Adaptive Anisotropic Diffusion for Ultrasonic Image Denoising and Edge Enhancement

Shujun Fu, Qiuqi Ruan, Wenqia Wang and Yu Li

Abstract**—**Utilizing echoic intension and distribution from different organs and local details of human body, ultrasonic image can catch important medical pathological changes, which unfortunately may be affected by ultrasonic speckle noise. A feature preserving ultrasonic image denoising and edge enhancement scheme is put forth, which includes two terms: anisotropic diffusion and edge enhancement, controlled by the optimum smoothing time. In this scheme, the anisotropic diffusion is governed by the local coordinate transformation and the first and the second order normal derivatives of the image, while the edge enhancement is done by the hyperbolic tangent function. Experiments on real ultrasonic images indicate effective preservation of edges, local details and ultrasonic echoic bright strips on denoising by our scheme.

Keywords**—**anisotropic diffusion, coordinate transformation, directional derivatives, edge enhancement, hyperbolic tangent function, image denoising.

I. INTRODUCTION

ULTROSONIC imaging extends its application to many fields of medical diagnosis, with its natures of low cost, portability, noninvasion and real time image formation, compared with other imaging techniques. Because ultrasonic image not only can observe shapes of human viscera, but also can examine their functions and blood stream states, it has become an important part of medical imaging. However, ultrasonic image may be contaminated by the speckle noise in its formation process, specially when the ultrasonic wavelength corresponds to the coarseness of the irradiated object surface, which can be interpreted by the stochastic scatter model. The presence of speckle noise will degrade image quality, and even conceal image details, which affects following image segmentation, feature extraction and recognition, quantitative analysis, and most importantly disease diagnosis. Thus, to compress speckle noise and to improve image quality are the main step of ultrasonic image pretreatment.

Denoising and edge detection on ultrasonic image lie on the understanding of the statistics of speckle noise. According to the scatterer number density and space distribution in the ultrasonic scan range, and the nature of one ultrasonic imaging system, we can categorize speckle noise into one of three classes, which can be modeled by the Rayleigh, the K or the Rician distribution [1] respectively. So different regions should be processed differently. At the same time, the ultrasonic imaging system often compress the ultrasonic echoic signal to adapt it to the display, because of its limited dynamic range, which has varied the probability density function of the signal and has transformed the multiplicative speckle noise into the additive one [2].

A number of methods have been proposed to address the problem of removing speckle noise including temporal averaging [3], median filtering [4], adaptive speckle reduction (ASR) [5,6] and wavelet shrinkage (WS) [7]. However, above methods could not succeed to balance between speckle suppression and feature preservation due to the complexity of speckle statistics. Therefore, a technique that relies on a more accurate model for removing speckle noise while preserving image feature well would be rather valuable for practical use.

In section II, we discuss the differential nature of a typical edge, and then we put forward an edge enhanced anisotropic diffusion (EEAD) scheme, where we design the local diffusion matrix using the first and the second order normal derivatives of the image, and we enhance image edges employing a hyperbolic tangent function. In section III, we implement the scheme using the explicit Euler format with the central differences scheme, and test it on real ultrasonic images. The validity of our scheme and future work are presented in section IV.

Manuscript received Novemble 4, 2004. This work was supported by the Key Laboratory Project of Information Science & Engineering of Railway of National Ministry of Railways, China (No.TDXX0510), the Technological Innovation Fund of Excellent Doctorial Candidate of Beijing Jiaotong University, China (No. 48007).
Shujun Fu and Qiuqi Ruan are with the Institute of Information Science, Beijing Jiaotong University, Beijing, 100044, China (corresponding author, E-mail: shujunfu@163. com).
Shujun Fu and Wenqia Wang are with the School of Mathematics and System Science, Shandong University, Jinan, 250100, China.
Yu Li is with Hospital of western section, Shandong University, Jinan, 250100, China.

II. ANISOTROPIC DIFFUSION AND EDGE ENHANCEMENT FOR ULTRASONIC IMAGE

In a medical ultrasonic image, edges and local details between heterogeneous organs are the most interesting part for clinicians. Therefore, to preserve and to enhance edges and local details on denoising are very important.

A. Edge enhanced anisotropic diffusion

The use of partial differential equations（PDEs）in image processing has grown significantly over the past years [8]. Its basic idea is to deform an image, a curve or a surface in a partial differential equation framework, and to approach the expected result as a solution to this equation.

P. Perona and J. Malik [9] put forward an anisotropic diffusion (AD) equation to smooth a noisy image:

$$\frac{\partial u(x,y,t)}{\partial t} = div(g(|\nabla u(x,y,t)|)\nabla u(x,y,t)), \quad (1)$$

where $u(x,y,t): \Omega \times [0,+\infty) \to R$ is a scale image, $g(|\nabla u|)$ is a decreasing function of the gradient. Their work made an important influence on this field.

In order to understand the diffusion action of above equation clearly, we analyze the diffusion filtering on edges. At point **o**, $n = \nabla u / |\nabla u|$, $t = \nabla u^{\perp} / |\nabla u|$ are the unit tangent and normal vectors, $t \perp n$ (see fig.1) .

