The Simple Practical Approach to HMT Realization Based on Behavioral Hypothesis

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***Abstract*—**Adoption of anti-monopoly measures depends on definition of boundaries of the market subject to analysis. For this purpose the so called "Hypothetical Monopolist Test" (hereinafter referred to as HMT) is widely applied. This report focuses on the HMT model development and algorithmization.

***Keywords*—**Hypothetical Monopolist Test, mathematical modeling, behavioral hypothesis.

I. Introduction

THE Hypothetical Monopolist Test is based on the concept which presumes the market to be a sphere where the monopoly power could appear.

According to HMT, the market is defined as a group of goods (in particular, such group may include only one product) and their geographical sales zones where a hypothetical company (striving for increase in profits and not subjected to anti-monopoly regulation measures) being a single seller of the said products

1) can raise the prices for this group of products (within the considered territorial boundaries) thus increasing its profits,

2) can not increase its profits by raising the prices for a part of this product group only (within the part of the said territory) leaving the prices for the rest of the products of this group unchanged.

The task of market boundaries definition consists in identification of the products (areas) making up the market from a wider range of products (areas). One has to find such a minimum group of products (areas) which can ensure increment in profits of the hypothetical monopolist once the latter undertakes a small but significant and long-term increase in prices for the mentioned products. This price growth can be considered as an increase in prices for all products of the subgroup by α percent, where $\alpha = 10\%$.

Until recently, the market boundaries definition was based on using a wide range of different procedures. Only recently, more precise definition techniques have been developed as a result of the works on hypothetical monopolist test operationalization carried out primarily by Y. Dobbs and G. Verden. At the same time, there is still a gap between theoretical developments and practical needs. Namely, application of existing techniques requires unreally detailed data on products, cross elasticity, demand functions, etc.

The techniques described herein are based on the assumptions and suppositions.

When defining the market boundaries by the hypothetical monopolist test, one considers some maximum set of products, the various subsets of which can be considered as a market. This maximum set will be called a *maximum group*, or a *group*. Prior to the conduct of the HMT, the maximum group of products should be preselected by anti-monopoly authority experts. The group products are assumed to be interchangeable which means that raising prices for some of them will result in the consumers switching to the rest of the products of this group.

To find out whether a subgroup of products is a market or not, one has to estimate the profit gained from selling the subgroup products as well as an increment in profit resulted from raising prices for these products. In this case gained profits stand for the difference between the income and the production cost. The income is calculated by multiplying the product price by its demand, while difference between the demand and sales is not taken into account. The product production cost is assumed to be independent on the product quantity. Thus, profit can be defined as a product of the demand by difference between the price and the production cost.

The price growth means a gradual (proportional) increase in prices for all products within the subgroup.

A customer is generally considered as a rational subject as well as a source of information for taking decisions in the field of market boundaries definition. Each customer may buy one or more products of the maximum group. If prices for the products the customer buys are raised, he may either switch to other products, or leave the market and reduce purchase volume of the products which are getting more expensive, or give no response to the price increase, or resort to a combined strategy. Customer's rational behavior consists in the fact that, e.g., if initially the customer preferred the goods defined as G_1 to the goods defined as G_2, after a price increase this preference will remain. The customer is an information source as during the survey he performs as a responder describing his behavior when the prices are raised for this or that subgroup of goods.

The above definition of the HMT implies that in order to identify the market boundaries the following steps will be taken:

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1) to perform a hypothetical increase in prices for the group of products and areas (i.e. the group which is a candidate for the relevant market) and calculate the increment in profit received by the hypothetical monopolist due to the price growth;

2) to perform a hypothetical increase in prices for all subsets within the group – a candidate for the relevant market, and calculate the corresponding values of increment in profit received by the hypothetical monopolist as a result of the above increase in prices;

3) to define a subgroup (including a group) meeting the conditions set by the HMT definition.

Thus, to ensure completeness of the research, one has to find out the consequences which may be caused by various hypothetical increases in prices. As these consequences are predicted by conducting surveys among the customers (or both sellers and customers) of the relevant goods followed by analysis of the survey results, the completeness of the research can hardly be achieved. In order to gather full information one has to find out customers' possible reactions to any possible variant of price growth. If the group consists of K products circulating within the same territory, one can identify 2^K-1 of product subgroups. If there are N different areas (zones) of goods circulation, the number of options increases up to 2^{KN}-1. The customers survey aimed at learning their possible reactions to such a high number of scenarios will lead to unjustifiably excessive pressure on the respondent at the best case, which may cause low reliability of survey results or may turn out to be impossible at the worst.

Therefore, the techniques described herein are based on the following strategy. The customers are surveyed as to their reactions for several price increase scenarios, which allow reliable approximation of the rest of possible scenarios. This information is enough to examine certain subgroups – candidates for the relevant market. To examine other subgroups a hypothesis is formulated concerning behavior of customers if prices go up for these subgroups.

This main purpose of this work is to desctibe three HMT techniques.

The first two of them, the so called "individual customer preference analysis" and "customer transport cost analysis" are intended to define product and geographical market boundaries correspondingly. These approaches are based on the practical experience of the anti-monopoly authority. The individual preference analysis requires conduct of a customer survey, while the transport cost analysis may use the existing well-known economic data on transport costs.

It is worth mentioning that the first one can also be applied for defining geographical boundaries, and the second – for determining the product market boundaries (provided the information is available on the costs associated with the customers switching from one type of goods to another).

The third approach, the so called "demand linear interpolation technique" is intended for preliminary assessment of the market situation. It is useful in case every customer purchases several goods from the group subject to examination. The customers are asked minimum set of questions aimed at establishing their reaction on increase in price for a single or for all goods of the group under analysis. These two cases are somewhat extreme in a certain sense. The increment in profit caused by increase in prices for the subgroup is evaluated by means of interpolation using the hypothesis that the demand fall depends on "economic freedom" enjoyed by the customers, i.e. on customers' ability to switch to other substitute goods.

II. The Technique based on Customer Individual Preferences Analysis ("Microapproach")

This section focuses on the market product boundaries definition based on the analysis of results of the customer survey. This approach is recommended to be applied when each customer purchases a single product (or two products in the extreme case). This technique is called "microapproach" since the point of gravity in this case is shifted to the microlevel, i.e. to preference analysis of a particular customer. Therefore, during the survey the number of questions in a questionnaire rises along with pressure exercised on a respondent. Moreover, the customer who buys a couple of products has to fill in a separate questionnaire for each procured product. Thus, this technique is recommended for the cases of high customer specialization.

We will rely on the following generally acknowledged definition of the market.

Let us consider the certain set of goods S_0. Its subset S is a market in the sense of α%-Hypothetical Monopolist Test, provided the following conditions are met.

1. A hypothetical company which is a single seller of subset S goods can raise price for these goods by α% thus gaining increment in profit.

2. For any subset S_1 within set S (where S_1 is strictly less than S itself) the hypothetical company which is a single seller of S_1 subset goods can not raise the price for the said goods by α% thus gaining increment in profit resulting from this price increase.

According to the offered technique, once experts have defined a list of relevant goods, each customer who bought these goods has to be interviewed. The questionnaire includes the following questions.

1. What goods out of those specified the list (and how many of them) do you buy?

2. What goods out of those specified the list will you switch to if prices for the goods you normally buy rise to an unacceptable extent?

3. Please, indicate your reaction if prices for the goods you normally buy rise by 10%.

- switch to the goods mentioned in the response to Question 2 (if you switch to a number of goods, please indicate the proportion);
- stop buying the goods specified in the list;
- reduce the purchase volume of the said goods (by how many percents?);
- will proceed buying the same volume.

