Generating an Acoustic Field for Measurement of Horizontally Stratified Sea Bottom Properties using Wavenumber Integration with Direct Global Matrix Approach

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Abstract-- The acoustic field is obtained in terms of the wave number integration for a horizontally stratified ocean. The Inverse Hankel Transform is obtained for shallow water over a Layered inhomogeneous elastic bottom using "Direct global Matrix approach" with Fast Field Approximation Technique. This paper presents the determination of total solution for the depth dependence of the field: the depth dependence Green's function for Self Source Acoustic field.

Key-Words-- Fast Field Approximation, Green's Function, Global Matrix, Geoacoustic, Hankel transforms, Helmholtz equation, and shallow water.

I. INTRODUCTION

The problem of determining sea floor geoacoustic properties from measured ocean acoustic fields has received high attention in recent years. Properly predicting acoustic propagation, especially in shallow water waveguides and/or at low frequencies is very essential in estimating the geoacoustic sea floor properties. Traditional methods for measuring seafloor properties are costly and time-consuming, hence the Inverse methods will be used for this purpose which reduces the problems of the traditional methods. Therefore, in the last few years there has been a growing interest in providing solutions to the inverse problem consisting of determining seafloor properties from the measurement of the acoustic field in the water column. This approach provide an advantage that there is no need for deploying any equipment in the bottom for measurement, and we can cover a much larger area in a single inversion methods as compare to traditional local methods[1].

Geoacoustic inversion represents a strongly nonlinear inverse problem with no direct solution. Matched field inversion (MFI) makes use of the pressure field received on an array of sensors. The measured acoustic field contains the information about the ocean environment, which can be extracted using MFI.

In general, all oceanographic structures have an effect on sound propagation, both as a source of attenuation and of acoustic fluctuations. Considering the upper and lower boundaries of the ocean waveguide, the sea surface is a simple horizontal boundary and nearly a perfect reflector. The sea floor on the other hand, is a lossy boundary with strongly varying topography across ocean basins. Both boundaries have small-scale roughness associated with them which causes scattering and attenuation of sound. The structure of the ocean bottom depends on the local geology, but in general it consists of a thin stratification of sediments overlying the oceanic crust in the deep ocean and relatively thick stratification over continental crust. The nature of stratification is dependent on many factors, including geological age and local geological activity.

The importance of treating the ocean bottom accurately in the numerical models depends on the factors such as source receiver separation, source frequency and ocean depth.

The acoustic signal produced by the source is realized in the form of pressure signal. A group of sensors which receives the pressure signal from the source eg. Hydrophone array. There is some attenuation when the signal is passed from source to receiver. The received signal is then passed through ADC which converts analog signal to digital samples. The sample thus obtained contains measurement noise, which is called real data.

Generating an acoustic field (Pressure field) using this real data is an important aspect in the measurement of stratified sea bottom properties. The best method to achieve this is wavenumber integration technique. This method is used in the 'inverse problem'. An inverse problem is the task that often occurs in many branches of science and mathematics where the values of some model parameter(s) must be obtained from the observed data.

Basically the wavenumber integration technique is a numerical implementation of the integral transform for horizontally stratified media. The field solution is in the form of a spectral (wavenumber) integral of solutions to the depth-separated wave equation. The wavenumber integration approach evaluates the integrals directly by numerical quadrature. In underwater acoustics, wavenumber integration approaches are often called FFPs (fast field programs) because of the use of FFTs for evaluation of the spectral integrals.

In the modified wavenumber integration technique, a series of integral transforms are applied to the Helmholtz equation to reduce the original four-dimensional partial differential equation (3 space dimensions and 1 time dimension) to a series of ordinary differential equations in the depth co-ordinate. These equations were then solved analytically within each layer in terms of unknown amplitudes which were determined by matching boundary conditions at the interfaces.