Aiming at different noise models of various domains in an ultrasonic image, we hope that an isotropic diffusion is practiced in homogeneous domains, while an anisotropic diffusion is done in domains of edges and local details, which diffuses along the tangent direction of edges, and does not diffuse across edges. Therefore, we can simply design a diffusion matrix using local coordinate transformation**.**

In fig.1, the coordinates relation between (n,t) and (x,y) is

$$\begin{pmatrix} n \\ t \end{pmatrix} = \frac{1}{|\nabla u|}\begin{pmatrix} u_x & u_y \\ -u_y & u_x \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

Further, we have

$$\begin{pmatrix} \frac{\partial}{\partial n} \\ \frac{\partial}{\partial t} \end{pmatrix} = \frac{1}{|\nabla u|}\begin{pmatrix} u_x & u_y \\ -u_y & u_x \end{pmatrix}\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}.$$

We adopt $f_1(x,y,t)$ and $f_2(x,y,t)$ as the diffusion coefficients along n and t. Then, the diffusion equation becomes:

$$\frac{\partial u}{\partial t} = div(D \bullet \nabla u), \quad (2)$$

where

$$D = \frac{1}{|\nabla u|^2}\begin{pmatrix} u_x & u_y \\ -u_y & u_x \end{pmatrix}^{\mathrm{T}}\begin{pmatrix} f_1 & 0 \\ 0 & f_2 \end{pmatrix}\begin{pmatrix} u_x & u_y \\ -u_y & u_x \end{pmatrix} = \frac{1}{|\nabla u|^2}\begin{pmatrix} f_1u_x^2 + f_2u_y^2 & (f_1 - f_2)u_xu_y \\ (f_1 - f_2)u_xu_y & f_1u_y^2 + f_2u_x^2 \end{pmatrix}. \quad (3)$$

Therefore, we put forward the following edge enhanced anisotropic diffusion (EEAD) model:

$$\begin{cases} \frac{\partial u}{\partial t} = \alpha(x,y,t)(div(D \bullet \nabla u)) - \beta(x,y,t)f_3(x,y,t)th(lv_{nn})|u_n| \\ v = \mathrm{G}_t * u \end{cases} \quad (4)$$

initial condition: $u(x,y,0) = u_0(x,y)$,

boundary condition: $u_n = 0$,

where $\alpha(x,y,t)$ and $\beta(x,y,t)$ are the control coefficients of the anisotropic diffusion and the edge enhancement respectively; D is the diffusion matrix of v (smoothed version of u); G_t is a Gaussian smoothing function; $f_3(x,y,t)$ is the edge enhancement coefficient; $th(x) = (e^x - e^{-x})/(e^x + e^{-x})$ is a hyperbolic tangent function, with l constant to control its gradient.

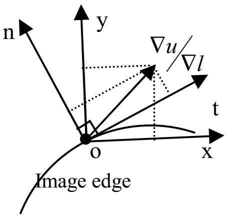
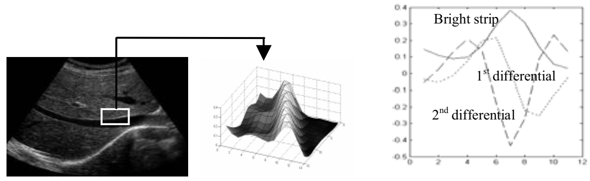


Fig.1 Decomposition of the directional derivative $\nabla u / \nabla l$ and the coordinate transformation on image edge.



(a) (b)

Fig.2 The ultrasonic echoic bright strip (a) and its differentials (b).

B. Analysis of our model and adoption of its parameters

1) Anisotropic diffusion

In the isotropic area (corresponds to a small gradient variety) we need an isotropic diffusion along all directions, so we adopt $f_1 \approx f_2$; in the area of edges and local details (corresponds to a big gradient variety), we need an anisotropic diffusion along the tangent directions of edges in order to preserve edges, so we adopt $f_1 < f_2$, $f_1, f_2 \to 0$ $(|v_n| \to +\infty)$.

From the principle of medical ultrasonic imaging system, we know that ultrasonic images catch useful medical information utilizing different intensities and distributions of echoic signals from various organs and local details. There may appear ultrasonic echoic bright strips with different intensities on edges and local details, which are important medical diagnostic message, and we should preserve them by all means in image processing (see fig.2).

When examining the differential nature of the echoic bright strip, we find that at its center the first order normal derivative of its profile approximates to 0, while its second order normal derivative reaches a minimum (see fig.2). In order to stop an excess polish at the echoic bright strip during smoothing, we can add a second order normal derivative term to the diffusion coefficients.

Based on above consideration, we adopt the following diffusion coefficients:

$$f_1 = 1/(1 + \mathrm{c}_1|v_n|^2 + \mathrm{c}_2|v_{nn}|^2), \quad f_2 = 1\Big/\sqrt{1 + \mathrm{c}_1|v_n|^2 + \mathrm{c}_2|v_{nn}|^2}, \quad (5)$$

where c_1 guarantees to preserve edges and local details, while c_2 to preserve echoic bright strips.

