

Shape Optimization of Permanent Magnet Motors Using the Reduced Basis Technique

A. Jabbari, M. Shakeri and A. Nabavi

Abstract—In this paper, a tooth shape optimization method for cogging torque reduction in Permanent Magnet (PM) motors is developed by using the Reduced Basis Technique (RBT) coupled by Finite Element Analysis (FEA) and Design of Experiments (DOE) methods. The primary objective of the method is to reduce the enormous number of design variables required to define the tooth shape. RBT is a weighted combination of several basis shapes. The aim of the method is to find the best combination using the weights for each tooth shape as the design variables. A multi-level design process is developed to find suitable basis shapes or trial shapes at each level that can be used in the reduced basis technique. Each level is treated as a separated optimization problem until the required objective – minimum cogging torque – is achieved. The process is started with geometrically simple basis shapes that are defined by their shape co-ordinates. The experimental design of Taguchi method is used to build the approximation model and to perform optimization. This method is demonstrated on the tooth shape optimization of a 8-poles/12-slots PM motor.

Keywords—PM motor, cogging torque, tooth shape optimization, RBT, FEA, DOE.

I. INTRODUCTION

RECENTLY PERMANENT magnet motors have been widely used in industrial applications because of their efficiency and power density. However, these have inherent torque ripples which cause vibration and noise. Since the cogging torque is induced by the magnetic coupling between the rotor and the stator, it is greatly affected by the configuration of the stator. The stator configuration effects on magnetic field distribution on a PM motor, therefore, proper tooth shape would improve the magnetic field distribution such that the flux leakage and core saturation can be avoided, i.e., reducing the cogging effect of motors.

The shape of the tooth is crucial in achieving the desired characteristics of the motor. Traditionally, an expertise designer uses his know-how for optimizing the tooth shape. With the advent of better computers, more robust and efficient

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TABLE I
SUMMARY OF LITERATURE REVIEW

Attributes Researcher	Finite Element Analysis		Optimization Method	NUMBER OF VARIABLES
	2D	3D		
Koh et al. [1]	×		DSA and ES	3
Chung et al. [2]	×		ES	3
Chun et al. [3]	×		ES	4
Wang et al. [4],[9]	×[4]	×[9]	DSA and ES	4/3(B-Spline)
Yao et al. [5],[6]	×		FEA	2(α, r)
Huang et al. [7],[10]	×[7]	×[10]	FEA	3(R ₁ ,R ₂ ,R ₃)
Yi et al. [8]	×		EMNC and GA	3
Lee et al. [11]		×	DOE and RSM	3

FEA=finite element analysis, DSA=design sensitivity analysis, ES=evolution strategy, EMNC=equivalent magnetic network circuit, DOE=design of experiments, RSM=response surface method, GA=genetic algorithm.

shape optimization techniques are developed for industrial applications. The sheer number of design variables required to define the tooth shape coupled with the huge computational time required to simulate the motor, make the application of most shape optimization algorithms impractical. There have been some developments for tooth shape optimization. Table I summarizes the literature by showing the type of optimization approach.

Two-dimensional Finite Element Analysis (FEA) was used to compute the cogging torque in permanent magnet motors by [1]-[8]. The cogging torque analysis, was carried out using three-dimensional FEA by [9]-[11]. Design of the tooth shape was performed using several optimization techniques. Design sensitivity analysis (DSA) and an evolution strategy (ES) is used to determine the proper tooth shape [1]-[4], [9]. FEA is also used for optimal design of the tooth shape [5]-[7], [10]. Tooth shape optimization was carried out using an equivalent magnetic network circuit (EMNC) and generic algorithm (GA) by [8]. Design of experiments (DOE) and response surface methodology (RSM) was also used as an optimization method by [11].

Two rectangular slots are placed on each tooth surface by Koh et al. [1] and Chung et al. [2]. They achieve to the optimal tooth shape by calculating the size of rectangular slots.