4. Please, indicate your reaction if prices for all listed goods rise by 10%.

- stop buying the listed goods;
- reduce (by certain percent) purchase volume of the said goods;

- will proceed buying the same volume.

When processing the questionnaires, the customers should be distributed between the «+» marked cells of the Table. In this manner the buyers are distributed between the subgroups of goods according to their responses.

Further on, "number of buyers" will mean the purchase volume. For example, a person who buys 100 product units per month is considered as 100 customers.

Let us demonstrate the microapproach technique on the example of the following three goods.

TABLE I
THE SETS OF CUSTOMERS

	S1	S2	S3	S4	S5	S6	S7
G1	+			+	+		+
G2		+		+		+	+
G3			+		+	+	+

Let N_1 be a number of customers who can satisfy their needs by G_1 good (product) only (namely those who replied in the questionnaire that: 1) they buy product G_1, 2) they can not replace it by any other goods), and let S_1 stand for a set of these customers. Designations N_2 , N_3 , S_2 , S_3 will have the corresponding meanings.

Let N_4 be a number of customers who can satisfy their needs by either product G_1 or G_2 (namely those who either buy product G_1 and can replace it by product G_2 or vice versa). S_4 will stand for a set of these buyers. N_i , S_i will have the corresponding meanings, while $i = 5, 6, 7$ as per the Table I.

Let Ω_j stand for the set of goods purchased by customers of group S_j and β_{ij} stand for the part of customers belonging to group S_j but buying good G_i .

Thus, $\sum_{i \in \Omega_j} \beta_{ij} = 1$.

In the following table each filled cell reads the corresponding number of customers.

TABLE II
NUMBERS OF CUSTOMERS

	S1	S2	S3	S4	S5	S6	S7
G1	N_1			$\beta_{14}N_4$	$\beta_{15}N_5$		$\beta_{17}N_7$
G2		N_2		$\beta_{24}N_4$		$\beta_{26}N_6$	$\beta_{27}N_7$
G3			N_3		$\beta_{35}N_5$	$\beta_{36}N_6$	$\beta_{37}N_7$

For example, let $\beta_{14}N_4$ stand for a number of customers who purchase the first product, however who think that they could switch to the second good (if, e.g., prices for the first one increase fivefold). And vice versa, let $\beta_{24}N_4$ stand for the number of customers who purchase the second product, however who assume they could switch to the first product (if, e.g., prices for the second one increase fivefold. Altogether (let their total number be N_4) they be a group of customers S_4 who consider (this is our model of a customer) the first and the second goods interchangeable. In other words, these customers use those properties of the first and second goods, which are characteristic of both of them.

Example.

Let us consider the set consisting of goods G1,G2,G3 where each product can be used for a number of purposes.

Depending on the purpose of buying product G1 the customer:

- either can not replace it (let this type of customers form group S1, where N_1 will stand for the group numerical strength).
- or can replace it by product G2 only (let this kind of customers form group S4, where $\beta_{14}N_4$ will stand for the group numerical strength).
- or can replace it by product G3 only (let this kind of customers form group S5, where $\beta_{15}N_5$ will stand for the group numerical strength).
- or can replace it both by product G2 and G3 (let this kind of customers form group S7, where $\beta_{17}N_7$ will stand for the group numerical strength.

At the same time we interview customers who buy product G2. A part of respondents will say that they are ready to switch to product G1 if prices for product G2 climb to an unacceptable level. These respondants belong to group S4, while $\beta_{24}N_4$ stand for this group numerical strength. Thus, S4 group including N_4 members (remember that $\beta_{14} + \beta_{24} = 1$) stands for the customers who use those consumer properties of products G1,G2 which are characteristic of both of them and buy these products according to some preferences unknown to us.

So, expressions for the demand for each mentioned product will look as follows:

$$q_1^0 = N_1 + \beta_{14}N_4 + \beta_{15}N_5 + \beta_{17}N_7$$

$$q_2^0 = N_2 + \beta_{24}N_4 + \beta_{26}N_6 + \beta_{27}N_7$$

$$q_3^0 = N_3 + \beta_{35}N_5 + \beta_{36}N_6 + \beta_{37}N_7$$

All values making up this formula are defined as a result of processing the questionnaires filled in by customers.

By answering the third question stated in the questionnaire the respondent provides information on how he would react on 10%-increase in prices for the goods which he usually buys.

Let γ_{ij} (calculated from the number of group S_j customers who buy product G_i (i.e. from $\beta_{ij}N_j$)) be the part of those respondents who reply they will continue to buy product G_i.

Let ν_{ij} be the part of those customers who reply they will stop buying the goods of the group under analysis.

Thus, the part of those who will switch to other goods of group Ω_j will be $1-\gamma_{ij}-\nu_{ij}$.

By answering the fourth question of the questionnaire the respondent provides information on how he would react on 10%-increase in prices for all listed goods.

Let ν_{ijm} (the value calculated from the number of group S_j customers who buy product G_i (i.e. from $\beta_{ij}N_j$) stand for the part of those who reply they will stop buying product G_i.

Calculation of increment in profit for minimum subgroups.

Let us consider the case of increase in prices for subgroup $\{G_1\}$.

Let us consider the situation when the price for a single product, e.g. G_i, was increase by 10%.

After the price increase for G_1, the value of the demand will be as follows:

$$q_1 = \gamma_{11}N_1 + \beta_{14}\gamma_{14}N_4 + \beta_{15}\gamma_{15}N_5 + \beta_{17}\gamma_{17}N_7$$

$$q_2 = N_2 + \beta_{24}N_4 + \beta_{26}N_6 + \beta_{27}N_7 + \left(1-\gamma_{14}-\nu_{14}\right)\beta_{14}N_4 + \omega_{12}\left(1-\gamma_{17}-\nu_{17}\right)\beta_{17}N_7$$

$$q_3 = N_3 + \beta_{35}N_5 + \beta_{36}N_6 + \beta_{37}N_7 + \left(1-\gamma_{15}-\nu_{15}\right)\beta_{15}N_5 + \omega_{13}\left(1-\gamma_{17}-\nu_{17}\right)\beta_{17}N_7$$

Where ω_{1i} stands for the part of those who will switch from G_1 (from the number of those who could switch from G_1) to both G_i, G_2 and G_3. In this case $\omega_{12}+\omega_{13}=1$. It should be noted, that to calculate increment in profit for product G_1 one is not required to know these values.

Having established the expression for demand for G_1 after the price increase, the expression for profit change can also be easily defined.

Before the price increase the profit for product G_1 was the following:

$$\Pi_1^0 = q_1^0\left(p_1^0 - c_1\right),$$

after the price increase -

$$\Pi_1 = q_1\left(p_1 - c_1\right).$$

Thus, the increment in profit is as follows:

$$\Delta\Pi = \Pi_1 - \Pi_1^0$$

The increment in profit for the cases of increase in prices for subgroups $\{G_2\}$, $\{G_3\}$ is considered in the same way.

Let us consider the price increase for subgroup $\{G_1, G_2\}$.

To avoid the excessive pressure on the respondent we have to adopt the certain model of a customer, i.e. to establish a hypothesis regarding his behavior. Within the offered approach the following buyer's behavior is assumed.

Let us calculate demand for G_1 product after the increase in prices for two products: G_1 and G_2.

The customers belonging to group S_1 are indifferent towards increase in prices for product G_2. Therefore, after the price increase for subgroup $\{G_1, G_2\}$, $\gamma_{11}N_1$ will remain for the buyers of product G_1 from this group, as if it is the case when prices increase for product G_1 only.