2) Edge enhancement

Then, we analyze a typical slope edge (see fig.3). **a** is the profile of one-dimension slope edge, whose center is **o**, and **b**, **c** are its first and second differential curves. It is evident that **b** increases from 0 gradually, reaches its maximum at **o**, and then decreases to 0; while **c** changes its symbol at **o**, from positive to negative. So we can control the variety of gray levels of image beside the edge center softly using a hyperbolic tangent function, to enhance the edge by minishing its breadth (see fig.4). Here we adopt the edge enhancement coefficient as:

$$f_3 = 1 - 1/(1 + c_3 |v_n|^2), \tag{6}$$

where c_3 controls selectively the area to be enhanced.

3) The control coefficients

With evolving the anisotropic diffusion equation, speckle noise becomes less and less. Therefore $\alpha(x, y, t)$ should decrease slowly, while $\beta(x, y, t)$ should increase from 0 slowly in order not to magnify noise.

According to the scale space theory and the literature [10], we can get the optimum smoothing time by estimating the variance of the speckle noise (for example, using wavelet decomposition coefficients):

$$T_0 = \sigma^2 / \mathrm{a}, \quad \mathrm{G}_t = e^{-\frac{x^2+y^2}{2\mathrm{a}t}} \Big/ 2\mathrm{a}\pi t, \tag{7}$$

and then, we adopt the following control coefficients:

$$\alpha = \begin{cases} 1 + l_1(1 - e^{l_2 t^2}), t \le T_0 \\ 0, \quad t > T_0 \end{cases}, \quad \beta = \begin{cases} 0, \quad t \le T_0 \\ 1 + l_1(1 - e^{l_2 t^2}), t > T_0 \end{cases}, \tag{8}$$

where l_1, l_2 are constants (see fig.5 for their profiles).

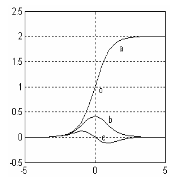


Fig.3 A typical edge and its differentials.

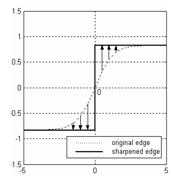


Fig.4 Edge enhancement.

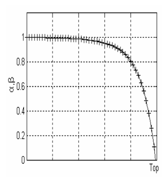


Fig.5 Control coefficients.

III. IMPLEMENTATION AND EXAMPLES

We used the explicit Euler format with the central difference scheme for model (4).

Our arithmetic has been tested by different ultrasonic images using MATLAB. At the same time we have compared it with others: AWMF (Adaptive Weighted Median Filter) [4], WSTS (Wavelet Soft Thresholding Shrinkage) [7], AD (Anisotropic Diffusion) [9]. Below we discuss an example by denoising an ultrasonic liver image (376×507).

Adoption of parameters of four different methods: AWMF, a =0.05, 5×5 stencil; WSTS, Symlets wavelet, level=2; AD, k =0.08, Δt =0.1; EEAD, (c_1, c_2, c_3, l) = (0.15, 1.4, 0.015, 0.015), Δt =0.07, T_0=1.4. All parameters have been optimized to approach best results.

From Fig.6 and Fig.7, it is clear that our method produces more promising results than others, both in denoising speckle noise (part **1**, **2**, **4**) and in preserving edges, local details and ultrasonic echoic bright strips (part **3**, **5**) (where broken lines are of original image, real lines are of the results obtained by four different methods respectively).

IV. CONCLUSIONS AND FUTURE WORK

A new nonlinear edge enhanced anisotropic diffusion model was proposed to reduce ultrasonic speckle noise while preserveing the edges, local details and ultrasonic echoic bright strips. The new technique has the advantages of denoising and preserving important features and organ surfaces well, which has a large potential in ultrasonic imaging enhancement and in assisting automated segmentation/ calculation techniques.

For the future, we shall apply the model to special local pathological changes, where we can adopt model parameters better, and can hope preferable results.

