# Optimization by ant colony hybryde for the bin-packing problem

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***Abstract*—The problem of bin-packing in two dimensions (2BP) consists in placing a given set of rectangular items in a minimum number of rectangular and identical containers, called bins. This article treats the case of objects with a free orientation of 90°.
We propose an approach of resolution combining optimization by colony of ants ($ACO$) and the heuristic method IMA to resolve this NP-Hard problem.

***Keywords*—Ant colony algorithm, Bin-packing problem, Heuristics methods.

## I. Introduction

Two-dimensional (2D) packing problems occur in a wide range of industries. The goal is simply the optimal utilization of space or material available.In the garment and paper industries the problem is often to cut smaller pieces from a large roll of cloth or paper while reducing the scrap. In the wood, glass, and metal industries the task is not to cut from a roll, but to cut from a fixed size sheet or plate. There are two major variants of the 2D packing problem: bin packing and strip packing. In the 2D bin packing variant, rectangles are to be packed in bins of given width and height, the goal being to minimize the number of bins used. In the 2D strip packing variant, rectangles must be packed in a fixed width, infinite height strip, the goal being to minimize the height. The bin packing variant is most suitable for the wood, glass, metal, and semiconductor industries, while the strip packing variant will generally apply to the paper and garment industries. An important consideration is whether or not the rectangles can be rotated as they are placed. In the wood and garment industries one may care about the grain of the material and rotations may not be permitted. While in the paper, glass, and semiconductor industries there may be no particular restrictions. Allowing rotations adds flexibility and can result in a better packing, while at the same time apparently complicating the task. Given the many permutations of 2D packing problems, most research focuses on a particular type of the packing problem: either bin packing or strip packing, with or without rotations. A survey of 2D packing problems is given by[11]. A typology of cutting and packing problems is defined in[15]. Being NP-hard, the 2D packing problems are an attractive challenge for evolutionary algorithms (EA)[10]. For application to strip packing problems, Jakobs developed a genetic algorithm (GA) using the bottom-left (BL) packing method with support for rotation of rectangles[6]. Hopper and Turton provide a comprehensive comparison of GA, simulated annealing (SA), naive evolution, and simple hill-climbing for the strip packing problem with rotations and for various BL methods [5]. A method to find strip packing solutions using an exact branch and bounds exhaustive search proposed in[9]. For application to bin- packing problems, El-Hayek developed a tabu searche (TS) using the IMA packing method with support for rotation of rectangles[8]. The problem of bin-packing in two dimensions (2BPD) is defined as follows: considering a set of $n$ rectangular items $A = [a_1, \ldots, a_n]$ and an unrestricted number of identical rectangles called bins[4], whose measurements are larger than those of objects, the problem consists in determining the minimal number of used bins to place the set of objects without overlap. Several real problems can be modelled as a problem of bin-packing.
The unidimensional problem of bin-packing is NP-hard[2]. This is valid for the 2BP which is a generalization of the unidimensional problem. In this article we treat the case in which the objects to place, have a free orientation of 90°. This version of the problem is designated by $2BP|R|F$ according to the classification of[11].

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## II. OPTIMIZATION BY COLONIES OF ANTS (ACO)

Optimization by colony of ants is a meta-heuristic method inspired of the real ant capacity to find the shortest path between their nest and the source of food. The first algorithm of optimization by colony of ants (ACO) has been proposed by Dorigo [3] to solve the Traveling Salesman Problem (TSP). This technique has since been used to solve a multitude of combinatorial optimization problems. The ants capacity to determine the shortest path is due to chemical marks (pheromones) they depose down on the ground. More a path is used by ants, more there are pheromones dropped off and more the path becomes attractive for the following ants. We define therefore $(i, j)$, the quantity of pheromones associated to the connection between 2 cities $i$ and $j$, as well as a probability $p_k(i, j)$ that has a $k$ ant to move from an $i$ city to a $j$ city. Every ant is placed at random in a city of departure and builds a solution by going from city to another. When all ants have constructs a circuit, the pheromones are updated. This updating includes two aspects. On the one hand pheromones evaporate and decrease with a speed of fading. On the other hand, they are proportional to the length of the path. So, it is necessary to increase pheromones between certain cities. The technique of this increase depends on the definition given to pheromones according to the treated problem and takes into account the order of cities visited in the best solutions obtained in the population of ants. Let's take for example the case of the one-dimensional Bin packing treated by Levine and Ducatelle [7]. The trail of pheromone $(i, j)$ represents the desire to have an