The same considerations are applicable to customers of group S_5. After the increase in price for product G_1, only $\gamma_{15}\beta_{15}N_5$ customers will be available.

Now let us consider the customers of group S_4

If prices for all three products of the maximum group go up, a part of S_4 group customers defined as ν_{14m} will leave the market. These customers view increase in prices for all products in the same way as increase in prices for products G_1 and G_2. That will be the part of customers who will leave the market in case of the said increase in prices for products G_1 and G_2. One should take into account that in case of this price increase, it will be unreasonable for the remaining customers to switch to G_2. Therefore, the part defined as $1-\nu_{14m}$ will still demand for the first product. Thus, a number of S_4 group customers defined as $\left(1-\nu_{14m}\right)\beta_{14}N_4$ will remain loyal to product G_1.

Let us consider now customers of group S_7

Those customers who would proceed buying product G_1 in case of an increase in prices for this product only (they total $\gamma_{17}\beta_{17}N_7$) will definitely remain loyal to this product if prices rise for both products. Those customers who, in case of an increase in prices for product G_1 only would either start buying product G_3 or leave the market, would behave in the same way if prices rise for both products.

Let us now consider those customers who in case of an increase in prices for product G_1 only would switch to product G_2. If prices rise for both products they could behave in either way: i.e. they could either remain loyal to G_1 or switch to G_3 or leave the market at all. *To elaborate the approximate solution it is hypothesized that in case of an increase in prices for both* G_1 *and* G_2 *a part of these customers will remain loyal to* G_1 *while the other part will quit (by either switching to* G_3 *or leaving the market); under* *this scenario the ratio of both parts will be equal to the ratio of parts of other customers who buy product G_1 from the same group of cell S_7 distributed in two parts*. Thus, we assume a certain homogeneity among the interviewed customers.

The number of customers who in case of an increase in prices for G_1 only switch to G_2 will be $\omega_{12}\left(1-\gamma_{17}-\nu_{17}\right)\beta_{17}N_7$. The rest of the buyers divide between those who "remain loyal to G_1" and "leave G_1" according to the ratio $\gamma_{17}/\left[\left(1-\omega_{12}\right)\left(1-\gamma_{17}-\nu_{17}\right)+\nu_{17}\right]$. Therefore, from this group of customers the following number will remain loyal to G_1

$$\begin{gathered}\omega_{12}\left(1-\gamma_{17}-\nu_{17}\right)\beta_{17}N_7\gamma_{17}/\{\gamma_{17}+\\ \left(1-\omega_{12}\right)\left(1-\gamma_{17}-\nu_{17}\right)+\nu_{17}\}\end{gathered}.$$

(provided both the fraction nominator and denominator are equal to zero, which means that all respondents said they would switch to G_2 if prices rise for G_1 only, let us accept that the part of customers defined as γ_{15} will remain loyal to G_1).

Thus, the demand for product G_1 after the increase in prices for subgroup $\{G_1, G_2\}$ will be expressed as follows.

$$\begin{aligned}q_1 &= \gamma_{11}N_1+\left(1-\nu_{14m}\right)\beta_{14}N_4+\gamma_{15}\beta_{15}N_5+\gamma_{17}\beta_{17}N_7+\\ &\omega_{12}\left(1-\gamma_{17}-\nu_{17}\right)\beta_{17}N_7\times\\ &\frac{\gamma_{17}}{\gamma_{17}+\left[\left(1-\omega_{12}\right)\left(1-\gamma_{17}-\nu_{17}\right)+\nu_{17}\right]}\end{aligned}$$

The demand for product G_2 will be calculated in the same manner.

$$\begin{aligned}q_2 &= \gamma_{22}N_2+\left(1-\nu_{24m}\right)\beta_{24}N_4+\gamma_{26}\beta_{26}N_6+\gamma_{27}\beta_{27}N_7+\\ &+\omega_{21}\left(1-\gamma_{27}-\nu_{27}\right)\beta_{27}N_7\times\\ &\frac{\gamma_{17}}{\gamma_{17}+\left[\left(1-\omega_{21}\right)\left(1-\gamma_{27}-\nu_{27}\right)+\nu_{27}\right]}\end{aligned}$$

The profit for subgroup $\{G_1, G_2\}$ before the price increase was as follows:

$$\Pi^0 = q_1^0\left(p_1^0-c_1\right)+q_2^0\left(p_2^0-c_2\right),$$

after the price increase:

$$\Pi = q_1\left(p_1-c_1\right)+q_2\left(p_2-c_2\right).$$

The increment in profit will be:

$$\Delta\Pi = \Pi-\Pi^0$$

The increase in prices for subgroups $\{G_1, G_3\}$ and $\{G_2, G_3\}$ are considered in the same way.

Let us consider the increase in prices for subgroup $\{G_1, G_2, G_3\}$.

It is evident that after the increase in prices for subgroup $\{G_1, G_2, G_3\}$ the demand for each product will be defined as follows

$$\begin{aligned}q_1 &= \gamma_{11}N_1+\left(1-\nu_{14m}\right)\beta_{14}N_4+\\ &\left(1-\nu_{15m}\right)\beta_{15}N_5+\left(1-\nu_{17m}\right)\beta_{17}N_7\\ q_2 &= \gamma_{22}N_2+\left(1-\nu_{24m}\right)\beta_{24}N_4+\\ &\left(1-\nu_{26m}\right)\beta_{26}N_6+\left(1-\nu_{27m}\right)\beta_{27}N_7\\ q_3 &= \gamma_{33}N_3+\left(1-\nu_{35m}\right)\beta_{35}N_5+\\ &\left(1-\nu_{36m}\right)\beta_{36}N_6+\left(1-\nu_{37m}\right)\beta_{37}N_7\end{aligned}$$

Before the increase in prices for subgroup $\{G_1, G_2, G_3\}$ the profit for this subgroup amounted to

$$\Pi^0 = q_1^0\left(p_1^0-c_1\right)+q_2^0\left(p_2^0-c_2\right)+q_3^0\left(p_3^0-c_3\right),$$

after the price increase the profit is defined as follows -

$$\Pi = q_1\left(p_1-c_1\right)+q_2\left(p_2-c_2\right)+q_3\left(p_3-c_3\right).$$

The increment in profit will be

$$\Delta\Pi = \Pi-\Pi^0$$

This is the way how increment in profit is calculated in case of an increase in prices for each subgroup.

Microapproach technique application example.

Considered here is the maximum group consisting of three products. The production cost of each product amounts to 4.3 rubles, while the selling price makes up 5 rubles. To define the market boundaries we surveyed 17 customers who were asked the questions listed in the questionnaire mentioned above.