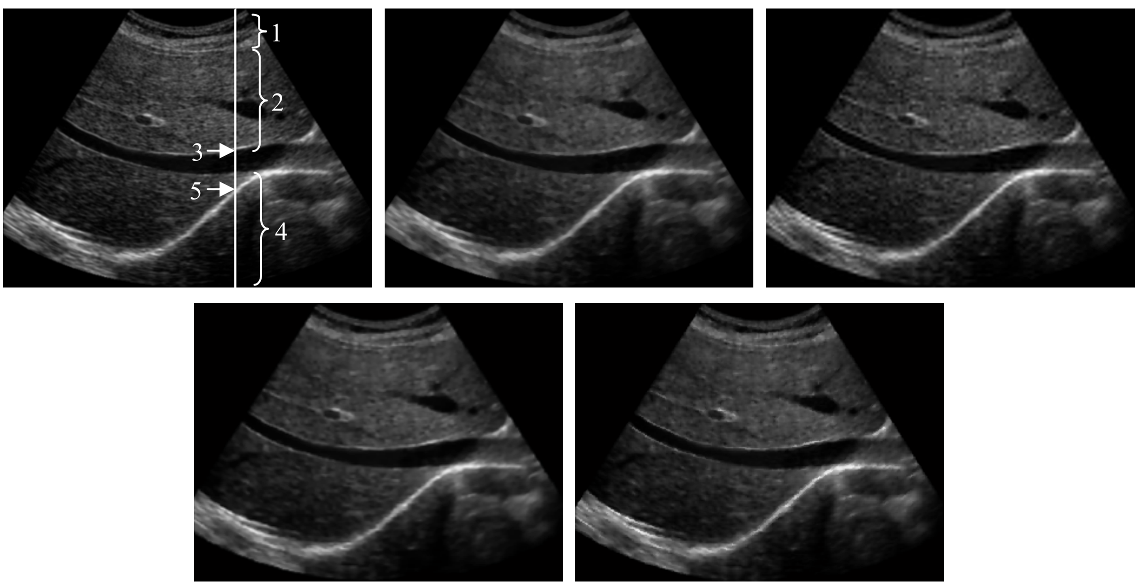


Fig.6 Denoising the liver image: original, AWMF, WSTS, AD and EEAD (from top left to bottom right).

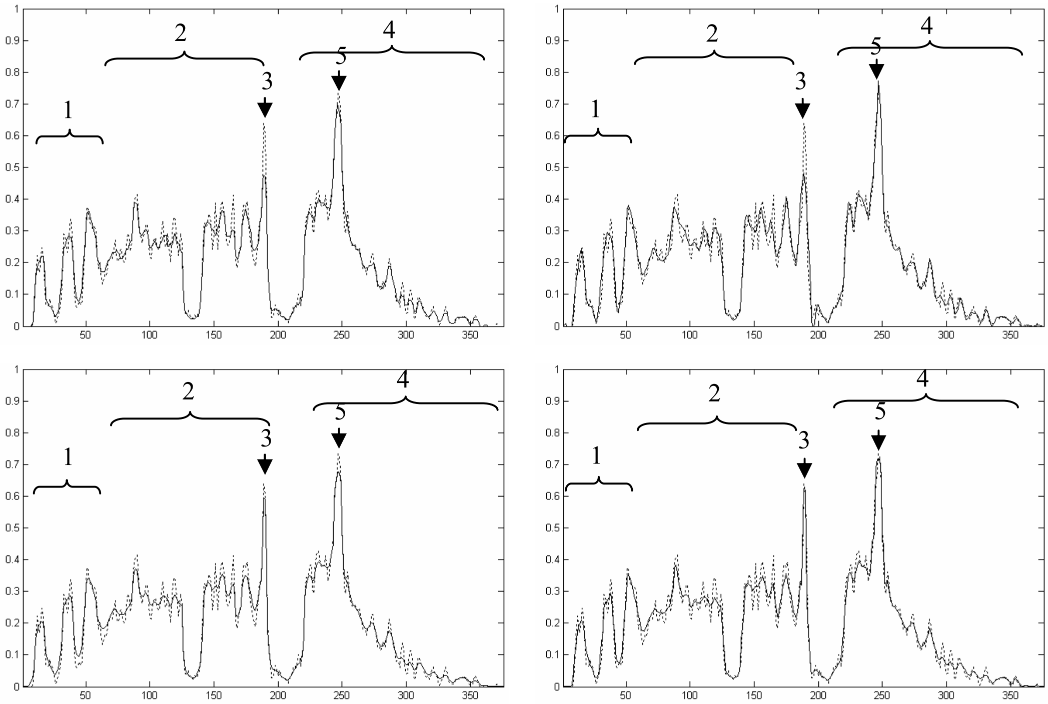


Fig.7 The 320th column profiles of above results: AWMF, WSTS, AD and EEAD (from top left to bottom right).

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Shujun Fu received the B.S. degree in power engineering and the M.S. degree in computational mathematics from Shandong University, Jinan, China, in 1990 and 1999 respectively. Since 1999 he is a lecturer of the School of Mathematics and System Science, Shandong University. He is currently pursuing the Ph.D. degree in signal and information processing at the Beijing Jiaotong University, Beijing, China. His research interests include image processing, partial differential equations, and wavelet analysis.

Qiuqi Ruan received the B.S. and M.S. degrees from Northern Jiaotong University in 1969 and 1981 respectively. From January 1987 to May 1990, he was a visiting scholar in the University of Pittsburgh, and the University of Cincinnati. Subsequently, he has been a visiting professor in USA for several times. Now he is a professor, doctorate supervisor and the head of the institute of information science in the school of computer and information technology, Beijing Jiaotong University, China. He is a senior member of IEEE. His main research interests include digital signal processing, computer vision, pattern recognition, and virtual reality etc.

Wenqia Wang is a professor and doctorate supervisor in the School of Mathematics and System Science, Shandong University, Jinan, China. His research interests include partial differential equations, numerical analysis.

