A Redefined Quasi-Sliding Mode Control Strategy

C. Vivekanandan, R. Prabhakar, and M. Gnanambigai

***Abstract*—**Discrete sliding mode control (DSCM) is one of the best techniques for analyzing the dynamics of nonlinear systems. This paper presents a modified discrete quasi-sliding mode control strategy. The *reaching law* condition defined in the presently available technique is redefined in this paper. The modified strategy improves the convergence rate of the system dynamics in the reaching mode. Due this, the system response is smoother during the transition from reaching mode to sliding mode with reduced settling time. This improvement is achieved at a lesser control effort compared to that required by the presently available techniques, thus improving the overall system efficiency. The proposed redefined strategy is applied to the same system, considered in the previous method and the results obtained demonstrate its effectiveness and improved efficiency.

***Keywords*—**Discrete time system, Sliding-mode control, Reaching condition.

I. Introduction

VARIABLE structure control (VSC) is a general approach for the design and control of a class of nonlinear systems that can be represented in state model. In variable structure control the system is allowed to vary its structure by properly and deliberately changing the sign and / or magnitude of the input, forcing discontinuities in the input with respect to time. The plane which separates these different structures is called as switching plane or switching surface. This discontinuous input makes the phase trajectory of the system to undergo two modes, *reaching mode* and *sliding mode*. In reaching mode, the system phase trajectory, starting from anywhere on the phase plane moves toward a switching plane and reaches it in finite time. This is followed by sliding mode in which the phase trajectory asymptotically tends to the origin of the phase plane [1], [2], [3]. The switching surface decides the closed loop dynamics of the system which are at the designer's choice and hence, also known as sliding mode control (SMC). The main advantage of SMC is its insensitivity to parameter variations, external disturbances and modeling errors [2], [3].
With the advent of digital computers and its widespread use in control systems, considerable efforts have been put in the study of discrete time VSC/SMC techniques, called as discrete sliding mode control techniques (DSMC) [5], [3], [6]. In discrete sliding mode the control input is constant over sampling periods. Hence, when the states reach the switching surface, the subsequent control would be unable to keep the states to be confined to the surface. This leads the system to undergo only quasi-sliding mode, i.e., the system states would approach the sliding surface but would generally be unable to stay on it [3]. Thus, DSMC does not posses the invariance property found in continuous time sliding mode.

Gao introduced '*reaching law'* method to design the controller for continuous-time VSC [4] and extended the same for discrete time counterpart [5]. This approach is found satisfactory when compared to the other methods proposed in [8]. The switching function $s(k)$ is effectively controlled to meet the required dynamics and also to satisfy the constraints of DSMC. This is followed by the derivation of the control law in conjunction with the known plant model and parameter variations.

The technique proposed by Gao is simpler and directly deals with the reaching process and makes it easy to obtain the control law [4]. However, chattering in the steady state is a major drawback, which is due to the discontinuous switching control applied to the plant, which excites the unmodelled high frequency dynamics of the system.

Bartoszewicz proposed a state-feedback-based control law for uncertain systems with *bounded uncertainties* that guarantee's discrete sliding mode [6]. This is an improved version of Gao's method and ensures finite time convergence of phase trajectories on the sliding plane without chattering.

By redefining the *priori* known function defined in [6], it is found that the system response is smoother during the transition from reaching mode to sliding mode. The redefined technique requires smaller control effort compared to the previous strategies, while retaining the finite time convergence of the state trajectory to the sliding surface.

The paper has been organized into six Sections. Section 2 gives an overview of DSMC and reaching condition details. Bartoszewicz technique is briefly discussed in Section 3 along with the redefined version of it. Section 4 explains the new improved quasi-sliding-mode control strategy for an uncertain system with unbounded uncertainties and the details of the two numerical examples considered in this paper. The responses of the example systems under different conditions and the improvements in the system dynamics and efficiency are discussed in Section 5. Section 6 provides the conclusion and future work.

