A Comparison of Real Valued Transforms for Image Compression

Shivali D. Kulkarni, Ameya K. Naik, and Nitin S. Nagori

***Abstract*—**In this paper we present simulation results for the application of a bandwidth efficient algorithm (mapping algorithm) to an image transmission system. This system considers three different real valued transforms to generate energy compact coefficients. First results are presented for gray scale and color image transmission in the absence of noise. It is seen that the system performs its best when discrete cosine transform is used. Also the performance of the system is dominated more by the size of the transform block rather than the number of coefficients transmitted or the number of bits used to represent each coefficient. Similar results are obtained in the presence of additive white Gaussian noise. The varying values of the bit error rate have very little or no impact on the performance of the algorithm. Optimum results are obtained for the system considering 8x8 transform block and by transmitting 15 coefficients from each block using 8 bits.

***Keywords*—**Additive white Gaussian noise channel, mapping algorithm, peak signal to noise ratio, transform encoding.

I. Introduction

DIGITAL image processing is a rapidly evolving field with extensive applications in the field of mobile technology. In an increasing number of applications [1] video and images are transmitted and received over portable wireless devices such as cellular phones, laptop computers and cameras used in surveillance. Although image transmission is highly desirable in many applications, a limiting factor has been the use of bandwidth and energy efficient methods for transmission. Also in most of the cases a reduction in signal bandwidth is generally accompanied with a decrease in the delivered image quality. Several image data compression techniques are discussed in [1]. Although the optimum image compression method largely depends on the type of image, recently however considerable attention has been given to the technique of transform encoding [2].

Transform coding methods provide high energy compaction within a small number of decorrelated coefficients thus eliminating redundancy. While the bit rate reduction is achieved, a strong data dependency is created between the pixels. This increases image sensitivity to channel noise and subsequently, affects the image quality considerably. A number of channel coding schemes have been proposed for reducing the channel noise. Channel coding schemes tend to introduce redundancy resulting in bandwidth expansion. Hence a trade off has to be achieved between the data compression obtained by source coding and data expansion due to channel coding. A better scheme would be to compress the source information as completely as possible and then to allow for the inherent redundancy due to channel coding. Although image transformation methods provide energy compaction in only a few coefficients, a critical issue is the transmission of high magnitude transform coefficients.

In this paper we present the mapping algorithm for transmitting transform coefficients using least number of bits. The performance of this algorithm is observed for different real valued transformations such as discrete cosine transform (DCT)[3-5], discrete sine transform (DST)[5-7] and discrete Hartley transform (DHT)[6,8]. It is seen that the discrete cosine transform gives a better energy compaction as compared to the other transforms. Furthermore the performance of these algorithms is evaluated for different sizes of transform block, number of coefficients transmitted from each block and number of bits used to represent each pixel coefficient. It is noted that the size of the transform block has more prominent effect on the performance of the transmission system as compared to the other parameters.

The remainder of the paper is organized as follows. Section II describes the process of image transmission using a channel encoding scheme. Section III discusses the mapping algorithm used for compression along with its performance measures. This is followed by a brief description of the various real valued image transforms used by the system (section IV). The simulation results are presented in section V and finally the concluding remarks are given in section VI.

II. System Model

The communication system shown in fig.1 is simulated using Matlab v. 7.6. The model (fig. 1) consists of a source encoder which accepts a still image as the input. The image undergoes a real valued image transformation such as DCT, DST and DHT. The coefficients obtained are then scanned [1] using zig-zag technique and only a few coefficients are considered from each scanning block for further processing.

Manuscript received June 30, 2008.