$i$ object and a $j$ object in the same bin. The probability so that a $k$ ant chose a $j$ object as next object for the considered bin $b$, in a solution partial $s$ is:

$$p_k(s,b,j) = \begin{cases} \frac{[\tau_b][\eta(j)]^\beta}{\sum_{g\in J_K}[\tau_b(g)][\eta(g)]^\beta} & \text{if } j \in J_k(s,b), \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

with

$$\tau_b(j) = \begin{cases} \frac{\sum_{i\in b}[\tau(i,j)]}{b} & \text{if } b \in J_k(s,b), \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Where $J_k(s,b)$ is the set of objects candidates (those whose size allows to place them there) to go in the current bin, $(j)$ is the weight of the $j$ object given by one heuristic that guides ants, and $b(j)$ is the value of the pheromone of an object $j$ for the bin b. $\beta$ defines the importance relative size of the heuristic. The updating of pheromones' trails is assured as follows: only the best ants are allowed to place pheromones after each iterations.The Pheromones are increased every time that $i$ and $j$ are combined in a bin. Considering the hypothesis that several identical items are there, them define $t(i,j)$ the number of times $i$ and $j$ go belong to the same best solution $s^{best}$.

$$\tau_{i,j} = \rho\tau(i,j) + t(i,j)f(s^{best}) \quad (3)$$

## III. THE HEURISTIC IMA

The heuristic IMA has been proposed by J. El Hayek [8] for the problem of $2BP|R|F$. Being given a process $I = (A,B)$ to solve, we consider two different lists. The first list $A$ contains the non orderly items. $A_0$ is Initially worth $A$, the set of all objects composing the I process. The second $Lma$ list is the list of the maximal areas available. It is initially constituted of only one area which is the first bin's area. At every step of an item seating, we choose the orientation of this one as well as the pair (the item to arrange, the maximal area used) from the lists A and $Lma$. This choice is based on an heuristic criteria takes in consideration features of items and those of the maximal areas. Let simultaneously $ai \in A$ an item non put away, which is considered in a defined orientation. The area $ma \in Lma$ is a maximal available area that can contains $ai$, according to the considered orientation and in a bl-steady position. Let $w_{ma}$ and $h_{ma}$ the ma edge projections on successively the axles $x$ and y. Let's note $q_1$, $q_2$, $q_3$, and $q_4$ four real numbers as: $0 \leq q_k \leq 1$; $k = 1,...,4$
$q_1 + q_2 + q_3 + q_4 = 1$
The pair (item to place, maximal area used) is then the pair of $(A, Lma)$ that maximizes the criteria mentioned below:

$$O(a_i, ma) = \frac{q_1(w_ih_i)}{(w_{ma}h_{ma})} + \frac{q_2(dxi)}{(w_{ma})} + \frac{q_3(dyi)}{(h_{ma})} + \frac{q_4(w_i^2+h_i^2)}{(w_{ma}^2+h_{ma}^2)}$$

The values given to parameters influence strongly the solution badly stored. For example, if the value given to $q_1$ is big enough (close to 1), the pair $(a_i, ma_i)$ indicated would be the one that present closest areas. Terms weighted by $q_1$ and $q_4$ are independent of the items orientation, while this orientation modifies terms weighted by $q_2$ and $q_3$.

## IV. ACO-IMA FOR THE $2BP|R|F$

We propose a hybrid algorithm of optimization by colony of ants using the mode of heuristic IMA placing. We use here a colony of ants in order to make vary the sequence of pieces placed in bins.

### A. *Pheromones*

The pheromones $\tau(i,j)$ put down by the ant represent the desire to place the $j$ piece after having placed the $i$ one. In order to update them at the end of each iteration, we suggest considering the $\alpha$ better solutions of the current iteration. The updating is assured according to the following formula:

$$\tau_{i,j} = \rho\tau(i,j) + \sum_{k=1}^{\alpha} u(i,j)(k)f(s^k) \quad (4)$$

$$u(i,j)(k) = \begin{cases} 1 & \text{if } j \text{ chosen after } i \text{ in } k \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$f(s^k) = \frac{\sum_{i=1}^{N}(N_i/N_{max})^\gamma}{N} \quad (6)$$

where:
- $N$ is the number of bin;
- $N_i$ is the number of items in the bin $i$;
- $N_{max}$ is the maximum number of objects per bin throughout the solutions iteration;
- $\gamma$ is a parameter that defines the importance of the nominator.

We have hoped to take in account the $\alpha$'s better solutions in order to be able to make vary this parameter and then the algorithm more aggressive in its research.