Wang et al. [4], [9], modeled the tooth shape by using nonuniform rational B-Spline (NURBS) with some control points which defined as design variables.

Yao et al. [5], [6], divided the tooth shape into two regions. Then, the proper angle and radii for each region calculated using optimization strategy.

Huang et al. [7], [10], proposed that the salient pole be divided into several regions. They state that by properly choosing the radii for these regions, the cogging torque can be greatly reduced.

Lee et al. [11], defined the tooth shape by five design variables. They state that, if many parameters are defined, it takes large simulation time.

[12]-[14] used a reduced basis technique to speed up the torque prediction.

The main challenge of current optimization methods, especially for complex one, is the number of design variables required for tooth shape optimization and the generality of the procedure, as described in table I.

The method presented in [12]-[14] reduces computation cost. In fact in this method “Basis” is “vector potential matrix and the forcing function”. “Reduced basis” means reducing the dimension of vector potential matrix and then forcing function. The method applied to magnet thickness and arc angle optimization of a spindle motor. A comparison of reduced basis technique presented in [12]-[14] and the proposed method is described in Table II.

TABLE II
COMPARISON OF THE METHOD PRESENTED IN [12]-[14] AND PROPOSED METHOD

Basis	Vector potential matrix/ forcing function	Tooth shape vector
Method	RBFEM [12]-[14]	RBT+FEM+DOE (proposed)
Basis	Vector potential matrix/ forcing function	Tooth shape vector
variable	Magnet thickness / arc angle	Weighting factors
objective	Reducing the dimension of Basis matrix considering only transformation region (on line) in each iteration and assembling this matrix with initial matrix (off line)	Reducing the number of design variable from n (number of boundary points) to m (number of basis shapes).
Generality	It is based on vector potential formulation, therefore can't be used in complex shape optimization. Because the method can be applied in optimization of design variables included in formulations.	It can be used in complex shape optimization.

RBT= reduced basis technique.

An integrated algorithm applicable for a larger class of problems is developed in this research and it is conducted to tooth shape optimization of a 3-phase radial flux PM motor with 8 poles-12 slots. An innovative, comprehensive way of using an efficient design variables linking method, termed as reduced basis technique [12], is demonstrated for tooth shape optimization. In the reduced basis technique, many initial tooth shapes, called basis shapes, are combined linearly by assigning weight factors. Different resultant shapes can be

generated by changing their weight factors. Therefore, the number of design variables required to define the shape is reduced to the number of basis shapes. So, the weights assigned for each basis shapes are the design variables and the optimization goal is to find the best possible combination of these weights to minimize a cost function.

The algorithm developed in this paper focuses on the Taguchi method which is the combination of mathematical and statistical techniques used in the empirical study of relationships and optimization, in which several independent variables influence a dependent variable or response. The choice of an experimental design to build the model depends on the objectives of the experiment and the number of factors to be investigated.

II. TOOTH SHAPE OPTIMIZATION METHODOLOGY

Although the reduced basis technique is widely used in metal forming process [15], but it is suggested for tooth shape optimization in this work. The reduced basis technique need to be adopted, because in spite of metal forming process, the tooth shape area is not constant.

A. Reduced Basis Technique

The main idea in this method is to construct basis vectors, $Y^1, Y^2, Y^3, \dots, Y^n$ with the large information content of each basis shape and to combine them linearly with the weighing factors, $a_1, a_2, a_3, \dots, a_n$, that correspond to each basis vector and $0 \leq a_i \leq 1$.

$$Y = Y^c + \sum_{i=1}^n a_i Y^i \quad (1)$$

where, Y^c is a vector added for generality.

The basis vectors, which represent each basis shape or initial guess shape, will have the co-ordinates or shape parameters that define the respective basis shape. If the number of shape variables required to define a basis shape is m , then by applying the reduced basis method, the number of design variables is decreased from m to n (equal to the number of weighing factors). Generally, the value of m is more than 30 even for a simple shape and the number of basis shapes n required is typically about 3 to define the optimum tooth shape. It is a common practice to define the basis vector by the node data of the basis shapes.

