MATLAB simulation of multiuser detection in CDMA

Halil Tanyer Eyyuboğlu

***Abstract*—**This paper describes the MATLAB simulation of multiuser detection in code division multiple access (CDMA). To reach low bit error rate (BER) levels within reasonable computation times, the method of importance sampling is utilized. Our design is based on the step by step practical implementation of matched filter (MF) detection, has therefore the flexibility of variations in all parameters of interest which are otherwise difficult to accommodate in a theoretical model. Our initial findings are compared and verified against theoretical predictions and against those available in the literature.

***Keywords*—**MATLAB simulation, CDMA, multiuser detection

I. INTRODUCTION

THE performance of a detection technique, which is usually expressed in terms of bit error rate (BER), may be evaluated via three options. The first one entails a mathematical formulation of the process, while the second course is based on computer simulation. The final method is the physical act of measuring BER out in the field. Here, it is the former two cases that are of interest to us. A mathematical formulation may be considered to be the excellent choice in the sense that it will provide exact results almost instantly, and if no restrictions exist, it will be applicable throughout the entire range of signal to noise ratio (SNR) values. For most situations, the method of simulation will serve to compliment the mathematical framework by verifying the derived results. On the other hand simulation approach will surpass the abilities of the mathematical formulation if it can reveal the variations against parameters not supported by the theoretical analysis.

Code division multiple access (CDMA) works on the principle of code multiplexing and its advanced version, named as W-CDMA is the candidate for future land mobile networks. Its detection techniques, broadly defined as multiuser detection, differ substantially from the conventional schemes. Over the years, numerous researchers have contributed to the development of detection in CDMA. Amongst these, the most notable ones are the pioneering works of Pursley and Verdu [1]-[3]. Some valuable textbooks have also been published recently [4]-[7].

Halil T. Eyyuboğlu is with Çankaya University, Electronic and Communication Engineering Department, Öğretmenler Cad. No:14 Yüzüncüyıl 06530 Balgat Ankara, Turkey. (Phone: (+90) 312-284-4500/360, Fax: (+90) 312-284-8043, e-mail: h.eyyuboglu@cankaya.edu.tr).

In simulating BER performance of a communications system, as an alternative to Monte Carlo (MC) method, importance sampling (IS) emerges as an effective tool to attain results in a shorter time. This is realized by biasing the probability distribution of white Gaussian noise (WGN) so that more errors are generated with lesser number of samples. Again, several studies may be cited on this subject [8]-[11].

The rest of our article proceeds as follows. In section II, we describe the construction of the initial model covering the estimation of error for binary PSK. Section III focuses on the same application for CDMA. Main results in the form of graphs and scope output are presented in section IV. We end with conclusion and suggestion for further studies in section V

II. CONSTRUCTION OF INITIAL MODEL

To ensure the reliability of our design and also to aid apprehension of the matter, we initially attempted to construct a model whose end results can easily be compared to those readily available in the literature. For this purpose, the selected platform was phase shift keying (PSK) detection in an additive WGN environment, where the degrading effects of the channel are completely ignored. Under such circumstances, the theoretical probability of error, which is essentially the quantification of BER, for antipodal and orthogonal binary PSK signals are respectively defined as

$$P_{BPSKap} = Q\left[\left(2SNR\right)^{0.5}\right],\ P_{BPSKort} = Q\left[\left(SNR\right)^{0.5}\right] \quad (1)$$

where Q is the complimentary error function, $SNR = E_b/N_0$ is the signal to noise obtained by dividing the energy of one bit, E_b, by two sided noise power spectral density, N_0.