The customers' replies were as follows:

TABLE III
CUSTOMER'S ANSWERS (EXAMPLE)

Customer	Purchased product, number	Under the scenario of unacceptable price increase	Under the scenario of 10%-increase in prices for the purchased product only	Under the scenario of 10%-increase in prices for all listed goods
1	G1 – 100 rubles	Can replace it by product G3	Reduce purchase volume by 10%	Reduce purchase volume by 10%
2	G1 – 500 rubles	Can replace it by either product G2 or G3	Switch to G3	Leave the market
3	G1 – 300	Cannot	Proceed	Proceed

	rubles	replace it	buying in the same amount	buying in the same amount
4	G2 – 300 rubles	Can replace it by product G1	Switch to G1	Proceed buying in the same amount
5	G1 – 200 rubles	Cannot replace it	Leave the market	Leave the market
6	G3 – 700 rubles	Can replace it by product G2	Proceed buying in the same amount	Proceed buying in the same amount
7	G3 – 300 rubles	Can replace it by product G2	Switch to G2	Proceed buying in the same amount
8	G2 – 100 rubles	Can replace it by product G3	Reduce purchase volume by 10%	Reduce purchase volume by 10%
9	G1 – 400 rubles	Can replace it by either product G2 or G3	Proceed buying in the same amount	Proceed buying in the same amount
10	G3 – 100 rubles	Cannot replace it	Leave the market	Leave the market
11	G2 – 300 rubles	Can replace it by either product G1 or G3	Switch to G3	Leave the market
12	G1 – 200 rubles	Can replace it by product G2	Switch to G2	Reduce purchase volume by 10%
13	G3 – 100 rubles	Can replace it by product G1	Switch to G1	Reduce purchase volume by 10%
14	G2 – 300 rubles	Can replace it by either product G1 or G3	Proceed buying in the same amount	Proceed buying in the same amount
15	G3 – 100 rubles	Cannot replace it	Reduce purchase volume by 50%	Reduce purchase volume by 50%
16	G1 – 200 rubles	Can replace it by either product G2 or G3	Reduce purchase volume by 10%	Reduce purchase volume by 10%
17	G2 – 200 rubles	Can replace it by product G3	Switch to G3	Leave the market

The following table provides information on which group each customer belongs to.

TABLE IV
GROUPS OF CUSTOMERS (EXAMPLE)

Customer	S1	S2	S3	S4	S5	S6	S7
G1	3.5			12	1		2,9.16
G2		+		4		8.17	11.14
G3			10.15		13	6.7	+

Let us take values of N_j and β_{ij}

$N_1 = 100$, $N_2 = 0$, $N_3 = 40$, $N_4 = 100$,

$\beta_{14} = 0,4$, $\beta_{24} = 0,6$, $N_5 = 40$ $\beta_{15} = 0,5$, $\beta_{35} = 0,5$

$N_6 = 260$, $\beta_{26} = 3/13 = 0,23$, $\beta_{36} = 10/13 = 0,77$

$N_7 = 340$, $\beta_{17} = 11/17 = 0,65$, $\beta_{27} = 6/17 = 0,35$,

$\beta_{37} = 0$

TABLE V
NUMBERS OF CUSTOMERS IN EACH SUBGROUP

$\beta_{ij} N_j$	S1	S2	S3	S4	S5	S6	S7
G1	100			40	20		220
G2		+		60		60	120
G3			40		20	200	+

The values of demand for each of three products will be defined as follows (sum up elements of each line)

$$q_1^0 = 380,\ q_2^0 = 240,\ q_3^0 = 260$$

The profit for each product will be as follows (the demand is multiplied by the difference between the selling price and the production cost).

$$\Pi_1^0 = 380(5-3) = 760$$

$$\Pi_2^0 = 240(5-3) = 480$$

$$\Pi_3^0 = 260(5-3) = 520$$

Now let us calculate parameters γ_{ij} (calculated from the number of S_j group customers who buy product G_i (i.e. from $\beta_{ij} N_j$), namely the part of those who replied they would proceed buying product G_i) as well as ν_{ij}, namely the part of those customers who would stop buying the products belonging to the set of products under analysis.

Now let us calculate γ_{11} and ν_{11}.

Out of 100 customers of subgroup G1S1:

60 will remain loyal (Customer3) while 40 will leave the market (Customer5).

Thus, $\gamma_{11} = 0,6$ and $\nu_{11} = 0,4$ (equation $\gamma_{11} + \nu_{11} = 1$ reflects the fact that one can switch to nothing).

In the same way we proceed calculating γ_{33} and ν_{33}.

As a result we get the following $\gamma_{33} = 0,25$ and $\nu_{33} = 0,75$.

Let us calculate γ_{14} and ν_{14}.

Out of 40 customers of subgroup G1S4 all 40 will switch to product G2 (Customer12).

Thus, $\gamma_{14} = 0$ and $\nu_{14} = 0$.

In the same way we proceed calculating other values γ_{ij} and ν_{ij}.

The received data are presented in the Tables VI and VII.

TABLE VI

VALUES OF γ_{ij}

γ_{ij}	S1	S2	S3	S4	S5	S6	S7
G1	0.6			0	0.9		0.53
G2		0		0		0.3	0.5
G3			0.25		0	0.03	0

TABLE VII

VALUES OF ν_{ij}

ν_{ij}	S1	S2	S3	S4	S5	S6	S7
G1	0.4			0	0.1		0.02
G2		0		0		0.03	0
G3			0.75		0	0	0

Now, let us consider the answers to the fourth question of the questionnaire and calculate ν_{ijm}. It stands for the part of those from the cell who will leave the market in case of a price increase for all three products.

Let us calculate ν_{11m}. Customer 3 (60 customers) will remain loyal, while Customer 5 (40 customers) will leave the market. Therefore, $\nu_{11m} = 0,4$

In the same way we calculate ν_{33m}, as a result we will get the following $\nu_{33m} = 0,75$

Now, let us calculate ν_{14m}. Customer12 (40 customers) will reduce the purchase volume by 10%. Therefore, $\nu_{14m} = 0,1$.

In the same way let us proceed calculating other values ν_{ijm}. The resulting data are shown in the Table IX.

TABLE VIII

VALUES OF ν_{ijm}

ν_{ijm}	S1	S2	S3	S4	S5	S6	S7
G1	0.4			0.1	0.1		0.47
G2		+		0		0.7	0.5
G3			0.75		0.1	0	+

Let us proceed to calculation of ω_{ij}

First, out of those customers who switch to other products, let us calculate ω_{12} - the part of customers who switch to product G1 of group S7 and who will switch to product G2. Since Customer 2 switches to product G3, $\omega_{12} = 0$.

Therefore, $\omega_{13} = 1$.

Let us calculate ω_{21}. As Customer 2 switches to product G3, $\omega_{21} = 0$. Therefore, $\omega_{31} = 1$

Now, let us proceed to calculation of the increment in profit for each of the subgroups.

Let us start with subgroup {G1}

We assume that the price was raised by 10% for product G1 only which costs now 5.5 rubles.

Now we calculate the demand for G1.

$$q_1 = \gamma_{11}N_1 + \gamma_{14}\beta_{14}N_4 + \gamma_{15}\beta_{15}N_5 + \gamma_{17}\beta_{17}N_7$$

$$q_1 = 0,6*100 + 0*40 + 0,9*20 + 0,53*220 = 194,6$$

The new profit will be

$$\Pi_1 = 194,6*(5,5-4,3) = 233,5.$$

Since $\Pi_1^0 = 380*(5-4,3) = 266$, $\Delta\Pi < 0$, then *set {G1} (including single product G1 only) is not a market.*

In the same way we proceed to consider subgroups {G2} and {G3} which lead to a conclusion that neither of them is a market.

Now let us consider subgroup {G1,G2}

Let us assume that the prices were raised by 10% for products G1,G2. After the increase they cost 5.5 rubles.