B. Basis Vector Definition

The geometrical features of the basis shapes can be defined in polar system by the r and θ co-ordinates of their boundary points. These co-ordinates define the basis vectors. All of the basis shapes have to be defined in the same fashion, and therefore, all the resulting basis vectors will have the same dimension. This will help to add them linearly with weights to each vector. The resulting vector will have a different shape than any of the basis shapes if at least two of the weights are non-zero. If the optimum tooth shape is any of the basis shapes, then the corresponding weight will be one and the others will be simply zero. The important factor that must be heeded is that the number of shape parameters should be as plentiful as possible. This means that the locations at which the co-ordinates are extracted should be as close to each other

as possible in order to facilitate splinting and to get the detailed profile. Fig. 1(a) shows an edge defined by a set of points. There are 19 points that define the edge and the coordinates of these points form the basis vector. If a less number of points are used, for example eight points, then the resultant edge would look much different than the original one (Fig. 1(b)). To avoid this type of error, it is always safe to define the basis shapes with a large number of boundary points. Since this will not increase the number of design variables, there is no extra computational cost incurred by increasing the dimension of the basis vectors. This type of basis vector definition is useful for generating various possible shapes for optimization, but scaling of the resultant shapes is essential to maintain area restriction.

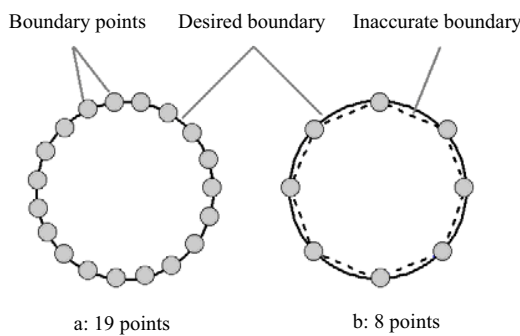


Fig. 1: Basis vectors definition

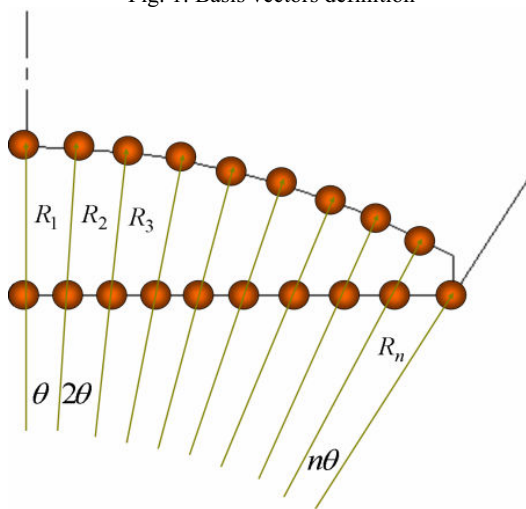


Fig. 2: Geometrical definition of half tooth shape.

C. Geometric Scaling

In order to understand the necessity of scaling, a simple problem of combining two basis shapes is considered. The unknown arc lengths (r_1, r_2) and unknown angles (θ_1, θ_2), which are defined by their respective basis shapes (Y^1 and Y^2), form the initial shapes. The optimum shape Y is defined as:

$$Y = a_1 \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix} + a_2 \begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} a_1 r_1 + a_2 r_2 \\ a_1 \theta_1 + a_2 \theta_2 \end{bmatrix} = \begin{bmatrix} R_i \\ \theta_i \end{bmatrix} \quad (2)$$

where a_1 and a_2 are the weights. The area of the resulting

shape may be greater than limit area; which is the maximum allowable tooth shape area. According to Fig. 2, R_1 and θ_1 considered to be fixed, then the resulting shape is scaled to a preset value ($1/\Sigma a_i$).