Direct Global matrix Approach is used to obtain the depth dependent solution. This approach uses a finite element solution of the depth separated wave equation, with each layer in the stratification being a finite element and with the unknown amplitudes of the homogenous solution within the layer being the degree of freedom[2].
The Fast Field Program were developed for more efficient analysis, which applies an elegant recursive technique to determine the depth-dependent solution for many horizontal wavenumbers simultaneously and is therefore extremely efficient.

The wavenumber integration approach is applicable to range independent or horizontally stratified environments. All interfaces are plane and parallel, and the layer properties are functions of depth only. This technique is based on the fact that for a horizontally stratified environment, it is possible to obtain exact integral transformations for the field within each layer, in terms of a set of unknown coefficients. These coefficients are found by matching the boundary conditions simultaneously at all interfaces, and the total field is determined by evaluation of the integral representation.

II. WAVENUMBER INTEGRATION

With the assumption of purely horizontal stratification and point sources and receivers, the in-homogenous form of the wave equation can be depth-separated in cylindrical coordinates (r, φ, z) with z axis passing through the sources making the field independent of angle φ to yield

$$\left[\frac{d^2}{dz^2} - [k_r^2 - k_m^2(z)]\right]\psi_m(k_r, z) = \frac{f_s(z)}{2\pi} \quad \text{--------- (1)}$$

Equation (1) represents the depth separated wave equation.
Where $\mathbf{k_r}$ is a horizontal wavenumber related to the medium wave number $\mathbf{k_m}$ and vertical wave number $\mathbf{k_z}$ via,

$$K_z = (k_m^2 - k_r^2)^{1/2} \quad \text{------------------------------ (2)}$$

The total solution for the depth dependence of the field i.e. depth dependent Green's function [2] is

$$\psi_m(k_r, z) = \hat{\psi}_m(k_r, z) + A_m^+(k_r)\psi_m^+(k_r, z) + A_m^-(k_r)\psi_m^-(k_r, z) \quad \text{------ (3)}$$

Where $A_m^+(k_r)$ and $A_m^-(k_r)$ are arbitrary coefficients to be determined from the boundary conditions at the interfaces between the layers. When the unknown coefficients are found, the total field at the angular frequency ω is found at the range r by evaluating the inverse Hankel transform.

2.1 Boundary conditions:

The field at each interface now has two distinct integral representations, one from the layer above and one from the layer below. Depending on the type of interface, a certain set of boundary conditions must be satisfied. [2]

- At a fluid-fluid interface, both the vertical displacement ω and the normal stress σ_{zz} must be continuous. If one of the media is a vacuum the normal stress vanishes.
- At a fluid-solid interface, both displacement and normal stress must be continuous while the tangential stress σ_{rz} vanishes. If the fluid layer is replaced by a vacuum, both the stress must vanish.
- At a 'welded' interface between two solid media, all the four parameters must be continuous.

2.2 Numerical Solution of the Depth Equation

The numerical solution of the full wave field problem divides naturally into two parts. First, the depth-dependent Green's function is found at a discrete number of horizontal wavenumber for the selected receiver depths. Secondly, the wavenumber integral is evaluated, yielding the transfer function at the selected depths and ranges. To yield the total response in time the above two steps are repeated and at selected frequencies, 'frequency integration' has to be performed. The overall efficiency of the wavenumber integration approach is closely related to the efficiency with which the depth equation is solved.

2.3 Direct global matrix approach:

The Direct Global Matrix (DGM) approach is a finite element solution of the depth separated wave equation, with each layer in the stratification being a finite element and with the unknown amplitudes of the homogeneous solutions within the layer being the degrees of freedom.

$$V(k_r) = \sum_{m=1}^{N-1} T^m \left[c_m^m(k_r)S_m - c_{m+1}^m(k_r)S_{m+1}\right] A(k_r) \quad \text{-------- (4)}$$

$$\widehat{V}(k_r) = \sum_{m=1}^{N-1} T^m \left[\hat{v}_m^m(k_r) - \hat{v}_{m+1}^m(k_r)\right] \quad \text{-------- (5)}$$