Yu Li is an assistant director physician in the hospital of western section, Shandong University, Jinan, China. His research interests include ultrasonic diagnosis of human abdomen and ultrasonic imaging.

Adaptive Bidirectional Flow for Image Interpolation and Enhancement

Shujun Fu, Qiuqi Ruan and Wenqia Wang

***Abstract*—**Image interpolation is a common problem in imaging applications. However, most interpolation algorithms in existence suffer visually the effects of blurred edges and jagged artifacts in the image to some extent. This paper presents an adaptive feature preserving bidirectional flow process, where an inverse diffusion is performed to sharpen edges along the normal directions to the isophote lines (edges), while a normal diffusion is done to remove artifacts ("jaggies") along the tangent directions. In order to preserve image features such as edges, corners and textures, the nonlinear diffusion coefficients are locally adjusted according to the directional derivatives of the image. Experimental results on synthetic images and nature images demonstrate that our interpolation algorithm substantially improves the subjective quality of the interpolated images over conventional interpolations.

***Keywords*—**anisotropic diffusion, bidirectional flow, directional derivatives, edge enhancement, image interpolation, inverse flow, shock filter.

I. INTRODUCTION

IMAGE interpolation (magnification) is an image processing to gain its high-resolution image from a low-resolution version. Conventional interpolations treat the problem primarily as either fitting a space-invariant function (e.g., bilinear and bicubic) or extrapolating in frequency domain [1]. The former employs similarly a low-pass filtering process and blurs the edges of the interpolated image; the latter introduces false high-frequency components and produces annoying artifacts ("jaggies" and "mosaics"). Adaptive interpolation techniques [2, 3] spatially adapt the interpolation coefficients to better match the local structures around the edges. However, it may bring errors in selecting and estimating the edges of interest. Edge-directed interpolations [4, 5, 6, 7] fit the sub-pixel edges of the image utilizing the limited quantification of the directions and the widths of the edges, while preventing interpolations from across the edges. Although they can produce sharp edges, they fit the edges too simply, and they may also lose some features of the image. Among others are projection onto convex sets (POCS) scheme, approaches based on wavelet analysis and morphological filtering, etc [8, 9, 10].

In section II, we presents an adaptive feature preserving bidirectional flow (BDF) process, that is, an anisotropic diffusion equation, where an inverse diffusion is performed to enhance edges along the normal directions to the isophote lines (edges), while a normal diffusion is done to remove artifacts ("jaggies") along the tangent directions. In section III, having investigated the character of the typical edge, we discuss the backward flow and the forward flow along different directions, and, to preserve image features, detail this section by properly designing the diffusion coefficients using the first and second order directional derivatives of the image. In section IV, we implement the scheme and test it on synthetic images and nature images. Conclusions are presented in section V.

Manuscript received Novemble 4, 2004. This work was supported by the Key Laboratory Project of Information Science & Engineering of Railway of National Ministry of Railways, China (No.TDXX0510), the Technological Innovation Fund of Excellent Doctorial Candidate of Beijing Jiaotong University, China (No. 48007).

Shujun Fu and Qiuqi Ruan are with the Institute of Information Science, Beijing Jiaotong University, Beijing, 100044, China (corresponding author, E-mail: shujunfu@163. com).

Shujun Fu and Wenqia Wang are with the School of Mathematics and System Science, Shandong University, Jinan, 250100, China.

II. A UNIFIED BIDIRECTIONAL FLOW

The use of partial differential equations（PDEs）in image processing has grown significantly over the past years [11]. Initially proposed by P. Perona and J. Malik [12], the nonlinear anisotropic diffusion filters have been widely used in image denoising, enhancement, and sharpening. The grey levels of an image $u(x,y,t):\Omega\times[0,+\infty)\rightarrow R$, are diffused according to:

$$\frac{\partial u(x,y,t)}{\partial t}=div(g(|\nabla u(x,y,t)|)\nabla u(x,y,t)) \quad (1)$$

The scalar diffusivity $g(|\nabla u|)$, chosen as a non-increasing function, governs the behaviour of the diffusion process. A typical choice for the diffusivity function is:

$$g(|\nabla u|)=1/(1+(|\nabla u|/K)^2), \quad (2)$$

with K some gradient threshold. Practical implementations of the P-M filter are giving impressive results, noise is eliminated and edges are kept or even enhanced provided that their gradient value is greater than K.

By formally developing the divergence term, (1) can be put in iterms of second order derivatives taken in the directions of the gradient vectors ($\vec{n}$) and in the orthogonal tangent ones ($\vec{t}$):

$$\frac{\partial u}{\partial t}=g(|\nabla u|)u_{tt}+(g'(|\nabla u|)|\nabla u|+g(|\nabla u|))u_{nn}, \quad (3)$$

$$u_{tt} = \frac{1}{|\nabla u|^2}(u_x^2 u_{yy} + u_y^2 u_{xx} - 2u_x u_y u_{xy}) = k|\nabla u|, \quad (4)$$

$$u_{nn} = \frac{1}{|\nabla u|^2}(u_x^2 u_{xx} + u_y^2 u_{yy} + 2u_x u_y u_{xy}). \quad (5)$$

with k the isophote curvature [11]. This expression allows an easier interpretation of the original equation: (1) acts like a low pass filter diffusing along the edge directions and, selectively, for diffusion functions as (2), can preserve edges without diffusing across edges. Results obtained with the P-M process paved the way for a variety of PDE-based methods that were applied to various problems in low-level vision.