Let us calculate the new value of demand for G1.

$$q_1 = \gamma_{11}N_1 + \left(1-\nu_{14m}\right)\beta_{14}N_4 + \gamma_{15}\beta_{15}N_5 + \gamma_{17}\beta_{17}N_7 + \\ + \omega_{12}\left(1-\gamma_{17}-\nu_{17}\right)\beta_{17}N_7 \times \\ \frac{\gamma_{17}}{\gamma_{17} + \left[\left(1-\omega_{12}\right)\left(1-\gamma_{17}-\nu_{17}\right)+\nu_{17}\right]}$$

$$q_1 = 0,6*100 + \left(1-0,1\right)*40 + 0,9*20 + \\ 0,53*220 + 0 = 230,6$$

Let us calculate the new value of demand for product G2.

$$q_2 = \gamma_{22}N_2 + \left(1-\nu_{24m}\right)\beta_{24}N_4 + \gamma_{26}\beta_{26}N_6 + \gamma_{27}\beta_{27}N_7 + \\ + \omega_{21}\left(1-\gamma_{27}-\nu_{27}\right)\beta_{27}N_7 \times \\ \frac{\gamma_{17}}{\gamma_{17} + \left[\left(1-\omega_{21}\right)\left(1-\gamma_{27}-\nu_{27}\right)+\nu_{27}\right]}$$

$$q_2 = 0 + \left(1-0\right)*60 + 0,3*60 + 0,5*120 + 0 = 138$$

Prior to the price increase, the profit for subgroup $\{G_1, G_2\}$ amounted to

$$\Pi^0 = 380\left(5-4,3\right) + 240\left(5-4,3\right) = 434,$$

after the increase the profit can be calculated as follows

$$\Pi = 230,6\left(5,5-4,3\right) + 138\left(5,5-4,3\right) = 442,3$$

The increment in profit will be

$$\Delta\Pi = 442,3 - 434 = 8,3$$

The increment in profit turns out to be positive. Therefore, subgroup $\{G_1, G_2\}$ does form a market.

III. DEFINITION OF MARKET GEOGRAPHICAL BOUNDARIES BASED ON CUSTOMER TRANSPORT COSTS ANALYSIS

The present Section is focused on presenting the technique for definition of market geographical boundaries which is based on the customer costs analysis. The basic idea of the technique consists in predicting the behavior of each single customer to price variations during estimation of the hypothetical monopolist profit change. At the same time, consideration is given to the customer expenses originating from the purchase value and transport costs.

The following behavior is attributed to the customer within the framework of the present approach. In case he encounters a price increase, all of his demand will either be switched to the sales point with invariable price level or will remain the same (it is noteworthy that more complicated behavior assuming the possibility for decrease in demand or its partial transfer to a different sales point is subject to the customer survey). Herein, the switching of demand actually results in the increase of the customer transport costs. Therefore, when encountering the price increase, each specific customer is facing the following alternative: purchase value increase or transport costs increase.

The technique assumes that the analyst resolves this alternative on the basis of the data available by choosing the least expensive option for the customer. Summarized data on all customers are used as a basis for definition of the price change occurred as a result of the hypothetical price increase. This provides for definition of the hypothetical monopolist profit change.

The present technique assumes there is some set of sales points, and the problem reduces to separating its subset which forms the market.

We proceed from the following market definition.

Some set S_0 of this product's sales points is taken for consideration. Its subset S is called a market in the sense of an a% hypothetical monopolist test in case the following conditions are met.

1. A hypothetical firm being the only seller of the given product in the sales points of subset S, can raise the price for this product by a% in all these sales points, thus achieving a profit growth which results from the said price increase.

2. For any subset S_1 of set S (which is strictly smaller than the S proper) a hypothetical firm being the only seller of the given product in the sales points of subset S_1 cannot raise the price for this product by a% in all these sales points, thus achieving a profit growth which results from the said price increase.

The sales points shall be specified as $A_1, A_2, \ldots, A_N$.

Geographically each customer is identified with the place whereto the purchased products shall be delivered at his cost, while the sales point is identified with the warehouse wherefrom the customer collects these products. The customers shall be specified as $B_1, B_2, \ldots, B_K$.

The set of customers shall be specified as Ω_0.

The transport costs for delivery from B_i to A_j shall be specified as d_{ij}.

Practical application of the technique implies that different sets of sales points shall be tested with a view to the fact whether the price increase in all points of this set is profitable. In particular, consideration shall be given to the sets consisting of a single sales point. Although this case is of no principal difference from the general one (when the tested set consists of an arbitrary quantity of points), we shall consider it just to show the basic idea of the technique in a simpler situation.

We shall specify as $m_1, m_2, \ldots, m_k$ the purchase volumes of customers $B_1, B_2, \ldots, B_k$ in point A_1.

We assume that the hypothetical monopolist exercises control over sales point A_1.

The product price in sales point A_j amounted p_i^0; in particular, in sales point A_1 it amounted p_1^0. After the hypothetical price increase in sales point A_1, the current price in it amounts $p_1 = p_1^0(1+z)$. We shall specify as c_j the expenses (production cost) of the hypothetical monopolist per unit of the product which he sells in sales point A_j.

The initial (i.e. prior to the price increase) profit of the hypothetical monopolist collected from sales point A_1 amounted:

$$\Pi^0 = \sum_{i=1}^{k} m_i \left(p_1^0 - c_1\right) = \left(p_1^0 - c_1\right) \sum_{i=1}^{k} m_i$$

Let us consider the customer behavior in case of a price increase in sales point A_1.

Customer B_i will switch to a different sales point in case:

$$p_1 + d_{i1} > \min_{j \neq 1}\left(p_j^0 + d_{ij}\right)$$

(i.e. if there is sales point A_j, where customer expenses are less than "new" expenses in sales point A_1).

We shall specify as $\Omega\{A_1\}$ the set of the customers who will switch their demand to different sales points in case this condition is met.

The profit of the hypothetical monopolist after the price increase will amount:

$$\Pi = \sum_{\Omega/\Omega\{A_1\}} m_i \left(p_1 - c_1\right) = \left(p_1 - c_1\right) \sum_{\Omega/\Omega\{A_1\}} m_i$$

Let us now consider a more complicated case, and namely the issue of whether set T which consists of n arbitrary sales points $\{A_{j1}, A_{j2} \ldots A_{jn}\}$ can be recognized as a market?

We shall assume that the hypothetical monopolist exercises control over a set of sales points: $T=\{A_{j1}, A_{j2} \ldots A_{jn}\}$.

The initial price in sales point A_j amounted p_i^0. After the hypothetical price increase in sales point A_j the current price in it amounts $p_j = p_j^0(1+z)$. We shall specify as c_j the expenses (production cost) of the hypothetical monopolist per unit of the product which he sells in sales point A_j.

We shall specify as m_{ij} the purchase volume (in physical indicators) of the product by customer B_i, in point A_j, belonging to set T.

The initial (i.e. prior to the price increase) profit of the hypothetical monopolist collected from all sales point of tested set T amounted:

$$\Pi^0 = \sum_{i=1}^{k} \sum_{j:A_j \in T} m_{ij} \left(p_j^0 - c_j\right) = \sum_{j:A_j \in T} \left(p_j^0 - c_j\right) \sum_{i=1}^{k} m_{ij}$$

Let us find out which conditions resulting from the price increase will make one or another customer switch to a sales point not belonging to tested set T.

Let us consider customer B_i.

Customer B_i will switch his demand to a sales point not belonging to tested set T if:

$$\min_{j \in T}\left(p_j + d_{ij}\right) > \min_{j \notin T}\left(p_j^0 + d_{ij}\right)$$

The customers who will, in case this condition is met, switch their demand to the sales points not belonging to set T form set Ω{T}.

The profit of the hypothetical monopolist after the price increase will amount:

$$\Pi = \sum_{i:B_i \in \Omega/\Omega\{T\}} \sum_{j:A_j \in T} m_{ij}\left(p_j - c_j\right)$$

After calculation of value Π^0 and estimation of value Π it is easy to calculate profit change $\Delta\Pi = \Pi - \Pi^0$. If $\Delta\Pi<0$, then tested set T is not a market. And if $\Delta\Pi>0$ for set T, but $\Delta\Pi<0$ for any of its smaller subset, then the T is a market.