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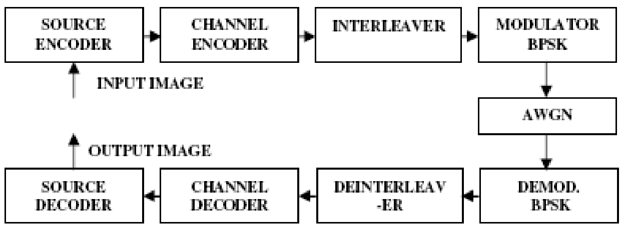


Fig. 1 System Model

The coefficients are modified according to the algorithm discussed in section III. The modified coefficients are converted into binary form and then passed on to the channel encoder. The encoder used is a low rate (1/8) maximum free distance convolutional encoder [10]. The encoded bits are then interleaved and modulated before transmission over the noisy channel. At the receiver side exactly reverse operation is performed to obtain the reconstructed image.

III. THE MAPPING ALGORITHM

The system described in fig. 1 uses a mapping algorithm to convert the transform coefficients into bits to be transmitted over the channel. The algorithm can be given as follows.

Step 1: Round the scanned transform coefficients (T) to the nearest higher integer.
Step 2: Find the minimum (minT) of the approximated transform coefficients.
Step 3: Convert the coefficients into positive integers by adding an offset equal to abs(minT) to all the coefficients.
i.e. T=T+ offset
where offset=abs(minT) and abs(x)=absolute value of x
Step 4: Let i=1 to nc
where nc= number of coefficients considered from each scan block
Step 5: Consider the ith value of the coefficients from each scan block (T_i)
Step 6: Find the maximum ($maxT_i$) and minimum ($minT_i$) value of the ith coefficients considered.
Step 7: Calculate the range of the scanned transform coefficients using the formula $rangeT_i=maxT_i-minT_i$.
Step 8: Decide the number of bits (n) used to represent the transform coefficients.
Step 9: Calculate the maximum and the minimum fraction less than 1 that can be represented using nc number of bits.
i.e. $nmaxT_i=(2^n-1)/2^n$ and $nminT_i=0$
Step 10: Calculate the new range using the formula
$nrangeT_i= nmaxT_i - nminT_i$
Step 11: Map the transform coefficients to the new range using the formula $n\,T_i =(nrangeT_i/ rangeT_i)*(T_i)$
Step 12: Convert the $nminT_i$, $rangeT_i$ and nT_i into binary form using only n number of bits. ($rangeT_i$ can however be represented using more number of bits).
Step 13: Repeat Step 5 to Step 10 for all the remaining values of i.
Step 14: Transmit all the binary representation of nT_i along with the offset converted into binary form
Step 15: For reconstruction exactly reverse of Steps 1 to 14 is performed i.e. inverse mapping is performed first followed by subtracting the offset from each value of inverse mapped coefficients. The coefficients are then placed at proper positions to obtain the reconstructed image.

The original image is compared with the reconstructed image using a metric known as peak signal to noise ratio ($PSNR$)[1]. Higher $PSNR$ values imply closer resemblance between reconstructed and original image.

If we denote the pixels of the original image by P_i and the pixels of the reconstructed image as Q_i (where $1 \le i \le n$), we first define the mean square error (MSE) between the two images as

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(P_i - Q_i\right)^2 \qquad (1)$$

The root mean square error ($RMSE$) is defined as the square root of the MSE, and the $PSNR$ is defined as

$$PSNR = 20\log_{10}\frac{\max_i |P_i|}{RMSE} \qquad (2)$$

IV. REAL VALUED IMAGE TRANSFORMS

For an $N \times N$ rectangular image, the transform pair is given by $V = ZUZ^T$ (3)
and $U = Z^{*T} V Z^*$ (4)
where Z is a $N \times N$ unitary matrix given by

$$Z = \begin{cases} C \; for \; DCT \\ \Psi \; for \; DST \end{cases}$$

Z^T is the transpose of Z
and Z^* is the conjugate of Z.

A. Discrete Cosine Transform

The $N \times N$ cosine transform matrix $C = \{C(u,v)\}$ is defined as

$$C(u,v) = \begin{cases} 1/\sqrt{N} \quad for\, u = 0, 0 \le v \le N-1 \\ \sqrt{\frac{2}{N}}\cos\left[\frac{\pi(2v+1)u}{2N}\right] \quad for\, 1 \le u \le N-1, 0 \le v \le N-1 \end{cases} \qquad (5)$$

The 2-dimensional DCT of an image can be generated using (3). The DCT has excellent energy compaction for highly correlated data.