G. Gilboa et al. [13] present a forward-and-backward (FAB) adaptive diffusion process, and apply it in signal and image enhancement and sharpening. They switch the diffusion process from a forward to a backward (inverse) mode according to local gradients of the image, which can enhance features while locally denoising smoother segments of the signal or image. Different from (2), they choose the following diffusion coefficient:

$$g(s) = 1/(1+(s/k_f)^n) - \alpha/(1+((s-k_b)/w)^{2m}), \quad (6)$$

with parameters (k_f, k_b, w) and (n, m), which are chosen such that a forward diffusion ($g(s) > 0$) is used to smooth low gradients, while a backward one ($g(s) < 0$) to enhance high gradients.

Another PDE-based enhancement process is proposed by L. Alvarez and L. Mazorra [14], which couples an anisotropic diffusion with the shock filter (we call it ADSF) of S. J. Osher and L. I. Rudin [15], yielding an equation of the form:

$$\frac{\partial u}{\partial t} = -sign(G_\sigma * u_{nn})|\nabla u| + cu_{tt}, \quad (7)$$

with c a positive constant, G_σ a Gaussian of standard deviation σ. The first term on the right side creates solutions approaching piecewise constant regions separated by shocks at the zero-crossings of the smoothed second derivative of the image along $\vec{n}$. The second term is an anisotropic diffusion along the level-set lines $\vec{t}$.

In fact, both (6) and (7) are based on the same idea, which can be shown more clearly below. Noticing the equation:

$$sign(s) = s/|s|, \quad s \neq 0, \quad (8)$$

we can now define a unified bidirectional flow (BDF) equation covering both (6) and (7):

$$\frac{\partial u}{\partial t} = \alpha(-c_n(u_n, u_{nn}, u_{tt})u_{nn}) + \beta(c_t(u_n, u_{nn}, u_{tt})u_{tt}), \quad (9)$$

where α, β are the backward and forward flow control coefficients, $c_n(s)$, $c_t(s)$ are diffusion coefficients of their arguments, which should be properly designed to preserve features of the image such as edges, corners and texture. We will discuss this issue immediately.

III. FEATURE PRESERVING BIDIRECTIONAL FLOW FOR IMAGE INTERPOLATION AND ENHANCEMENT

Image interpolation means "reading between the original pixels", which also can be considered as a diffusion process: "to diffuse gray levels from pixels of the original image to the blank interpolated pixels between them". Therefore, this paper further extends the nonlinear PDE-based flow methods, and applies them to image interpolation and enhancement.

We divide our BDF process into two steps. First, the image is interpolated to the new desired size. In our implementation, we use bilinear interpolation. The first step provides good results over smooth areas, but edges are smeared, and artifacts ("jaggies") are also introduced. Then, we perform the BDF process to enhance the edges and smooth the interpolation byproducts. Most importantly, we detail this section by designing the diffusion coefficients c_n and c_t properly to preserve image features.

A. Backward flow

1) 1D backward flow. For interpreting the backward flow clearly, we begin with one dimensional (1D) signal case, then extend to 2D image. Because conventional interpolations (e.g., bilinear and bicubic) result in inevitably blurred edges of the image, we first analyze the differential properties of 1D typical slope edge. In Fig.1, **a** is a slope edge, whose center is **o**, and **b**, **c** are its first and second order differential curves. It is evident that **b** increases from 0 gradually, reaches its maximum at **o**, then decreases to 0; while **c** changes its symbol at **o**, from positive to negative. Here we want to control the variety of gray levels beside the edge center **o**. More precisely, we want to minish gray levels of pixels on the left of **o**, while to add that on the right of **o**, by which we can enhance the edge reducing its width (see Fig.2).

Thus, being contrary to the classic non-linear anisotropic diffusion, here we perform a backward flow (inverse diffusion):

$$\frac{\partial u}{\partial t} = -c_x sign(u_{xx}), \quad (10)$$

with c_x the positive diffusion coefficient.

We can further understand (10) clearly by discretizing u_{xx} at a point (i) in the central difference scheme:

$$(u_x)_i = (u_{i+1} - u_{i-1})/2h, \quad (11)$$

$$(u_{xx})_i = (u_{i+1} + u_{i-1} - 2u_i)/h^2 = 3(\bar{u}_i - u_i)/h^2, \quad \bar{u}_i = (u_{i-1} + u_i + u_{i+1})/3, \quad (12)$$

with h the spacial step. When $u_i > \bar{u}_i$ (the local mean of u_i), u_i will increase by evolving (10); contrariwise, u_i will decrease. For **o**, it does not vary because $(u_{xx})_o \approx 0$. This makes edges and corners (singularities) emphasized ("stretch- ed") while preserving the locations of edges. At the same time, we can see that the forward flow means "flowing to the local mean", while the backward flow means "flowing from the local mean".