Probability of error can also be computed by creating the message signal, $s(t)$, the noise signal, $n(t)$, and the matched detection (MF) process in MATLAB. To this end, the intrinsic MATLAB functions of *randint* is used for message signal, $s(t)$, and *wgn* for noise signal, $n(t)$. For detection process, we have employed the correlation metrics, $C\left(\boldsymbol{r}, \boldsymbol{s}_m\right)$, given in [12], restated here as

$$C\left(\boldsymbol{r}, \boldsymbol{s}_m\right) = 2\boldsymbol{r}\boldsymbol{s}_m - \lVert \boldsymbol{s}_m \rVert^2 \quad (2)$$

where $\boldsymbol{r}$ is the vector obtained from the samples of MF detector after multiplying the incoming signal, $r(t) = s(t) + n(t)$ by orthogonal basis functions, integrating the result over the bit period, T, and sampling it at the end of T. Similarly $\boldsymbol{s}_m$ is the vectorial representation of $s_m(t)$, where the index m, refers to which message signal was sent. In binary PSK, m = 1, 2. The operation of $\|\boldsymbol{s}_m\|^2$ gives the square of the length for vector $\boldsymbol{s}_m$ or the energy in $s_m(t)$. During detection process, $C(\boldsymbol{r},\boldsymbol{s}_m)$ is evaluated by sequentially substituting s_1 and s_2 in (2). Then a decision is made in favor of that message signal, $s_1(t)$ or $s_2(t)$, yielding the greatest numerical value when substituted in (2). Finally, this experimental probability of error is to be calculated from simply dividing accumulated number of errors by the total number of received bits, n, expressed formally as

$$P_{BPSKexper} = \sum_{i=1}^{n} d_e / n \qquad (3)$$

where $d_e = 1$ if an error has occurred, and $d_e = 0$ otherwise. A plot of probability of error versus SNR is presented in Fig.1 for the case of antipodal BPSK signal. Here theoretical graph refers to using (1), whereas the experimental graph is obtained from (3), with the runs of the constructed MATLAB model. The results displayed in Fig. (1) agree with closely with those given in [13]. But note that in our simulation, the detection process functions as exactly defined in (2), also allowing more than one noise sample per message signal, indicated by the parameter f_s in Fig. (1).

Establishing the BER via the course of (3) is widely known as MC method. This method gives the most exact results. But, the number of samples (n) to be used in the simulation is inversely proportional to BER figures achievable. For instance, to produce a BER = 10^{-k} that will lie within a confidence interval of 90 %, n should be at least 10^{k+1} [8]. Choosing a high n, on the other hand, dramatically increases the computation time. For instance, Fig. (1) was acquired at the end of one week run on a PC equipped with 3 GHz CPU and 512 Mbyte RAM. For this reason, we have to resort to importance sampling that will be somewhat elaborated in section IV.

III. CDMA MODEL FOR MULTIUSER DETECTION

Using the experience of the initial model, we have constructed a simulation model for CDMA detection. In CDMA, there is a similar correlation metrics, $C(\boldsymbol{r}_K,\boldsymbol{b}_K)$ to be written as follows [14]

$$C(\boldsymbol{r}_K,\boldsymbol{b}_K) = 2\boldsymbol{b}_K\boldsymbol{r}_K^t - \boldsymbol{b}_K\boldsymbol{R}_S\boldsymbol{b}_K^t \qquad (4)$$

Here, $\boldsymbol{r}_K$ is the MF processed row vector of received signals (message signal plus noise) from K number of users, $\boldsymbol{b}_K$ is the row vector of possible transmitted sequence multiplied by the received amplitude levels of the users in question, $\boldsymbol{R}_S$ is the correlation matrix containing the cross correlation of the spreading codes and the letter t, as a superscript indicates the transpose operation. Analogous to the procedure outlined in the previous section, the detection of CDMA signals may be carried out via testing all likely sequences of $\boldsymbol{b}_K$ in (4) and deciding on the $\boldsymbol{b}_K$ sequence that maximizes the numeric value of $C(\boldsymbol{r}_K,\boldsymbol{b}_K)$. This detection mechanism yields the best, i.e., the optimum detection, but its complexity grows in the order of 2^K. As an alternative, suboptmimum methods have been developed, some of which are decorrelating detector, minimum mean square error (MMSE) detector and blind force detector. For our application, we have the adopted decorrelating detector where the detection process suitably takes into account the interference from other users, hence it has multiuser capability. In the case of decorrelating detector, the received bits are resolved as $\boldsymbol{b}_K'$, by computing the sign (denoted by the operator sgn) of the product $\boldsymbol{R}_S^{-1}\boldsymbol{r}_K$, i.e.,