Where
$V(k_r)$ is the global discontinuity vector,
$\widehat{V}(k_r)$ is the global source field discontinuity vector,
$A(k_r)$ is the global degree of freedom vector,
$c_m(k_r)$ is a local coefficient matrix i.e. a function of the horizontal wavenumber k_r,
S_m and T^m are topology matrices that extremely sparse, containing only ones and zeros.
Now, $C(k_r)$, the global coefficient matrix is given by

$$C(k_r) = \sum_{m=1}^{N-1} T^m \left[c_m^m(k_r)S_m - c_{m+1}^m(k_r)S_{m+1}\right] \quad \text{----------- (6)}$$

Because, $C(k_r)A(k_r) = -\widehat{V}(k_r)$ ------------ (7)

The setup of the global coefficient matrix requires only the calculation of the elements of the local coefficient matrices $c_m(k_r)$. The subsequent solution of the Eq. (7) then yields the unknown wavefield amplitudes in all the layers simultaneously.

To determine the acoustic field parameters at a particular receiver range r and depth z, we must numerically evaluate the inverse Hankel transform of the solution to the depth separated wave equation at depth z [2],

$$g(r,z) = \int_0^{\infty} g(k_r,z) J_m(k_r r) k_r dk_r \quad \text{---------- (8)}$$

Where,

$g(r,z)$ represents the field parameter of interest, e.g., acoustic pressure, or a particular displacement or stress component.

$g(k_r,z)$ is the associated wavenumber kernel.

J_m is the m^{th} order Bessel function, m=0 except for horizontal displacement and shear stress.

2.4 Fast Field Approximation:

The Fast Field Approximation Technique can be used to accurately evaluate the inverse Hankel Transform,

First the Bessel function is expressed in terms of Hankel functions, and finally we arrive at the following expression for the inverse Hankel transform.

$$g(r,z) = \sqrt{\tfrac{1}{2\pi r}} e^{-i(m+\frac{1}{2})\frac{\pi}{2}} \int_0^{\infty} g(k_r,z)\sqrt{k_r} e^{ik_r r} dk_r \quad \text{-------- (9)}$$

The approximation of Eq. (8) by Eq. (9) does not remove any of the complications concerning the integration interval or the oscillatory nature of the integrand. However the exponential function is more suitable for numerical integration than the Bessel function, particularly in terms of computation time.

To numerically evaluate the FFP integral of(9), we use either a quadrature scheme for semi-infinite integration intervals or truncate the integration interval at a wavenumber beyond which the contribution to the integral is insignificant.

2.5 FFP Integration

In the Fast-field-program (FFP) approach, the truncated wavenumber space is discretized equidistantly,

$$k_l = k_{min} + l\Delta k_r,\ l = 0, 1, \dots (M-1) \quad \text{-------- (10)}$$

Where M is the total number of sample points, and $\Delta k_r = (k_{max} - k_{min})/(M-1)$.

The field solution is often required at a large number of ranges r_j in particular in connection with the underwater acoustics problem of determining transmission loss as function of range. In these cases the Fourier integral in Eq. (9) is very efficiently evaluated by means of an FFT (Fast Fourier Transform), which is the original reason for naming this approximate Hankel transform solution the FFP. The range axis r is then discretized as

$$r_j = r_{min} + j\Delta r,\quad j = 0, 1, .., (M-1) \quad \text{------- (11)}$$

Where the range step Δr is determined by the relation

$$\Delta r \Delta k_r = \frac{2\pi}{M} \quad \text{--------------------------- (12)}$$

And M is an integral power of 2. the following discrete approximation of Eq. (9) is then obtained

$$g(r_j,z) \backsimeq \frac{\Delta k_r}{\sqrt{2\pi r_j}} e^{i[k_{min} r_j - (m+\frac{1}{2})\frac{\pi}{2}]} \sum_{l=0}^{M-1} \left[g(k_l,z) e^{i r_{min} l \Delta k_r} \sqrt{k_l}\right] e^{i\frac{2\pi t j}{M}} \quad \text{------- (13)}$$

where the summation can be performed by means of an FFT, yielding the field at M ranges simultaneously

III. RESULT

The following wave forms shows the acoustic field created for various ranges.