IV. Estimation of Market Product Boundaries Using Demand Interpolation Technique

Within the framework of this approach the customers are presented a list of relevant products and are offered to answer the following questions.

1. Which products out of those specified in the list do you buy?
2. What is your purchase volume of each of the offered products?
3. How much will this product purchase volume reduce in case of a 10% price increase for this product only (specify the purchase reduction volume in percents within the range from 0% to 100%)?
4. How much will this product purchase volume reduce in case of a 10% price increase for all products in the list (specify the purchase reduction volume in percents within the range from 0% to 100%)?

The customer with number j answers the specified questions by filling out the table.

TABLE IX
The Customer's Answers

Product	Purchase volume	Purchase reduction in case of price increase for this product only	Purchase reduction in case of price increase for all products in the list
G_1	P_{1j}	ν_{1j}	μ_{1j}
G_2	P_{2j}	ν_{2j}	μ_{2j}
G_3	P_{3j}	ν_{3j}	μ_{3j}
…	…	…	…
G_K	P_{Kj}	ν_{Kj}	μ_{Kj}

Here and further on in the expressions of type $P_{ij}, \nu_{ij}, \mu_{ij}$, the first index means the product number (from 1 to *K*), while the second one, the customer number (from 1 to *N)*.

The consolidation table is drawn on the basis of the questionnaires filled out by customers *N*.

TABLE X
The Consolidation Table

Product	Purchase volume	Purchase reduction in case of price increase for this product only	Purchase reduction in case of price increase for all products in the list
G_1	q_1^0	ν_1	μ_1
G_2	q_2^0	ν_2	μ_2
G_3	q_3^0	ν_3	μ_3
…	…	…	…
G_K	q_K^0	ν_K	μ_K

The data of the second column (purchase volume) are calculated by summing up the quantity of this product purchases effected by all customers:

$$q_i^0 = P_{i1} + P_{i2} + P_{i3} + \ldots + P_{iN}\,.$$

The data of the third and forth columns (purchase reduction shares) are calculated as weight-average shares of customers' purchase reduction with weights being equal to the purchase volumes:

$$\nu_i = \frac{\nu_{i1}P_{i1} + \nu_{i2}P_{i2} + \nu_{i3}P_{i3} + \ldots + \nu_{iN}P_{iN}}{q_i^0}$$

$$\mu_i = \frac{\mu_{i1}P_{i1} + \mu_{i2}P_{i2} + \mu_{i3}P_{i3} + \ldots + \mu_{iN}P_{iN}}{q_i^0}$$

The minimal subgroups being subject to analysis are the subgroups from a single product. For calculation of a profit change on the basis of these subgroups no additional hypotheses or assumptions are required since the customers' answers to the first question in the questionary provide the information necessary for the analysis of these subgroups.

The maximal subgroup being subject to analysis is the subgroup from all relevant products, i.e. the one coinciding with the given maximal product group. For calculation of a profit change on the basis of this subgroup no additional hypotheses or assumptions are required either since the customers' answers to the second question in the questionary provide the information necessary for the analysis of this subgroup.

In order to give a sound estimate of the increment in profit of the hypothetical monopolist in case of a price increase for different product subgroups (except for the subgroups from a single product and the maximal subgroup coinciding with the initial product group) it is necessary to formulate the fundamental behavior hypothesis which is essentially a customer rational behavior hypothesis.

Let us consider the situation when the price is raised only for product G_1. The customers will redistribute a part of their demand for other products of this group and will partially reduce purchases of this product.

On the other hand let us assume that the price is raised only for two products G_1 and G_2. In this case redistribution of demand from G_1 to G_2 will be less than in case the price is raised only for G_1 (probably there will be no redistribution between G_1 and G_2 at all and there is a possibility for the demand backflow from G_2 to G_1). At the same time redistribution of demand from G_1 to other products of this group as well as the share of the customers of product G_1 who stopped purchasing the group products due to the price increase do not depend on the fact whether the price for product G_2 has increased or not.

Thus, the decrease in demand for product G_1 in case of a price increase only for G_1 will not be less than in case of a price increase for G_1 and G_2. The more so, the wider is the product subgroup (including G_1) which is subject to the price increase, the lower is the decrease in demand for product G_1. In other words, the possibilities for the customer to switch to the consumption of a different product are governed by the freedom of choice, which in its turn is defined by the market share of the products being subject to price increase.

This assertion makes up the idea of the behavior hypothesis adopted within the framework of the present approach.

We shall formulate it as follows.

Let Ω_1 *and* Ω_2 *be some product subgroups including product* G_k. *We shall specify as* s_1 *(*s_2*) the share of the group products in the total purchase volume which is accounted for the products of subgroups* Ω_1, *(*Ω_2*), i.e.*

$$s_1 = \frac{\sum_{\Omega_1} q_i^0}{\sum_{i=1}^{N} q_i^0}, \quad s_2 = \frac{\sum_{\Omega_2} q_i^0}{\sum_{i=1}^{N} q_i^0}$$

where in the numerator of each of the expressions the summation of all products of the respective subgroup is performed.

The fundamental behavior hypothesis is formulated as follows: if $s_1 \geq s_2$, *then the decrease in demand for product* G_k *in case of a price increase for all products of subgroup* Ω_1 *will not be more than in case of a price increase for all products of subgroup* Ω_2.

In particular, decrease in demand for a certain product will be minimal in case of a price increase for all products of the considered group and maximal in case of a price increase for this product only. Therefore, if ν is a decrease in demand for certain product G_i in case of a price increase for subgroup Ω which include this product, then $0 < \mu_i < \nu < \nu_i$.

Basing on the behavior hypothesis we shall give estimation to the increment in profit of the hypothetical monopolist in case of a price increase for different product subgroups (except for the subgroups from a single product and the maximal subgroup which coincides with the initial product subgroup). In order to calculate the profit change on the basis of a certain subgroup of products in case of a price increase for them, it is necessary to calculate the demand change for each product from this subgroup.

To substantiate the behavior hypothesis we shall introduce the quantitative degree of dependence of the demand decrease on share s. And namely, *in case of a price increase for certain subgroup* Ω *let us assume that the decrease in demand for the products included in it has a linear dependence on the total sales share of products not included in this subgroup.*

In this case we shall speak about the linear interpolation of demand.

Let us specify as Δq_k the change in demand for the k^{th} product in case of a price increase for certain product subgroup Ω which include this product, while specifying as s the share in the total purchase volume of the group products which is accounted for the products of subgroup Ω:

$$s = \sum_{\Omega} q_i^0 \Big/ \sum_{i=1}^{N} q_i^0$$

We shall write down the fundamental behavior hypothesis in the following form:

$$\Delta q_k(s) = q_k^0 \left[-a(1-s) + b \right],$$

where $a > 0$ and b are some unknown for the moment coefficients.

The "minus" sign before coefficient *a* means that the change in demand is negative (according to our terminology, the change in demand is equal to its decrease which is taken with the opposite sign).

The numerical values of coefficients a, b can be defined on the basis of the consolidated questionary analysis (Ref. the Table).