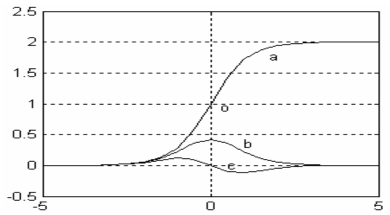


Fig.1 The differentials of 1D typical slope edge **a**, with center **o**, and the first and second order differential curves **b**, **c**.

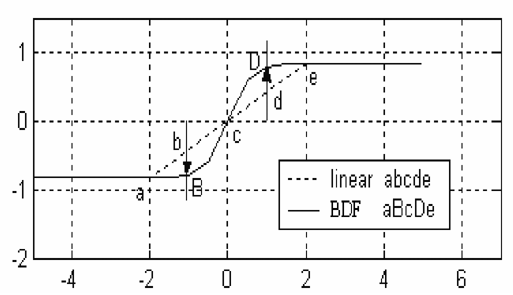


Fig.2 Edge enhanced BDF process (the solid line **aBcDe**), compared with linear interpolation (the broken line **abcde**).

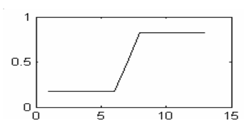
2) Comparison with FAB. Here we will clarify the differences between our scheme and the forward-and-backward (FAB) diffusion process [13]. In [13], FAB performs selectively forward or backward diffusion according to local gradients of the image. However, the bidirectional flow (BDF) performs differently forward or backward diffusion according to tangent or normal directions to the local isophote lines of the image.

In 1D signal case, when we adopt FAB to a step edge, we find that overshoot or ringing artifacts appear on the edge (see Fig.3). Thus, to decide diffusion speed c_x by the gradient information does not work. In section 1), we see that the backward flow means "flowing from the local mean", which manifests itself as increasing of u_{xx} at overshoot pixels more and more largely with iteration times. For suppressing this plague, we add the second order derivative information to c_x:

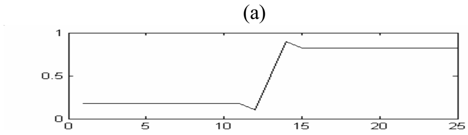
$$c_x = |u_x| / (1 + l_1 u_{xx}^2), \tag{13}$$

with l_1 a constant.

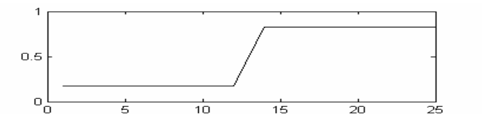
In fig.3, a step edge (a) is interpolated by FAB (b) and BDF (c) respectively. Having suppressed overshoots successfully by our scheme is shown.



(a)



(b)



(c)

Fig.3 BDF processing of a 1D step edge: (a) original 1D step edge; (b) result by FAB; (c) result by BDF, $l_1 = 0.008$. Both are obtained after 25 time steps.

3) Comparison with ADSF. Now we proceed our discussion with 2D image interpolation and enhancement. In [14], indicating edges by the zero-crossing is a binary decision process, by which, unfortunately, the obtained result is a false piecewise constant image whose texture and fine details are lost (see Fig.5). For this reason, we substitute $sign(s)$ by a hyperbolic tangent function $th(s)$, controlling softly the variety of gray levels of the image beside the edge center, and propose a backward flow of the form:

$$\frac{\partial u}{\partial t} = -c_n th(l u_{nn}), \tag{14}$$

with diffusion coefficient:

$$c_n = |u_n| / (1 + l_1 u_{nn}^2), \tag{15}$$

where $th(x) = (e^x - e^{-x}) / (e^x + e^{-x})$ is a hyperbolic tangent function, l is a constant to control its gradient.

B. Forward flow

1) Forward flow. Conventional interpolations also result in artifacts ("jaggies") in the image [16]. Now we perform a forward flow (normal diffusion) along the tangent directions to the isophote lines (edges):

$$\frac{\partial u}{\partial t} = c_t u_{tt}, \tag{16}$$

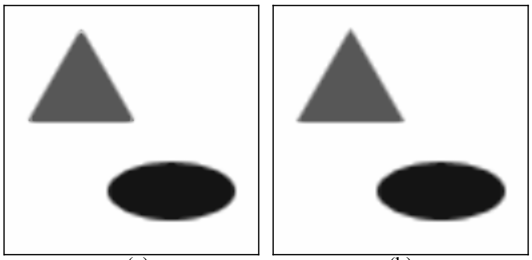
with c_t the diffusion coefficient. Because the forward flow means "flowing to the local mean", (16) can produce smooth level curves of the interpolated image [see Fig.4].

2) Comparison with level-set reconstruction. An image magnification method using level-set reconstruction is presented in [16], where instead of assuming a smoothness prior for the underlying intensity function, it assumes smooth- ness of the level curves, and produces appealing visually images. However, with $c_t = 1$, it may smooth away corners and small details at the same time. From (12), we can see that u_{tt} at the angle is much bigger than one at the edge line in value along the tangent direction. Therefore, for preventing over smoothness to corners, we add the second order derivative information to c_t:

$$c_t = 1 / (1 + l_2 u_{tt}^2), \tag{17}$$

with l_2 a constant.