### B. *General principle of a solution construction.*

We consider a population of $K$ ants. At each iteration of the algorithm, every $k$ ant $(k \in K)$ is going to begin with a set of $n$ items to place and an empty bin. Each one is going to build a solution $s$ by choosing pieces to place one by one at random. The choice of items takes in consideration the probability $p_k(i,j)$ that an ant $k$ chooses the piece $j$, knowing that it has chosen the piece $i$ before.

$$p_k(i,j) = \begin{cases} \frac{\tau(i,j)[\eta(j)]^\beta}{\sum_{g\in J_k}\tau(i,g)[\eta(g)]^\beta} & \text{if } j \in J_k(i) \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where:
- $\tau(i,j)$ are the pheromones put down by ants between $i$ and $j$;
- $\eta(j)$ a parameter that guides the heuristic;
- $J_k(i)$ the set of the even taken pieces.

In this section we present the numerical results. We tested our algorithms on benchmarks proposed in literature. These benchmarks are factorized in ten classes of processes, generated at random. The first six classes have been proposed in [1] while the last four ones have been proposed in[14]. Each class is composed of five groups which differ by the number of items ($n = 20, 40, 60, 80, 100$). Each group contains ten different processes. The benchmark contains in total 500 different processes. Features of processes, which compose the six classes of [1][12], are as follow:

Class 1: $w_i$ and $h_i$ generated at random following a uniform law in $[1, 10]$,
$W = H = 10$,
Class 2: $w_i$ and $h_i$ generated at random following a uniform law in $[1, 10]$,
$W = H = 30$,
Class 3: $w_i$ and $h_i$ generated at random following a uniform law in $[1, 35]$,
$W = H = 40$,
Class 4: $w_i$ and $h_i$ generated at random following a uniform law in $[1, 35]$,
$W = H = 100$,
Class 5: $w_i$ and $h_i$ generated at random following a uniform law in $[1, 100]$,
$W = H = 10$,
Class 6: $w_i$ and $h_i$ generated at random following a uniform law in $[1, 100]$,
$W = H = 300$.

In each of the above classes, all the item sizes are generated in the same interval. Martello and Vigo have proposed the following classes, where a more realistic situation is considered. The items are classified into four types:

Type 1: $w_i$ uniformly random in $[\frac{2}{3}W, W]$, $h_i$ uniformly random in $[1, \frac{1}{2}H]$,
Type 2: $w_i$ uniformly random in $[1, \frac{1}{2}W]$, $h_i$ uniformly random in $[\frac{2}{3}H, H]$,
Type31: $w_i$ uniformly random in $[\frac{1}{2}W, W]$, $h_i$ uniformly random in $[\frac{1}{2}H, H]$,
Type 4: $w_i$ uniformly random in $[1, \frac{1}{2}W]$, $h_i$ uniformly random in $[1, \frac{1}{2}H]$,

The bin sizes are $W = H = 100$ for all classes while the items are as follows:

Class 7: type 1 with probability $70\%$, type 2, 3, 4 with probability $10\%$ each;
Class 8: type 2 with probability $70\%$, type 1, 3, 4 with probability $10\%$ each;
Class 9: type 3 with probability $70\%$, type 1, 2, 4 with probability $10\%$ each;
Class 10: type 4 with probability $70\%$, type 1, 2, 3 with probability $10\%$ each.

## V. Parameter Settings

The different parameters of the algorithm are the size of the population K, the number of $\alpha$ better solutions we have to take in account in the pheromones updating, the $\rho$ parameter of pheromones fading. The parameters $\gamma$ and $\beta$, which intervene respectively in the calculation of the mark given to each solution and the one of probabilities, are others algorithm's parameters. We tested, for each parameter, different values. We chose to generate populations of K = 20 solutions for numbers of pieces of 20 and 40, the increase of the population's size don't improve the solution obtained.Concerning problems with 60 and 100 pieces, the population is to K = 40 solutions. The number of pieces being bigger, the one of combination is big as well. Therefore, it is interesting to have a variety sufficiently big at each iteration, before updating pheromones. For the $\alpha$ parameter, we tested different values from 1 to 7 for a population of 20 solutions and from 1 to 10 when we have a population of 40. It results from this study is that the increase of this parameter doesn't improve the quality of results and we finally chose to take $\alpha = 3$ for a population of 20 solutions and $\alpha = 5$ for a population of 40 solutions. Concerning the $\gamma$ parameter, we tested values 1, 2, 3 and 4

TABLE I
Comparison of the number of bins to pack the item.