$$\boldsymbol{b}_K' = \mathrm{sgn}\left(\boldsymbol{R}_S^{-1}\boldsymbol{r}_K\right) \qquad (5)$$

Compared with (3), this treatment is much simpler and only involves taking the inverse of the correlation matrix $\boldsymbol{R}_s$ and multiplying it by the MF processed received signals, $\boldsymbol{r}_K$. Performing the detection using (5), the MC based experimental probability of error for CDMA will be

$$P_{CDMA\text{exper}} = \sum_{i=1}^{n} d_e / n \qquad (6)$$

The theoretical probability of error, BER, for CDMA applications may be stated as follows

$$P_{CDMA} = Q\left[\left(2SNR\right)^{0.5} \Big/ \left(1 + SNR\left\{\sum_{k=2}^{K} P_k\left[2m_{k,1}(0) + m_{k,1}(1)\right] / \left(3P_1N^3\right)\right\}\right)^{0.5}\right] \qquad (7)$$

where N is the period of the spreading code, SNR is defined with respect to the intended user, in our case we have selected the first user as the intended user, therefore $SNR = E_1/N_0$, P_k with $k = 1\ldots K$ indicates the received power level of the signal

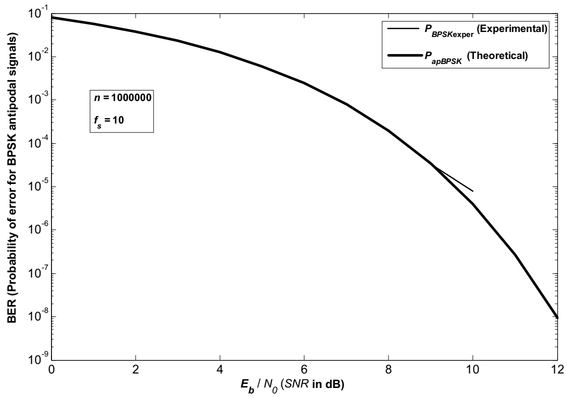

Fig. 1. Plots of experimental and theoretical BER for antipodal BPSK signals.

belonging to kth user, the terms $m_{k,1}(0)+m_{k,1}(1)$ are related to the interference arising from the nonzero cross correlation coefficients of the spreading codes and are the same as those defined in [1], [15]. It is easy to conclude that when only one user is active, i.e., K = 1, then the interference terms, subsequently the sum in the denominator of (7) collapse to zero. This way, (7) becomes identical to first equation of (1). In the literature, there exists a number of different versions of the theoretical probability of error for CDMA, including those independent of the characteristics of spreading code and the one named as improved Gaussian approximation. Another version is possible for P_{CDMA}, if all the received power level of all users are equal and SNR >> 1. For this specific case, (7) will reduce to [16]

$$P_{CDMA} \approx Q\left[\left(\frac{3N}{K-1}\right)^{0.5}\right] \quad (8)$$

IV. RESULTS AND DISCUSSIONS

In the construction of our simulation model for CDMA detection, we have used the *mdl* file utility of MATLAB to generate the GOLD spreading sequences. To the same file, a scope and signal from workspace blocks were added to visualize waveforms of message signal, gold sequences, transmitted signal as a the sum of signals belonging to all active users and received signal. A glimpse from these waveforms is shown in Fig. 2. By examining these illustrations, one may appreciate the miracle of detection mechanisms and how powerful they are in extracting the desired message signal out of a real mess of received waveforms. As may be detected from this figure, in the context of CDMA, N (period of spreading code) = T (bit duration of the message signal).