B. Discrete Sine Transform

The $N \times N$ sine transform matrix $\psi = \psi(u,v)$ is given as

$$\psi(u,v) = \sqrt{\frac{2}{N+1}}\sin\left\{\frac{\pi(u+1)(v+1)}{N+1}\right\} \; for \; 0 \le u, v \le N-1 \qquad (6)$$

It is seen that similar to DCT a large fraction of the total energy is concentrated in a few transform coefficients.

C. Discrete Hartley Transform

Hartley transform is a substitute for Fourier transform in many filtering applications. The discrete two dimensional Hartley transform is defined as

$$F(u,v)=\frac{1}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)cas\left\{\frac{2\pi}{N}\left(ux+vy\right)\right\} \quad (8)$$

The inverse discrete Hartley transform is

$$f(x,y)=\frac{1}{N}\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} F(u,v)cas\left\{\frac{2\pi}{N}\left(ux+vy\right)\right\} \quad (9)$$

where $cas\theta=\cos\theta+\sin\theta$.

V. RESULTS AND DISCUSSION

In this section, we investigate the bit error rate (BER) and $PSNR$ performance of a transmission system using mapping technique for source encoding. Simulations have been run to demonstrate the performance of mapping algorithm under the following conditions

i) Results are presented for gray scale and color image of the size 256x256 pixels.
i) Number of bits used to represent each transform coefficient = 7, 8, 9 and 10.
ii) Number of coefficients considered from each transform block for transmission = 10, 15 and 21.
iii) Size of the transform = Size of the scan block=8x8 (64 coefficients),16x16 (256 coefficients),32x32 (1024 coefficients) and 64x64 (4096 coefficients).
iv) Channel encoder used is a maximum free distance convolutional encoder with rate =1/8 and constraint length=8.
v) Results are presented for noiseless condition and in the presence of additive white Gaussian noise (AWGN).

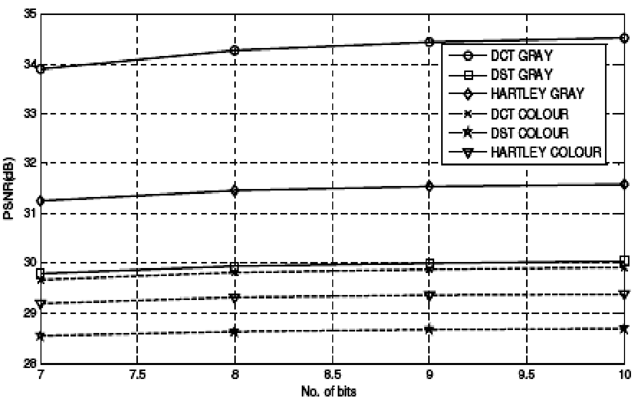


Fig. 2 PSNR vs number of bits for mapping algorithm for DCT, DST and DHT under noiseless conditions.

Fig.2 shows the plot of PSNR versus number of bits for mapping algorithm. Results are obtained for different transform methods such as DCT, DST and Hartley under noiseless conditions. It can be seen that the performance of the system is better when DCT is used. Also the variation in the number of bits has significant improvement only in the initial stage (7bits to 8 bits). The PSNR value remains fairly constant thereafter. Similar results are obtained for the color image under the same conditions. Although the performance of the algorithm is degraded, DCT still performs better.

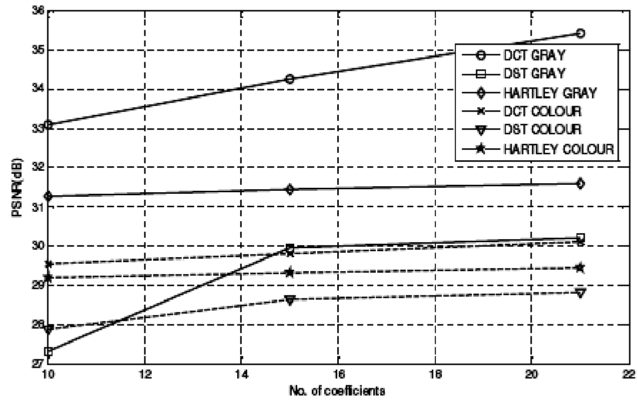


Fig. 3 PSNR vs number of coefficients for mapping algorithm for DCT, DST and DHT under noiseless conditions.