| Class | $W \times H$ | n | nBin (IMA) | time (IMA) | nBin (SACO) | time (SACO) |
|---|---|---|---|---|---|---|
| 1 | $10 \times 10$ | 20 | 6.6 | 0.003 | 6.6 | 0.012 |
| | | 40 | 12.9 | 0.067 | 12.9 | 0.075 |
| | | 60 | 19.5 | 0.001 | 19.5 | 0.004 |
| | | 80 | 27.0 | 0.110 | 27.0 | 0.200 |
| | | 100 | 31.3 | 0.321 | 31.1 | 0.330 |
| 2 | $30 \times 30$ | 20 | 1.0 | 0.001 | 1.0 | 0.000 |
| | | 40 | 1.9 | 0.003 | 1.9 | 0.007 |
| | | 60 | 2.5 | 0.000 | 2.5 | 0.000 |
| | | 80 | 3.1 | 0.015 | 3.1 | 0.020 |
| | | 100 | 3.9 | 0.024 | 3.9 | 0.024 |
| 3 | $40 \times 40$ | 20 | 4.7 | 0.001 | 4.7 | 0.001 |
| | | 40 | 9.4 | 0.126 | 9.4 | 0.126 |
| | | 60 | 13.5 | 0.242 | 13.5 | 0.242 |
| | | 80 | 18.4 | 0.359 | 18.4 | 0.359 |
| | | 100 | 22.2 | 1.173 | 21.9 | 1.250 |
| 4 | $100 \times 100$ | 20 | 1.0 | 0.000 | 1.0 | 0.000 |
| | | 40 | 1.9 | 0.001 | 1.9 | 0.001 |
| | | 60 | 2.5 | 0.432 | 2.5 | 0.445 |
| | | 80 | 3.1 | 0.323 | 3.0 | 0.400 |
| | | 100 | 3.7 | 0.032 | 3.68 | 0.400 |
| 5 | $100 \times 100$ | 20 | 5.9 | 0.000 | 5.9 | 0.000 |
| | | 40 | 11.4 | 0.067 | 11.9 | 0.067 |
| | | 60 | 17.4 | 0.334 | 17.4 | 0.334 |
| | | 80 | 23.9 | 0.228 | 23.9 | 0.228 |
| | | 100 | 27.9 | 1.154 | 27.87 | 1.154 |
| 6 | $300 \times 300$ | 20 | 1.0 | 0.000 | 1.0 | 0.000 |
| | | 40 | 1.7 | 0.304 | 1.7 | 0.300 |
| | | 60 | 2.1 | 0.009 | 2.1 | 0.0100 |
| | | 80 | 3.0 | 0.012 | 3.0 | 0.020 |
| | | 100 | 3.2 | 0.300 | 3.1 | 0.400 |
| 7 | $100 \times 100$ | 20 | 5,2 | 0,072 | 5,2 | 0.1 |
| | | 40 | 10.4 | 0.309 | 10.4 | 0.351 |
| | | 60 | 14.7 | 0.615 | 14.7 | 0.623 |
| | | 80 | 21.2 | 1.317 | 21.2 | 1.413 |
| | | 100 | 25.3 | 1.928 | 25.3 | 2.013 |
| 8 | $100 \times 100$ | 20 | 5,3 | 0.063 | 5.3 | 0.102 |
| | | 40 | 10.4 | 0.354 | 10.4 | 0.391 |
| | | 60 | 15.0 | 0.704 | 15.0 | 0.805 |
| | | 80 | 20.8 | 1.168 | 20.7 | 1.280 |
| | | 100 | 25.7 | 1.904 | 25.6 | 2.01 |
| 9 | $100 \times 100$ | 20 | 14.3 | 0.001 | 14.3 | 0.004 |
| | | 40 | 27.5 | 0.003 | 27.5 | 0.007 |
| | | 60 | 43.5 | 0.004 | 43.5 | 0.334 |
| | | 80 | 57.3 | 0.012 | 57.3 | 0.029 |
| | | 100 | 69.3 | 0.014 | 69.2 | 0.104 |
| 10 | $100 \times 100$ | 20 | 4.1 | 0.040 | 4.1 | 0.06 |
| | | 40 | 7.3 | 0.146 | 7.3 | 0.200 |
| | | 60 | 10.1 | 0.533 | 10.00 | 0.670 |
| | | 80 | 12.8 | 0.876 | 12.70 | 0.920 |
| | | 100 | 15.8 | 1.089 | 15.7 | 1.23 |

and could notice that the best results have been obtained for $\gamma$ = 3. As for the $\rho$ parameter, the best results have been gotten when $\rho = 0.7$. Concerning the $\beta$ parameter, we tested a range of values between 2 and 10 and it appears that this parameter influences the solution quality, but can't determine for a set of problem or for a whole class, an optimal value. For every process, we tested values 2 and 5 and carried the best solution at every time.

## VI. Conclusion

In this paper, we have proposed new pretreatments for the two-dimensional bin-packing problem, the non-oriented case. We have proposed a new heuristic method (SACO) for the resolution of the problem. We have tested the pretreatments on benchmarks from the literature and shown their efficiency to simplify the instances. We have also compared the results obtained by our heuristic (SACO) with those of the recent literature and shown that it gives better results in average, and that it is competitive in terms of speed.

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