D. Optimization Problem Definition

Appropriate starting basis shapes are required to employ the algorithm to find optimum tooth shape design. But in some instances in which the design is completely new, a different material, or a different number of slots per poles per phases, it may not be possible to begin with a reasonable set of starting basis shapes. Our goal is to obtain the basis shapes for a new type of motor design problem.

In those cases, we may not obtain the optimum tooth shape by solving the shape optimization problem just once. The problem can be solved in multiple levels as shown in Fig. 3 in which the optimization procedure guides the designer progressively in selecting viable basis shapes. In first Level, the basis shapes may not be anywhere near to what they are supposed to be, but by the first set of basis shapes the one can determine a best combination from the first trial shapes.

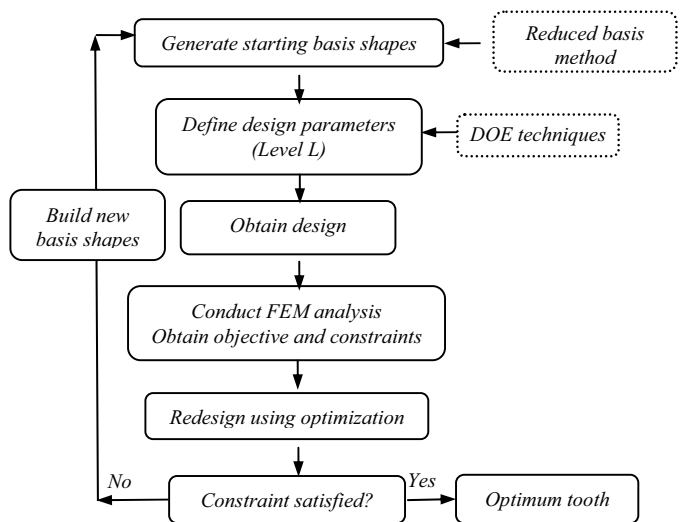


Fig. 3: Multi-level design process.

Three or four shape resulted from first level are constructed as starting shapes for second level. This process may be repeated typically three to four times before suitable basis shapes are found. Irrespective of how impractical the starting shapes in any level are, the optimum shape in that level will give a better minimization than the starting basis shapes, or may the one will normally set one the weighting factor for each basis shape. After the completion of each level, the next level is started as a new problem and the best shape of the previous level becomes one of the basis vectors (Basis 1). A few additional basis shapes are chosen, which will be variants of Basis 1. Thus the designer is guided into the right path to reach the optimum shape because the basis shapes are selected by the modification from the previous level. Even if all the additional shapes are inappropriate in second Level, one can give Basis 1 as the optimum shape, as it was the best shape in the previous level. It must be noted here that the modification

procedure of the algorithm will take the designer to the optimum tooth shape. The computational time increases because a new surrogate model has to be built in every level. An experienced designer can start from an intermediate level with practical basis shapes and reach the optimum solution in a single level. There is room for making use of a designer's experience can be used for similar design.

III. CASE STUDY

The tooth shape optimization of a 8 poles-12 slots permanent magnet motor is demonstrated in this work. The finite element package Maxwell 2D is used to calculate the cogging torque in order to conduct DOE. In the finite element model, the tooth shape is meshed with a triangular mesh. Nd-Fe-B material is assigned to the permanent magnets. M-19 (3.25% silicon) steel, listed in the material library of software, is assigned to the stator and hub. A cross-section of the motor is shown in Fig. 4. Specifications of the investigated motor are presented in Table III. Fig. 5 shows three curves selected as initial trial basis shapes. Basis 1 assumed as Polyline, Basis 2 as Arc and Basis 3 as Spline.

2-D FEA simulations of the basis shapes are performed to find the cogging torque for preliminary analysis as a sample of analysis as shown in Fig. 6. The peak to peak cogging torques of the Basis shapes are 8.72, 11.96, and 8.08 mN.m, respectively. From this preliminary analysis, it can be said that the Basis 3 is more successful than the other two shapes in reducing the cogging torque. Therefore, the contribution of Basis 3 must be more than the other basis shapes, which must be recognized by the optimizer.

