: Trend algorithm and a sample result; RR_mean = mean calculated in reference region.

algorithm. Developed methodology tested on more than 40 data sets gave encouraging results. Though the utility of wavelet approximation and the proposed criterion is established, further development of trend algorithm to include more number of trends need to be performed to reveal the true capability of the proposed methodology.

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