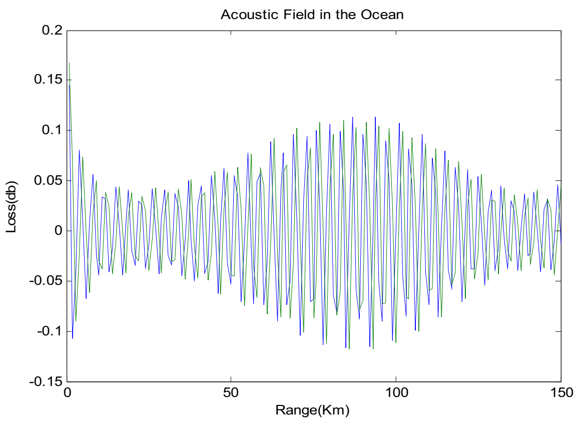

Fig.1 Acoustic field for frequency of 225Hz, source depth Zs = 7 km, receiver depth Zr = 45 km and Range varying between 0 and 150 km.

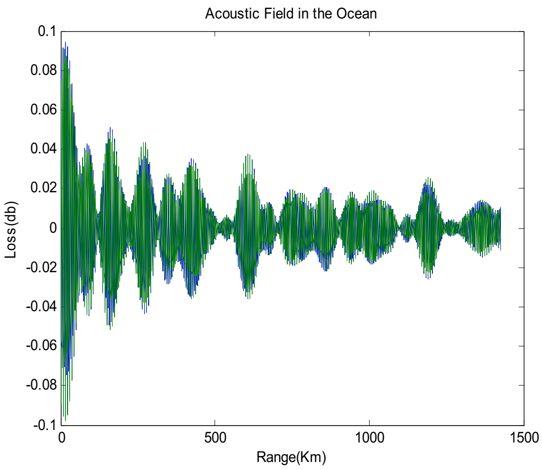

Fig.2 Acoustic field for Frequency = 225Hz , source depth z_s = 7km, receiver depth z_r = 45km and Range varying between 0 and 1500km

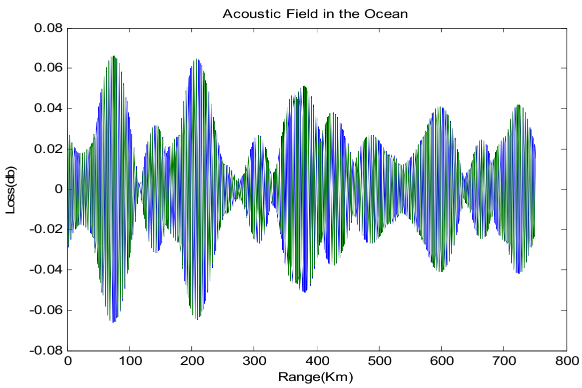

Fig.3 Acoustic field for Frequency = 200 Hz, source depth z_s = 7km, receiver depth z_r = 45km And Range varying between 0 and 800km.

IV. CONCLUSION

This paper describes wavenumber integration Technique, which can be used to model the acoustic field in ocean generated by ship generated source in shallow water, the DGM approach is highly efficient in cases where total wavefield is to be determined at many depths. Multiple sources can be treated simply by superimposing their contributions on the right side of Eq. (7). The basic solutions obtained by the DGM approach are the amplitudes of the up and down going waves in each layer; the total solution is directly decomposed into components. It is analytically well conditioned.

DGM requires a relatively complicated coding to establish the local-to-global mapping. Memory requirement of the DGM approach is proportional to the number of layers, which causes practical limitations.

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