II. Discrete Quasi-Sliding Mode and Reaching Conditions

A. Preliminaries:

Consider a discrete-time system represented by,

$$x(k+1) = Ax(k) + \Delta Ax(k) + bu(k) + f(k)$$
$$y(k) = h^T x(k) \quad (1)$$

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where x is the $n \times 1$ state vector, A is an $n \times n$ system matrix, b and h are input and output vectors of appropriate dimensions, u is the system input, and y is the system output. The pair (A, b) must be controllable. Further, the $n \times n$ parameter variation matrix ΔA and the $n \times 1$ external disturbance vector f satisfy the matching conditions [3],

$$\Delta A = b\bar{A} \qquad \bar{A} \text{ - a row vector}$$

$$f = b\bar{f} \qquad \bar{f}\text{- scalar}$$

Define the switching function

$$s(k) = c^T x(k) \tag{2}$$

with vector c such that $c^T b \neq 0$ and the resulting quasi-sliding motion is stable. Disturbances and parameter variations are bounded, so that the following relation holds:

$$d_l \leq d(k) = c^T \Delta A x(k) + c^T f(k) \leq d_u \tag{3}$$

where the lower bound d_l and the upper bound d_u are known constants. Further more define $d_0 = 0.5(d_l + d_u)$ and $\delta_d = 0.5(d_u - d_l)$, where d_0 the average is value of $d(k)$ and δ_d is the maximum possible deviation as shown in Fig.1. The quasi-sliding mode is defined as the motion such that $|s(k)| \leq \epsilon$, where the positive constant ϵ is called the quasi-sliding-mode bandwidth.

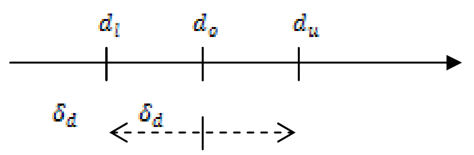


Fig. 1 Deviation of $d(k)$

B. Reaching Condition and Reaching Mode

The condition under which the system states starting from any initial state, move towards the sliding surface and reach it in finite time, called as *reaching condition* or *reaching law*. The system trajectory under the reaching condition is called the *reaching mode* or *reaching phase*. So, under ideal conditions there exists a finite time t_s such that for all $t \geq t_s$, the sliding function $s(x) = 0$.

In continuous time the reaching law is a differential equation which specifies the dynamics of a switching function $s(x)$. A simple reaching law may be written as [4]

$$\frac{ds}{dt} = -q\,(sgn(s)) \tag{4}$$

In the above condition the parameter q decides the rate of convergence of $s(x)$ towards the sliding surface and the magnitude of chattering in the sliding mode. Hence, by proper choice of the parameter q in (4), the dynamic quality of VSC system in the reaching mode can be controlled.

In case of discrete mode the condition given in (4) is written as:

$$s(k+1) - s(k) = -qT\left(sgn(s(k))\right) \tag{5}$$

where T is the sampling period. The condition specified by (5) assures that

$$|s(k+1)| < |s(k)| \tag{6}$$

However, this condition fails to ensure the finite time convergence. Hence, the objective is to determine a suitable function that steers the system to sliding surface in finite sampling period. Gao in [5] suggested a condition

$$s(k+1) - s(k) = -qT\left(sgn(sk)\right) - \epsilon T s(k)$$
$$\epsilon > 0,\; q > 0 \text{ and } 1 - \epsilon T > 0$$

which results in chattering and its magnitude is limited with in the so called quasi-sliding mode bandwidth (QSMB), given by

$$2\Delta = 2\frac{qT}{1 - \epsilon T} \tag{7}$$

Bartoszewicz suggested a new reaching law [6]

$$s(k+1) = d(k) - d_0 + s_d(k+1) \tag{8}$$

where $s_d(k)$ is a *priori* known function. In this technique the system state is steered to the sliding plane $s(k) = 0$, and do not allowed cross it, and hence, chattering is eliminated and ideal quasi-sliding mode can be achieved for certain systems. Following section gives a brief overview of this technique.

III. Bartoszewicz's Reaching Law

A. Fundamentals

Bartoszewicz proposed a reaching law, given in (8) in which the unknown $d(k)$ is defined by (3) and $s_d(k)$ is an a *priori* known function such that the following applies [6].