In our simulation runs, we observed that from the viewpoint of computation time and memory, it is impossible to go beyond K = 10, n = 1000, and N = 31 using the optimum detection technique supplied by (4). Up to these limits, the produced BER results conformed perfectly to the theoretical predictions of (7) in the range, BER = 10^{-1} to 10^{-2}. Secondly, the detection mechanism based on (5) was tried. In this particular case, n = 10000 was achievable provided that K < 10 and $N \le 31$. To proceed further and reach lower BER figures, it was decided to introduce importance sampling (IS) at this stage. The theory behind IS is that the probability density function governing the noise samples is artificially biased in such a way that more errors are produced with lesser n values. Hence in this situation, the above mentioned MC restriction that, in a 90 % confidence interval, 10^{k+1} samples are required to reach a BER up to 10^{-k} no longer applies. This bias is appropriately embedded into the formulation as follows. Assume that the initial noise pdf is to be biased by increasing its variance, hence noise power spectral density, N_0. Then we may introduce a weight function, w_i, in the following manner

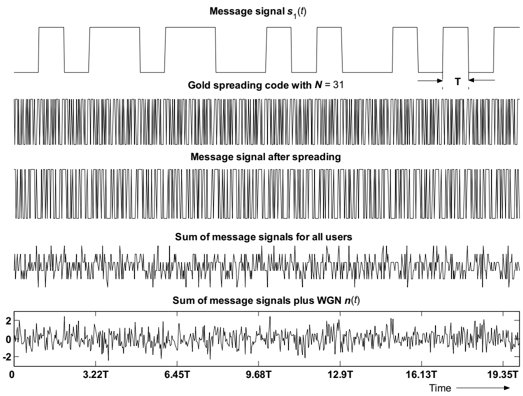

Fig. 2. Typical CDMA waveforms at transmitter and receiver.

$$w_i = f_{phy}(n_i)/f_{bias}(n_i) \quad (9)$$

where $f_{phy}(n_i)$ and $f_{bias}(n_i)$ are respectively the pdf of noise source before and after biasing. Now the new BER estimator for probability of error, P_{bias}, will be in the form of

$$P_{bias} = \sum_{i=1}^{n} w_i d_e / n \quad (10)$$

The above equation differs from (3) and (6) in the sense that the error counting includes its associated weighting factor. This action is necessary to offset the original biasing applied. By carefully adjusting w_i, it is possible to attain really low BER figures with a much smaller number of samples than actually required. This process is known as the classical importance sampling (CIS). Altering the noise distribution pdf by a shift in the mean value is named as improved importance sampling (IIS). It is the former that is used in our graphs.

Fig. 3 illustrates the probability of error plotted according to (10) and (7), these are consecutively experimental BER, i.e., P_{bias} and the theoretical BER with biasing, i.e., P_{CDMA}. With the present computer resources, it was impossible to go beyond the settings of this figure, that is K = 5, n = 20000 and T (or N) = 31. Nevertheless, this graph proves that by using IS, much lower BER levels than those of MC method are attainable. But of course at larger SNR values, the deviation from the theoretical curve is also visible from Fig. (3).

V. CONCLUSION AND SUGGESTION FOR FURTHER STUDIES

In this study, starting with a simple initial model of BPSK, we have demonstrated the feasibility of constructing a CDMA multiuser detection process in a MATLAB environment. For Monte Carlo type of runs, reasonably correct BER results are achieved provided that the number of samples used is one order higher than the inverse of the expected BER figure. For all cases examined a close agreement has been