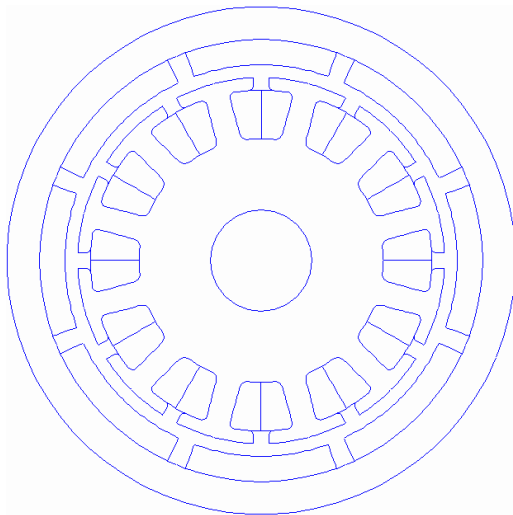


Fig. 4. Cross-section of 8P12S permanent magnet motor.

TABLE III
SPECIFICATIONS OF THE INVESTIGATED MOTOR

Field		Armature	
Outer Diameter	32 mm	Outer Diameter	27.6 mm
Inner Diameter	28.2 mm	Inner Diameter	10 mm
Number of poles	8	Number of Slots	12
Magnet Arc	42°	Tooth Width	1.8 mm
Magnet Thickness	1.2 mm	Yoke Width	4 mm
Magnet Arc R	28.2~37.2 mm	Slot Open	1.18 mm
Magnet material	TDK-fb9b	Tip Thickness	0.92 mm
		Slot Skew	0°
		Stack Width	6.25 mm
		Lamination material	M19-0.5mm

Each Basis shape is defined by 30 shape variables, (r and co-ordinates). These shape variables form the respective Basis vector. The reduced basis technique is applied to three basis vectors and the number of design variables is decreased to three, which are the weights for each basis vector. By changing these weights, it is possible to obtain various resultant tooth shapes for the optimizer to find the best combination of these weights. 9 DOE points are generated to conduct simulation. All of the resultant tooth shapes are scaled to maintain in a limited area. Simulations are conducted at these DOE points to find the cogging torque and to build the Taguchi models for optimization. Optimization is performed in QualiTek-4 to minimize the cogging torque.

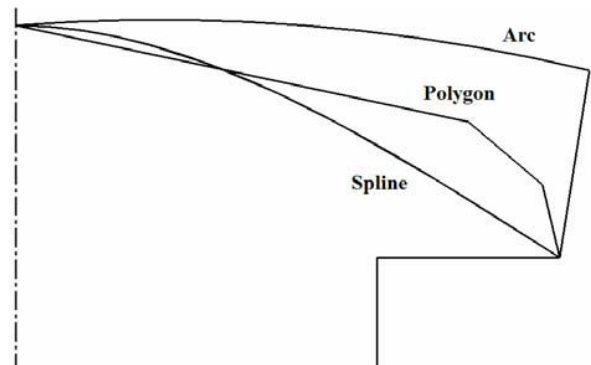


Fig. 5. The selected Basis shapes.

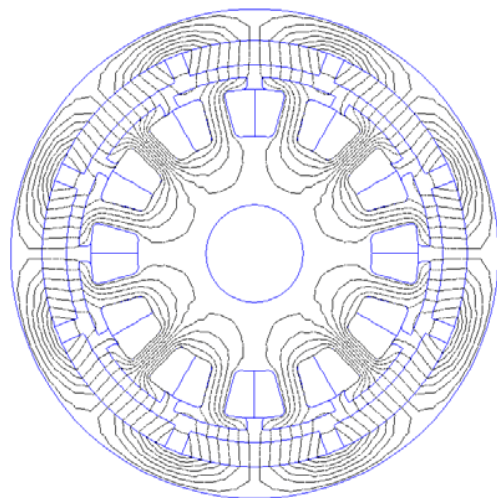


Fig. 6. Finite element analysis of the model.