1. If $s(0) > 2\delta_d$, then

$$s_d(0) = s(0)$$

$$s_d(k).s_d(0) \geq 0, \quad \text{for any } k \geq 0$$

$$s_d(k) = 0, \quad \text{for any } k \geq k^*$$

$$|s_d(k+1)| < |s_d(k)| - 2\delta_d, \text{ for any } k < k^*. \tag{9}$$

2. Otherwise, i.e., if $s(0) \leq 2\delta_d$, then $s_d(k) = 0$ for any $k \geq 0$.

The constant k^*, in relations (9), is a positive integer chosen by the designer in order to achieve good trade-off between the fast convergence rate of the system and the magnitude of the control u required to achieve this convergence rate. The definition chosen in [6] for $s_d(k)$, when $s(0) > 2\delta_d$ is

$$s_d(k) = \frac{k^* - k}{k^*} s(0)$$

$$k^* < \frac{s(0)}{2\delta_d} \qquad (10)$$

where $k = 0,1,\cdots,k^*$.

In order to determine the control u which drives the system in such a way that the reaching law (8) is satisfied, (1) and (2) are used to calculate $s(k+1)$:

$$s(k+1) = c^T Ax(k) + c^T \Delta Ax(k) + c^T bu(k) + c^T f(k)$$

$$s(k+1) = c^T Ax(k) + d(k) + c^T bu(k) \qquad (11)$$

Comparing this equation with the reaching law (8), the following is obtained.

$$s_d(k+1) - d_0 = c^T Ax(k) + c^T bu(k) \qquad (12)$$

Hence, the input is given by

$$u(k) = -(c^T b)^{-1}[c^T Ax(k) + d_0 - s_d(k+1)] \qquad (13)$$

By this means, a control law is designed, which guarantees that, for any $k \geq k^*$, the system state satisfies the following inequality:

$$|s(k)| = |d(k-1) - d_0| \leq \delta_d \qquad (14)$$

IV. Redefined Bartoszewicz Law

In the above discussion the value $s_d(k)$ for any given $k < k^*$ calculated using the initial value of the switching function $s(k)$. Hence, the rate at which the system dynamics move towards the sliding mode *i.e.* the rate at which the value of $s_d(k)$ decreases, is decided by the present value of k^* alone.

Now, let the priori known function given in [10] be redefined as

$$s_d(k) = \frac{k^* - k}{k^*} s(k) \qquad (15)$$

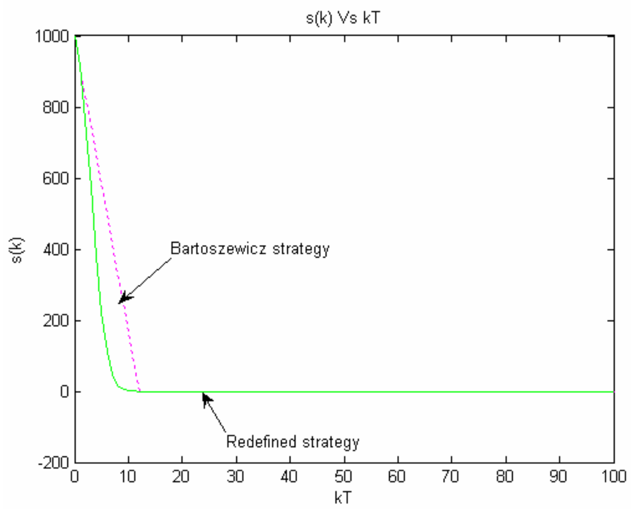
The value of the switching function $s(k)$ also decreases in each control step, and hence the value of $s_d(k)$ decreases at a faster rate than before, driving the system in to the sliding mode quickly. Due to this the settling time is reduced and transition from reaching mode to sliding mode is smoother. The control effort is calculated as $\sum_{k=0}^{n} |u(k)|$ in both cases and it is found that the lesser control effort is required in the proposed technique than in the previous techniques *viz.* Gao's reaching law and Bartoszewicz law. In what follows, the same problem considered by the presently available technique is used to prove the effectiveness and the efficiency of the proposed technique.