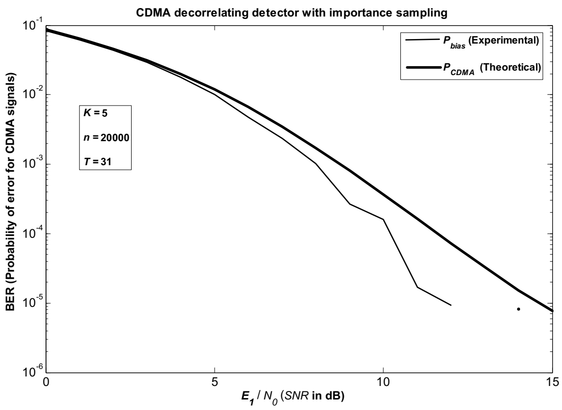

Fig. 3. Plots of experimental and theoretical BER for CDMA signals.

observed between the theoretical and the experimental results. In our terminology theoretical results are those obtained from the available analytic formulation, whereas the experimental results are computed from the model constructed in the MATLAB environment. To reach low BER figures, we have resorted to importance sampling.

We note that our present experimental results cover only the fundamental detection strategy in CDMA. By incorporating other features such as synchronization, the characteristics of the communication channel, it will be possible to set up a simulation environment much closer to practical situations. This way, we will also be able observe the effects of parameters that may not be accounted for theoretically. Presently, we are engaged in this direction.

References

[1] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication-Part I," *IEEE Trans. Commun.* vol. COM 25, pp. 795-799, Aug. 1977.

[2] S. Verdu, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory* vol. IT 32, pp. 85-96, Jan. 1986.

[3] M. Honig, U. Madhow and S. Verdu, "Blind adaptive multiuser detection," *IEEE Trans. Inform. Theory* vol. 41, pp. 944-960, July 1995.

[4] R. E. Ziemer and R. L. Peterson, *Introduction to Digital Communications*, 2nd ed. New Jersey: Prentice Hall, 2001, ch. 9.

[5] X. Wand and H. V. Poor, *Wireless Communication Systems Advanced Techniques for Signal Reception*, New Jersey: Prentice Hall, 2004, ch. 1-6.

[6] W. H. Tranter, K. S. Shanmugan, T. S. Rappaport and K. L. Kosbar, *Principles of Communication System Simulation with Wireless Applications*, New Jersey: Prentice Hall, 2004, ch. 9-11, 16.

[7] S. G. Glisic, *Adaptive CDMA Theory and Practice*, Chichester: Wiley, 2002, ch. 13-14.

[8] M. C. Jeruchim, "Techniques for estimating the bit error rate in the simulation of digital communication systems," *IEEE J. Select. Areas Commun.* vol. SAC 2, pp. 153-170, Jan. 1984.

[9] P. J. Smith, M. Shafi and H. Gao, "Quick simulation: a review of importance sampling techniques in communication systems," *IEEE J. Select. Areas Commun.* vol. 15, pp. 597-613, May 1997.

[10] K. B. Letaief, "Efficient evaluation of the error probabilities of spread-spectrum multiple-access communications," *IEEE Trans. Commun.* vol. 45, pp. 239-246, Feb. 1997.

[11] D. Lu and K. Yao "Improved importance sampling technique for efficient simulation of digital communication systems," *IEEE J. Select. Areas Commun.* vol. 6, pp. 67-75, Jan. 1998.

[12] J. G. Proakis and M. Salehi, *Communication Systems Engineering*, 2nd ed., Prentice Hall, 2002, ch. 7.

[13] J. G. Proakis, M. Salehi and G. Bauch, *Contemporary Communication Systems using MATLAB and Simulink*, 2nd ed., Belmont, CA: Thomson, 2004, ch. 5.

[14] J. G. Proakis, *Digital Communications*, 3rd ed., New York: McGraw-Hill, 1995, ch. 15.

[15] S. R. Park, I. Song, S. Yoon and J. Lee, "A new polyphase sequence with perfect even and good odd cross-correlation functions for DS/CDMA systems," *IEEE Trans. Vehic. Tech.* vol. 51, pp. 855-866, Sept. 2002.

[16] T. F. Wong, Course notes on Spread spectrum and CDMA, ch. 6, Available : http://wireless.ece.ufl.ed/~twong.