A. Procedure of ANOVA

Designing the Experiments

A DOE/Taguchi approach is used to study the effects of multiple variables simultaneously. Three factors including Basis shapes weighting factor will be investigated. Based on known variation of cogging torque with respect to different factors, each factor is considered to have three levels. Therefore, a L-9 orthogonal array have been selected to run the experiments. Table IV shows the factors and their levels and the layout for the selected array is also presented in Table V.

TABLE IV
WEIGHTING FACTORS AND THEIR SELECTED VALUES

Levels	a ₁	a ₂	a ₃
1	0.85	0.1	0.9
2	0.8	0.12	0.88
3	0.75	0.14	0.86

The experiments are carried out using FEA and the cogging torque obtained from each experiment is shown in Table V.

TABLE V: THE VALUES OF COGGING TORQUE IN ALL 9 TRIALS.

Experiments by 2D FEA	T _{c-p} [mN.m]
1	1.748
2	1.932
3	2.108
4	1.708
5	1.868
6	1.852
7	1.632
8	1.588
9	1.772

Analyzing the Results

Considering cogging torque as target function, the results are investigated. The main effects table, which presents the mean value of cogging torque for each factor at all levels, is shown in Table VI. By the analysis of means (ANOVA), it is observed that for each weighting factor, its averages in three levels are close. In other words, parameters a₁, a₃ and especially a₂ have very low variances and this means that the optimization is successful and the values of parameters in three levels are properly selected. This result confirmed through FEA. If a parameter has high variances, the optimum shape is considered as a new basis shape instead of the shape with high variances. This procedure is repeated according to multi-level design process until all factors have low variances.

TABLE VI
COGGING TORQUE FOR ALL LEVELS OF ALL FACTORS

Level	T _{c-p} of a ₁ [N.m]	T _{c-p} of a ₂ [N.m]	T _{c-p} of a ₃ [N.m]
1	1.929	1.695	1.729
2	1.809	1.796	1.803
3	1.664	1.91	1.869

Predicted optimum combination of factors is shown in Table VII. The predicted result at optimum combination and confidence interval (CI) are calculated from the following relations:

$$Y_{opt} = \bar{T} + (\bar{A}_i - \bar{T}) + (\bar{B}_i - \bar{T}) + (\bar{C}_i - \bar{T}) \tag{3}$$

$$C.I. = \sqrt{\frac{F(1, n_e) \times V_e}{N_e}} \tag{4}$$

TABLE IX
OPTIMUM CONDITION FOR COGGING TORQUE

	A	B	C	T _{c-p} [mN.m]
Level	3	1	1	
value	0.75	0.1	0.9	
Expected result at optimum				1.486

In the equation (3), \bar{T} is the grand average of performance and \bar{X} , denotes the average effect of factor X at its optimum level. In the equation (4), F(1,n_e) is the F value from the F Table [16] for a given confident level and V_e denotes the variance of the error term and N_e is calculated by:

$$N_e = total \ DOFs / (1 + DOF_{all})$$

Now, the predicted optimum value and confidence interval for 90% confidence level are obtained from equation (3).

Confirming the Predicted Result

Having performed the analysis of results, the predicted optimum result must be verified through carrying out experiments at optimum combination of factors. If the result of optimum experiment is within the permissible limit, the predicted result will be verified and otherwise, the DOE experiments must be redesigned and return considering interactions between factors.

Running the experiments at optimum combination of factors for cogging torque result (1.495 mN.m), this value is within the permissible limit as shown in equation (5) and the predicted result is confirmed. Here confirmation means that for 90% confidence level, there is no need to repeat the procedure of DOE with counting for interactions between factors.

$$\begin{aligned} Y_{opt} &= 1.486 \\ C.I. &= \pm 0.014 \\ 1.472 &\leq E \leq 1.5 \end{aligned} \tag{5}$$

The optimum values for weighting factor a₁, a₂ and a₃ are 0.75, 0.1, and 0.9, respectively. The cogging torque of the resultant shape is 1.495 mN.m. It is clear that most of the contribution is from Basis 3 (a₃=0.9). There is also a significant contribution from Basis 1 (a₁=0.75). Fig. 7 shows the optimum resultant tooth shape achieved by the reduced basis technique. A comparison of cogging torque for the selected Basis shapes and optimum shape is shown in Fig. 8.