Fig. 3 shows the plot of PSNR versus number of coefficients for the same algorithm under similar conditions. The increase in the number of coefficients leads to an increase in the quality of transmission both for gray scale and color images. However the variation in the number of coefficients has less effect on the transmission quality after 15 coefficients.

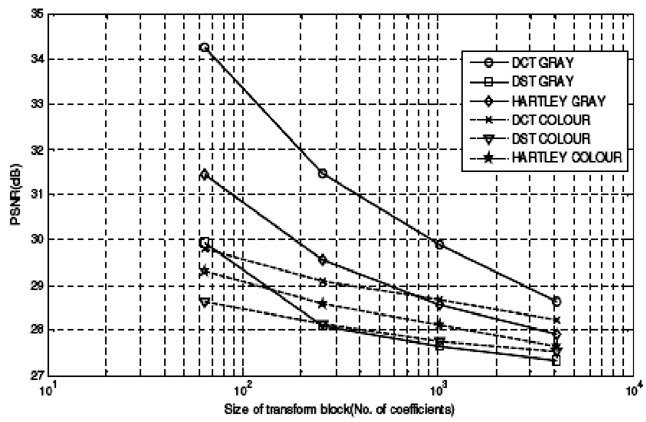


Fig. 4 PSNR vs size of transform block for mapping algorithm for DCT, DST and DHT under noiseless conditions.

The system performance is also evaluated by varying the size of the transform block (fig. 4). It can be noted that the performance of the system degrades severely as the size of the transform block is increased. The decrease in the size of the transform block is however associated with increase in the computational complexity. Hence a block size of 8 x 8 (64 coefficients) is normally taken to provide optimum performance.

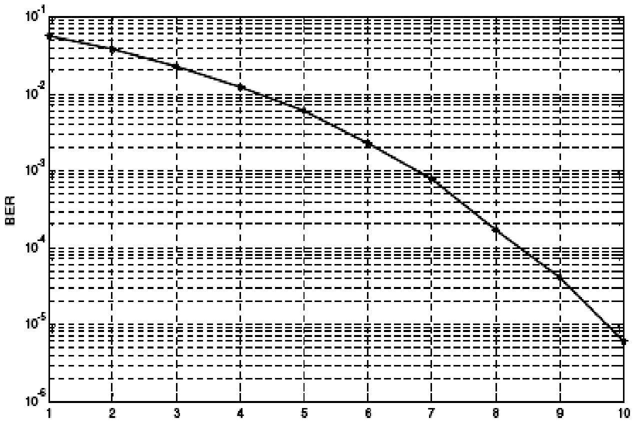


Fig. 5 BER vs E_b/N_o in dB for AWGN channel.

Results are also depicted for the system in the presence of noise. Fig. 5 shows the variation of BER versus E_b/N_o for AWGN channel. Results (fig. 6-8) are plotted again by varying the same parameters and observing the effect on the value of PSNR. It is found that the effect of the BER has negligible effect on the system performance inferring that the system is more robust to channel noise.

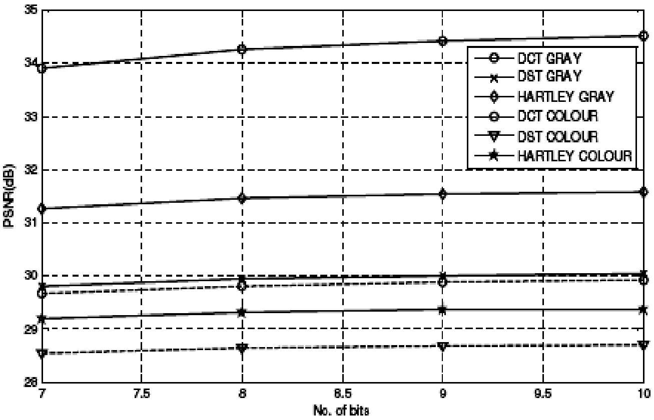


Fig. 6 PSNR vs number of bits for mapping algorithm for DCT, DST and DHT in the presence of AWGN.