V. Simulation Example

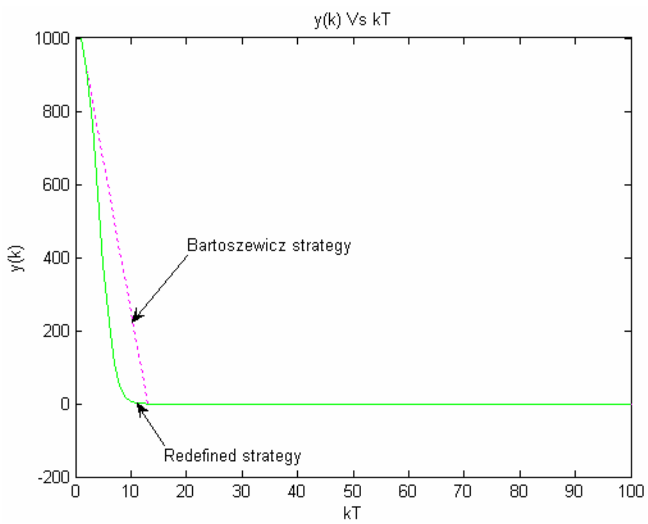
To prove the advantages of the redefined reaching law, the system considered by Bartoszewicz in [6] is used and the parameters of that system are,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0.5 \end{bmatrix}; \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } \quad h = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

The sliding line considered is $c^T = [1 \quad 1]$ and the initial condition assumed is $x(0) = [1000 \quad 0]^T$. The values of k_f and k are set to 12 and 100 respectively, similar to that of Bartoszewicz, so that they reflect fairly the performance of the controlled system, both during the transient period and in the steady state. The parameter variation is assumed to be zero *i.e.* $\Delta A = 0$ and the external disturbance vector considered is $f(k) = [0 \quad 1]^T$. With the above assumptions the system is simulated using both Bartoszewicz and redefined techniques and the results obtained are given in Fig. 2 (a) and (b).



(a)



(b)

Fig. 2 System dynamics with controllers designed using Bartoszewicz reaching law and redefined reaching law (a) Evolution of $s(k)$, (b) System output $y(k)$

It is clear from the plots that the transition from reaching mode to sliding mode is sharp with Bartoszewicz law method where as, it is smooth with the proposed technique. Further, the system output $y(k)$ settles at 13th sample $(kT = 13)$ in case of Bartoszewicz technique and at 9th sample $(kT = 9)$ in the proposed strategy, the clear indication of the improved settling time. The control effort, calculated as $J = \sum_{k=0}^{100} |u(k)|$, required by Bartoszewicz law method is 682.33 and the same required by proposed technique is 611.0295. The control quality criterion defined as $Q = \sum_{k=0}^{100} y(k)$ is 7600 in Bartoszewicz method and the same is 5036 in our method. Hence, a better quality control is achieved with a lesser control effort in our proposed technique. The following Table I summarizes the performance parameters of the controllers designed using Gao's reaching law, Bartoszewicz reaching law and our redefined reaching law.

TABLE I
COMPARISON OF PERFORMANCE PARAMETERS OF CONTROLLERS DESIGNED USING DIFFERENT TYPES OF REACHING LAW METHODS

	Settling Time (t_s) kT	Control Effort $J = \sum_{k=0}^{100} \|u(k)\|$	Quality Factor $Q = \sum_{k=0}^{100} y(k)$
Gao	17	5590.00	10200
Bartoszewicz	13	682.33	7600
Proposed	**9**	**611.03**	**5036**

VI. CONCLUSION

In this paper the redefined discrete-time sliding mode strategies based on the technique proposed by Bartoszewicz is presented. Better system dynamics are obtained with lesser control effort, and the overall system performance improves with the technique proposed in this paper, which is proved by simulating the same system considered by Bartoszewicz. The simulation is done for SISO systems and can easily be extended for MIMO also.

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