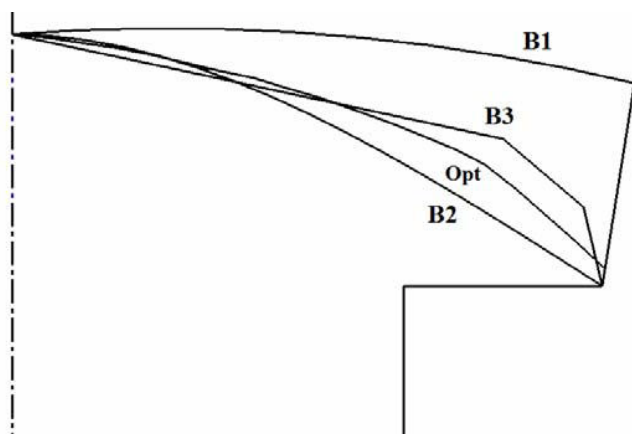


Fig. 7. Optimum resultant shape,
B1: Arc curve, B2: Spline, B3: Polyline, Opt: Optimum curve.

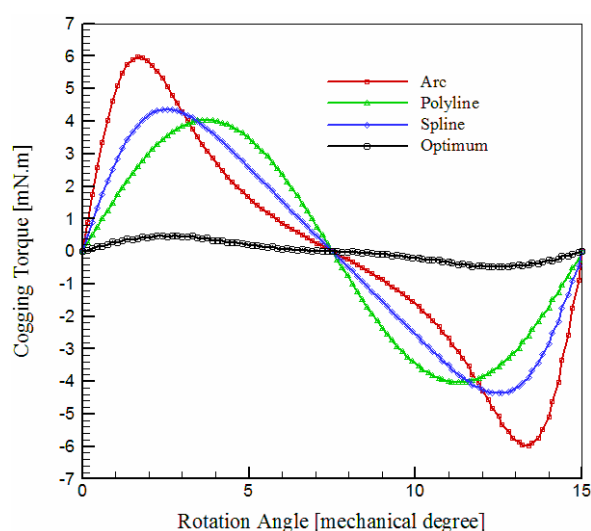


Fig. 8: A comparison of cogging Torque for Basis shapes and optimum shape.

As shown in Fig 8, the peak to peak value of the cogging torque for Basis shapes are 8.72, 11.96, and 8.08 mN.m, which it has been reduced significantly by this method to 1.495 mN.m.

IV. DISCUSSION AND CONCLUSION

A two-dimensional tooth shape optimization method for permanent magnet motors is introduced in this paper using the reduced basis technique. This design technique can be used for both 2-D and 3-D tooth shape optimization. The concept of a multi-level design process is introduced, which aids the designer in the selection of practical basis shapes that will give cogging torque reduction, but this will also increase the number of FEA simulations. It is important to mention that if expert knowledge is available, then practical basis shapes can be selected and the optimum tooth shape can be obtained in a single level. Increasing the number of basis shapes also enables the designer to obtain a better tooth shape, but the computation time also increases to build an approximation model. The reduced basis method aids in the use of the ANOVA models for optimization. Most tooth shapes obtained by this method are practical. However, if the motor geometry

is complicated, it is prudent to start from very simple starting shapes. The presented algorithm has been applied on a 8poles-12slots PM motor, as a case study. An optimum tooth shape has been achieved by the implemented algorithms, starting from three basis shapes such as polyline, arc, spline. The cogging torque has been reduced significantly (to 1.495 mN.m) by this optimization method.

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