In case of a price increase only for certain product G_k (i.e. when $s = q_i^0 \Big/ \sum_{i=1}^{K} q_i^0$), the change in demand is equal to $-\nu_i q_k^0$. At the same time, in case of a price increase for all products of a large group subject to the analysis (i.e. when $s = 1$), the change in demand is equal to $-\mu_i q_k^0$. Proceeding from these two statements it is easy to find the values of coefficients a, b:

$$a = \frac{\nu_i - \mu_i}{1 - q_k^0 \Big/ \sum_{i=1}^{K} q_i^0}, \qquad b = -\mu_i.$$

Therefore, the expression for the change in demand for product G_k being the result of a price increase for the products of subgroup Ω, which has share s in the total product purchase volume, has the following form:

$$\Delta q_k(s) = q_k^0 \left[\frac{v_i - \mu_i}{1 - q_k^0 \Big/ \sum_{i=1}^{K} q_i^0} (s-1) - \mu_i \right] \quad (1)$$

Let us calculate the change in profit on the basis of product G_k, which occurred as a result of the specified price increase.

We shall specify as p_k^0 the price for product G_k before the price increase, and as c_k the production cost of a unit of this product. Then the profit from the sales of product G_k before the price increase has the following form:

$$\Pi_k^0 = q_k^0 \left(p_k^0 - c_k \right),$$

while after a z-percent price increase for the products of subgroup Ω –

$$\Pi_k = \left(q_k^0 + \Delta q_k \right)\left((1+z)\, p_k^0 - c_k \right)$$

The increment in price on the basis of product G_k has the following form:

$$\Delta\Pi_k = \Pi_k - \Pi_k^0.$$

The overall increment of the hypothetical monopolist profit on the basis of product subgroup Ω is calculated by summation of increments in profit on the basis of each product from this subgroup:

$$\Delta\Pi = \sum_{k \in \Omega} \left[\left(q_k^0 + \Delta q_k \right)\left((1+z)\, p_k^0 - c_k \right) - q_k^0 \left(p_k^0 - c_k \right) \right] \quad (2)$$

The technique application algorithm stipulates that the increment in profit is calculated according to the specified formula on the basis of product subgroup Ω which is subject to analysis. If it is negative, the Ω is not a market. But if the increment in profit is positive, then the calculation of the increment in profit is performed on the basis of each of subsets Ω which include one product less than the Ω. If all these increments in profit are negative, the Ω is a market. Otherwise, the Ω is a subgroup which is larger than the market.

V. Simultaneous Estimation of Market Product and Geographical Boundaries Using Demand Interpolation Technique

The idea of market boundaries estimation using the demand interpolation technique is applied within the present Section to solution of the following problem. There are several sales points where several products are sold. It is necessary to simultaneously define the market product and geographical boundaries, i.e. the set of products and sales points which, if controlled by the hypothetical monopolist, allow him to raise the prices and achieve a profit growth. In this case the answer can have, for example, the following form: profit growth ensures control over the sales of product G1 in five sales points A1, A2, A3, A4, A5 and control over the sales of products G1, G2 in three sales points A1, A2, A3 as well as control over sales of products G1, G2, G3 in one sales point A1, etc.

The information required for application of the technique shall be obtained through the customer survey. The questionary shall describe a variety of products and a variety of sales points and shall include the following questions.

1. Which products, in which sales points and in what quantity do you buy?

After that the respondent answers questions with regard to each product and each sales point.

2. If the price for this product in this sales point has increased by 10% how much will the reduction of this product purchase in this point will amount?

3. If the price for all products in this sales point has increased by 10% how much will the reduction of these products purchase in this point will amount?

4. If the price for this product in all sales points has increased by 10% how much will the reduction of this product purchase in this point will amount?

Let us assume it is necessary to estimate the change in demand for a certain product in a certain sales point in case of a 10% price increase for a certain product group in certain sales points (including the product which is subject to demand change estimation). We shall use the idea of performing the estimation of the basis of the demand linear interpolation.

During estimation of only market product boundaries the change in demand for a product was considered as a linear function of one variable, and namely, the share of the group (subject to the price increase) in the total sales volume.

During simultaneous estimation of the market product and geographical boundaries we consider the change in demand for product G_j in point A_i as a linear function of two variables, and namely the (*) share of the product group (which include product G_j) in the total sales volume in point A_i and the (**) share of the point group (which include point A_i) in the total sales volume of product G_j.

Thus, the change in demand (and consequently the demand proper) is shown geometrically as a plane of three space variables. The vertical axis has a meaning of the demand change proper, while the horizontal axes mean the respective shares.

Answers to questions 2 through 4 (summated for all customers) provide for drawing three points which define the required plane.

The plane which is drawn on the basis of these three points is actually a two-dimensional linear interpolation of demand. It is used for estimation of a decrease in demand for the given product in the given sales point.

So let us assume that it is necessary to estimate the change of a seller's profit in case of a price increase for certain products in certain sales points. The decrease in demand is estimated for each product included in that group and for each of its sales points using the specified technique. After that, using the standard technique, we shall perform calculation of the profit change on the basis of this product in this point, etc.

Let us perform particular calculation for the following example.

In each of three sales points A1, A2, A3 products G1, G2 are sold. One unit of product G1 from the consumer's point of view is equal to one unit of product G2 (i.e. if the products are substitutional for a certain consumer, he will replace one unit of one product by one unit of other product). The production cost of each product amounts c=4 rubles for the seller, while the sale price is equal to p^0=10 rubles.

In sales point A1 10 units of product G1 (for convenience we shall call the relevant customers as B1) and 10 units of product G2 are sold per month (for convenience we shall call the relevant customers as B4), etc. The respective data are summarized in the Table XII where each cell contains the name of the customers and the sales volume.

TABLE XI
THE SALES VOLUMES

	G1	G2
A1	B1, 10	B4, 10
A2	B2, 20	B5, 20
A3	B3, 20	B6, 20

The survey of the customers gave the following results.

TABLE XII
THE SAMPLE DATA

Customer category	Decrease in demand in case of price increase (in %)		
	Only for the given product in the given point	For both products in the given point	For the given product in all the three points
B1	40	20	10
B2	40	20	20
B3	40	10	20
B4	40	20	10
B5	40	10	20
B6	40	20	20

We shall draw a linear function of decrease in demand for customers B1. Let us specify the share of demand decrease as z ($0 < z \le 1$), the share of the product group in sales point A1 as x ($0 < x \le 1$) and the share of the product group in the total sales volume of product G1 as y ($0 < y \le 1$). The linear function of decrease in demand is defined as a plane passing through three points:

$$x = 0,5;\ y = 0,2;\ z = 0,4$$

(in case of a price increase only for product G1 only in sales point A1, the share of the product in the point amounts 0.5 (10 out of 20), the share in the total sales volume of product G1 amounts 0.2 (10 out of 50), while the decrease in demand amounts 40%);

$$x = 0,5;\ y = 1;\ z = 0,1$$

(in case of a price increase only for product G1 in all sales points, the share of the product in point A1 amounts 0.5 (10 out of 20), the share in the total sales volume of product G1 amounts 1 (50 out of 50), while the decrease in demand amounts 10%);

$$x = 1;\ y = 0,2;\ z = 0,2$$

(in case of a price increase for both products only in sales point A1, the share of the product in the point amounts 1 (20 out of 20), the share in the total sales volume of product G1 amounts 0.2 (10 out of 50), while the decrease in demand amounts 20%).

Using a well-known mathematical formula we shall draw an equation of the plane passing through the three points specified:

$$\begin{vmatrix} x-0,5 & y-0,2 & z-0,4 \\ 0,5-0,5 & 1-0,2 & 0,1-0,4 \\ 1-0,5 & 0,2-0,2 & 0,2-0,4 \end{vmatrix} = 0.$$

After simple transformations we obtain a linear function equation for a decrease in demand of customers B1:

$$0,16x + 0,15y + 0,4z - 0,27 = 0.$$

Using the similar procedure we obtain a linear function equation for a decrease in demand of other customers:

B2: $0,12x + 0,1y + 0,3z - 0,22 = 0$.