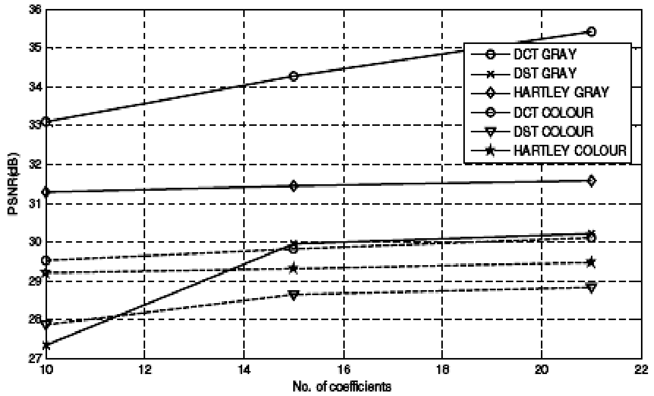


Fig. 7 PSNR vs number of coefficients for mapping technique for DCT, DST and DHT in the presence of AWGN.

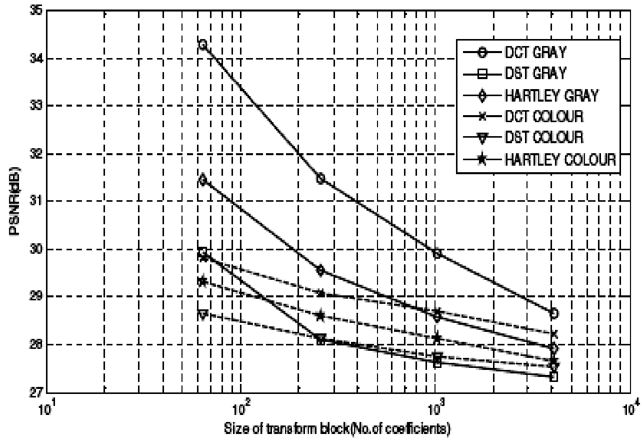
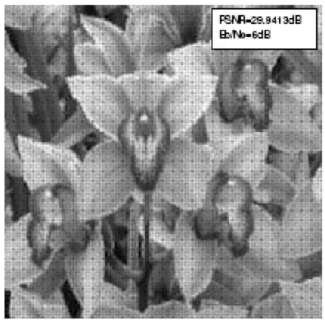


Fig. 8 PSNR vs size of transform block for mapping algorithm for DCT, DST and DHT in the presence of AWGN.

Fig. 9 show the original and a sample of reconstructed images for DCT, DST and Hartley for bit energy to noise ratio of 6dB.

(a)

(b)



(c)

(d)

Fig. 9 (a) Original Image
(b) Reconstructed image with DCT for AWGN (E_b/N_o=6dB).
(c)Reconstructed image with DST and AWGN (E_b/N_o=6dB).
(d) Reconstructed image with DHT for AWGN (E_b/N_o=6dB).

It is seen that the reconstructed image shows more resemblance to the original image when DCT (PSNR=34.267dB) is used. The performance of the system is poor when DST (PSNR=29.841dB) is used. Hartley transform (PSNR=31.4520dB) gives performance slightly better than DST.

VI. CONCLUSION

It can be concluded that the mapping algorithm makes the image transmission more robust to the channel noise. Also the effect of varying the scan block is more prominent rather than when the number of bits or the number of coefficients is varied. An optimum system performance can be obtained for number of bits =8, number of coefficients=15 and size of the transform block =8 x 8 (64 coefficients).

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