In Fig.4, it shows that the weighted forward flow (17) not only has better smoothed image contours, but also has preserved three corners of the triangle in a synthetic image (b), and yet which has been over smoothed by the level-set reconstruction method (a).



(a) (b)

Fig.4 Interpolating a synthetic image: (a) the level-set reconstruction, (b) the weighted forward flow (17).

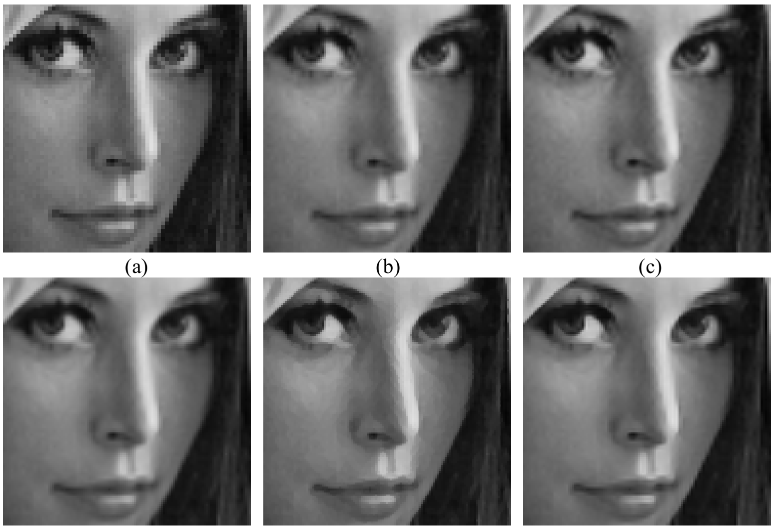
C. *Bidirectional flow*

Based on preceding discussion, we write the adaptive feature preserving bidirectional flow as:

$$\frac{\partial u}{\partial t} = \alpha(-c_n th(lu_{nn})) + \beta(c_t u_{tt}), \tag{18}$$

with Neumann boundary condition, backward and forward flow control coefficients α, β, where we adopt the following diffusion coefficients:

$$c_n = |u_n| / (1 + l_1 u_{nn}^2), \quad c_t = 1/(1 + l_2 u_{tt}^2). \tag{19}$$



(a) (b) (c)

(d) (e) (f)

Fig.5 BDF processing of Lena image, compared with others: (a) nearest, (b) bilinear, (c) bicubic, (d) the level-set reconstruction, (e) ADSF, and (f) BDF.

Ⅳ. EXPERIMENTAL RESULTS

We used the explicit Euler method with the central difference scheme. A number of images have been used to test our scheme (18). Examples shown in Fig.5 are Lena image, where we interpolate it by a factor 2 with parameters: $[l, l_1, l_2]$= [200, 0.008, 0.003], $[\alpha, \beta]$ = [3, 2].

It is generally agreed that peak signal-to-noise ratio (PSNR) does not always provide an accurate measure of the visual quality for natural images [6, 7]. Therefore, we shall only rely on subjective evaluation to assess the visual quality of the interpolated images in this paper. Comparing with the level-set reconstruction, ADSF, and conventional interpolation methods: nearest, bilinear and bicubic, it can be seen that the best visual quality is obtained by interpolating the image using the proposed method, which preserves most features of the image, and produces sharp edges and smooth contours (see Lena's brim, cheek and eyeballs).

Ⅴ. CONCLUSIONS

This paper presents an adaptive feature preserving bidirectional flow process, by which we not only can effectively sharpen edges, but also can smooth contours of the image. Preserving image features such as edges, corners and textures, this method produces better visual results of the interpolated images than conventional interpolations.

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Shujun Fu received the B.S. degree in power engineering and the M.S. degree in computational mathematics from Shandong University, Jinan, China, in 1990 and 1999 respectively. Since 1999 he is a lecturer of the School of Mathematics and System Science, Shandong University. He is currently pursuing the Ph.D. degree in signal and information processing at the Beijing Jiaotong University, Beijing, China. His research interests include image processing, partial differential equations, and wavelet analysis.

Qiuqi Ruan received the B.S. and M.S. degrees from Northern Jiaotong University in 1969 and 1981 respectively. From January 1987 to May 1990, he was a visiting scholar in the University of Pittsburgh, and the University of Cincinnati. Subsequently, he has been a visiting professor in USA for several times. Now he is a professor, doctorate supervisor and the head of the institute of information science in the school of computer and information technology, Beijing Jiaotong University, China. He is a senior member of IEEE. His main research interests include digital signal processing, computer vision, pattern recognition, and virtual reality etc.

Wenqia Wang is a professor and doctorate supervisor in the School of Mathematics and System Science, Shandong University, Jinan, China. His research interests include partial differential equations, numerical analysis.