B3: $0,18x + 0,1y + 0,3z - 0,25 = 0$.

B4: $0,16x + 0,15y + 0,4z - 0,27 = 0$.

B5: $0,18x + 0,1y + 0,3z - 0,25 = 0$.

B6: $0,12x + 0,1y + 0,3z - 0,22 = 0$.

Using the two found linear functions we shall test different product groups and sales points.

Let us, for example, consider product A1 and sales point G1. Is it possible for a hypothetical firm being the only seller of this product in this point to raise the price by 10% to obtain additional profit?

The initial demand is 10 units, the initial price is 10 rubles, while the production cost is 4 rubles. The seller's profit is therefore:

Π=10*(10-4)=60 rubles.

After a hypothetical price increase for product G1 of up to 11 rubles the demand will decrease by 40% and will amount 6 units. The profit will turn out to be equal to:

Π=6*(11-4)=42 rubles.

The decrease in demand testifies to the fact that product G1 in sales point A1 is not a market.

Let us now consider a set containing both products which are sold in sales point A1.

The initial profit of the seller amounts:

Π=20*(10-4)=120 rubles.

After the hypothetical price increase for both products of up to 11 rubles the demand of customers B1 for the first product will decrease by 20% and will amount 8 units, while the demand of customers B4 for product G1 will also decrease by 20% and will amount 8 units. The profit will appear to be equal to:

Π=16*(11-4)=112 rubles.

The decrease in demand testifies to the fact that sales point A1 is not a market.

Let us consider product G1 which is sold in all the three points.

The initial seller's profit amounts:

Π=50*(10-4)=300 rubles.

After the hypothetical price increase for product G1 in all the three points of up to 11 rubles the demand of customers

B1 will decrease by 10% and will amount 9 units, the demand of customers B2 in sales point A2 will decrease by 20% and will amount 16 units, while the demand of customers B3 in sales point A3 will decrease by 20% and will also amount 16 units. The profit will appear to equal to:

П=41*(11-4)=287 rubles.

The decrease in demand testifies to the fact that product G1 does not form a market.

For analysis of the above mentioned market candidates no interpolation was required. We will demonstrate the way it is used by the example of the following two market candidates.

Let us consider a set consisting of both products which are sold in sales point A2 and of product G1 which is sold in sales point A3.

The initial seller's profit amounts:

П=60*(10-4)=360 rubles.

After the hypothetical price increase of up to 11 rubles the demand of customers B2 for the first product will be defined using the following equation:

B2: $0,12x + 0,1y + 0,3z - 0,22 = 0$.

Where x=1 (40 units out of 40), y=0.8 (40 units out of 50). Having solved the equation we will find that z=0.07, i.e. the demand of customers B2 will decrease by 7% and will amount 18.7 units.

The demand of customers B3 for the first product will be defined using the following equation:

B3: $0,18x + 0,1y + 0,3z - 0,25 = 0$.

Where x=0.5 (20 units out of 40), y=0.8 (40 units out of 50). Having solved the equation we will find that z=0.27, i.e. the demand of customers B3 will decrease by 27% and will amount 14.7 units.

The demand of customers B5 for product G2 will be defined using the following equation:

B5: $0,18x + 0,1y + 0,3z - 0,25 = 0$.

Where x=1 (40 units out of 40), y=0.4 (20 units out of 50). Having solved the equation we will find that z=0.1, i.e. the demand of customers B5 will decrease by 10% and will amount 18 units. The overall demand in the considered group will amount:

18.7+14.7+18=51.4 units.

The profit will appear to be equal to:

П=51.4*(11-4)=359.8 rubles.

The decrease in demand testifies to the fact that the considered group is not a market.

Let us now consider the set consisting of both products which are sold in point A2 and of product G2 which is sold in point A3.

The initial seller's profit amounts:

П=60*(10-4)=360 rubles.

After the hypothetical price increase of up to 11 rubles the demand of customers B2 for the first product will be defined using the following equation:

B2: $0,12x + 0,1y + 0,3z - 0,22 = 0$.

Where x=1 (40 units out of 40), y=0.4 (20 units out of 50). Having solved the equation we will find that z=0.2, i.e. the demand of customers B2 will decrease by 20% and will amount 16 units.

The demand of customers B5 for product G2 will be defined using the following equation:

B5: $0,18x + 0,1y + 0,3z - 0,25 = 0$.

Where x=1 (40 units out of 40), y=0.8 (40 units out of 50). Having solved the equation we will find that z=0, i.e. the demand of customers B5 will not decrease (in the considered approximation) and will amount 20 units.

The demand of customers B6 for product G2 will be defined using the following equation:

B6: $0,12x + 0,1y + 0,3z - 0,22 = 0$.

Where x=0.5 (20 units out of 40), y=0.8 (40 units out of 50). Having solved the equation we will find that z=0.27, i.e. the demand of customers B6 will decrease by 27% and will amount 14.7 units.

The overall demand in the considered group will amount:

18+20+14.7=52.7 units.

The profit will turn out to be equal to:

П=52.7*(11-4)=368.9 rubles.

The increase in profit testifies to the fact that the considered group is a market (it is easy to demonstrate that a price increase with regard to any of its smaller subsets results in the decrease in profit).

So it is possible for a firm which is the only seller of both products in point A2 and of product G2 in point A3 to raise the price for these products, thus achieving the increase in profit.

VI. Conclusion

The hypothetical monopolist test has obtained a wide-spread occurrence as an instrument for definition of product market boundaries. Its principal advantage consists in minimizing the subjectivity which can be shown by an antimonopoly body. It is based not on expert judgments and the opinion of several interested customers but on the fact of how the market responds to a price increase on the whole.

At the same time, in practice the HMT has the form of not so much an algorithm as a paradigm. The case of defining the market boundaries with two products (territories) available – market inclusion candidates – is trivial, while in case of a greater number of options available the test application is still more an art rather than a trade for antimonopoly bodies. The task we are facing at the present development stage of the HMT theory is to formalize the test to the extent which would allow the market analysis using a unified standard pattern and future development of a software ensuring the required calculations (processing of the customer-respondent answers). The objective to be finally reached by the HMT is the development of the "model-algorithm-program" standard triad of mathematical modeling (*Samarskii and Mikhailov*, 2002)

The existing works on the issue of the HMT algorithm presentation consider the situation of complete information availability (the demand elasticities are known including the cross price elasticities which define the similarity of products and the required exogenous parameters). Our work is focused on the cases of information incompleteness.

The present work has also considered three approaches to the problem of the HMT practical application. Assumptions on the customer rational behavior have been made within each of them. It has been assumed that the antimonopoly body obtains part of the required information from the customers, while the other part (for the case when the number of products or areas exceeds two) is defined at the cost of our hypotheses on the customer behavior. This is necessary for the case when a great number of products or territories (K,N>2) are available since it provides a possibility to avoid the necessity to obtain the answers from each customer to the questions on his possible behavior in 2^{NK}-1 cases.

Applicability of the HMT for the (K,N>2) case is of critical importance for antimonopoly bodies which have always considered the market as consisting of substitutional products (customer accessible territories). Since in the case when one product has different groups of customers, while each group has its own set of substitutional products (accessible territories), a problem appears from the legal point of view. The antimonopoly legislation does not per se answers the question as to how individual customer (customer group) opinions shall be aggregated into the market opinion as a whole. It is the threshold value allowing for market delineation (for discarding the opinion of the customers who fail to exert principal influence on the market) which is defined by the legal formula of the hypothetical monopolist test.

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