Weakly generalized closed map

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Abstract: In this paper we introduce a new class of mg-continuous mapping and studied some of its basic properties.We obtain some characterizations of such functions. Moreover we define sub minimal structure and further study certain properties of mg-closed sets.

Keywords: m-structure, mg-continuous mapping, minimal structure, mg $T_2 space$,
sub minimal structure, $T_{\frac{1}{2}}$ space, mg-compact set

1. Introduction

Levine [9] introduced the concept of g-closed sets and studied their properties. A subset A of a space X is g-closed if and only if $cl(A) \subset O$ whenever $A \subset O$ and O is open. Hence every closed set is a g-closed set. The union and intersection of two g-closed set is g-closed set. Regular open sets and stronger regular open sets have been introduced and investigated by Stone[19] and Tang[21] respectively. Complements of regular open sets and strong regular open sets are called regular closed sets and strong regular closed sets. Andrijecvic [1], Arya and Nour[2], Bhattacharya and Lahiri[5], Levine[9],[10],Mashour et al[13] and Njastad[17] introduced and investigated semi-preopen sets, generalized semi open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets, generalized open set, semi-open sets pre-open sets and α-open sets which are some of the weak forms of open sets and the complements of theses sets are called the same types of closed sets respectively. Ganster and Reilly [8] have introduced locally closed sets which are weaker than both open and closed sets. Cameron[6] has introduced regular semi-open sets which are weaker than regular open sets.

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2. Preliminaries

In this section we begin by recalling some definitions and properties.

Let (X, τ) be a topological spaces and A be a subset. The closure of A and interior of A are denoted by $cl(A)$ and $int(A)$ respectively. We recall some generalized open sets.

Definition [9] 2.1: A subset A of a space X is g-closed if and only if $cl(A) \subset G$ whenever $A \subset G$ and G is open.

Definition [20]2.2: A map $f : X \rightarrow Y$ is called g-closed if each closed set F of $X, f(F)$ is g-closed in Y.

Definition[18]2.3: A map$f : X \rightarrow Y$ is called semi-closed if each closed set F of $X, f(F)$ is semiclosed in Y.

Definition [15] 2.4 : A map $f : X \rightarrow Y$ is called α-open if each open set F of X, f(F) is α-set in Y.

Definition [7]2.5 : A map $f : X \rightarrow Y$ is called pre-closed if for each closed map F of X, f(F) is pre-closed in Y.

Definition [12]2.6: A map $f : X \rightarrow Y$ is called regular-closed if for each set F of $X, f(F)$ is regular closed in Y.

Definition (11)2.7: A map $f : X \rightarrow Y$ is said to be strongly continuous if $f^{-1}(V)$ is both open and closed in X for each subset V of Y.

Definition [4] 2.8:A map $f : X \rightarrow Y$ is said to be generalized continuous if $f^{-1}(V)$ is g-open in X for each set V of Y

Definition [15] 2.9 A subset $A$of a topological space X is said to be weakly generalized closed (wg-closed) set in X if G contains $cl(int(A))$ whenever G

contains A and G is open in X.

Definition[9] 2.10A topological space X is said to be *T1/2*-space if every g-closed set is closed.

Remark:2.11: The following diagram are well known.

$closed \Rightarrow g - closed \quad w - closed$

$\swarrow$

$regularclosed \Rightarrow wg - closed \quad \Leftarrow \alpha - closedset$

$\swarrow \nwarrow$

$gsp - closedset \quad Pre - closedset$

3.Properties of Weakly generalized closed

In this section we studied some of wg-closed sets properties.

Definition 3.1: A map $f : X \rightarrow Y$ is called wg-closed map if for each closed set F of X, f(F) is wg-closed set.

Remark 3.2: Every g-closed map is a wg-closed map and the converse is need not be true from the following example.

Example3.3: Let $X = \{a, b, c\}$ and $\tau_1 = \{\phi, x, \{a\}, \{b\}, \{a, b\}\}$,$\tau_2 = \{\phi, X, \{a\}, \{a, b\}\}$ be topologies on X. Let $\{a, c\}$ is T_1-closed but not T_2-closed.

Theorem 3.4: A map $f : X \rightarrow Y$ is wg-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a wg-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$

Proof: Suppose f is wg-closed. Let S be a subset of Y and U is an open set of X such that $f^{-1}(S) \subset U$. Then $V = Y - f^{-1}(X - U)$is a wg-open set containing S such that $f^{-1}(V) \subset U$.

For the converse suppose that F is a closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ and $X - F$ is open. By hypothesis there is wg-open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore $F \subset X - f^{-1}(V)$. Hence $Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$ which implies $f(F) = Y - V$. Since $Y - V$ is wg-closed if $f(F)$ is wg-closed and thus f is a wg-closed map.

Theorem 3.5: If $f : X \rightarrow Y$ is continuous and wg-closed and A is a wg-closed set of X then $f(A)$ is wg-closed.

proof: Let $f(A) \subset O$ where O is an open set of Y. Since f is g-continuous, $f^{-1}(O)$ is an open set containg A. Hence $cl(A) \subset f^{-1}(O)$ is A is wg-closed set. since f is wg-closed, $f(cl(A))$ is a wg-closed set contained in the open set O which implies than $cl(f(Cl(A)) \subset O$ and hence $clf(cl(A)) \subset O$ and hence $cl(f(A)) \subset O$ so f is a wg-closed set.

corollary 3.6: If $f : X \rightarrow Y$is g-continuous and closed and A is g-closed set of X the $f(A)$is wg-closed.

Corollary 3.7: If $f : X \rightarrow Y$ is wg-closed and continuous and A is wg-closed set of X then $f_A : A \rightarrow Y$ is continuous and wg-closed set.

Proof Let F be a closed set of A then F is wg-closed set of X. From above theorem 3.5 follows that $f_A(F) = f(F)$ is wg-closed set of Y. Here f_A is wg-closed and continuous.

Theorem 3.8 If a map $f : X \rightarrow Y$ is closed and a map $g : Y \rightarrow Z$ is wg-closed then $f : X \rightarrow Z$is wg-closed.

Proof Let H be a closed set in X. Then $f(H)$ is closed and $(g \circ F)(H) = g(f(H))$ is wg-closed as g is wg-closed. Thus $g \circ f$ is wg-closed.

Theorem 3.9: If $f : X \rightarrow Y$ is continuous and wg-closed and A is a wg-closed set of X then $f_A : A \rightarrow Y$is continuous and wg-closed.

Proof: If F is a closed set of A then F is a wg-closed set of X. From Theorem 3.4, It follows that $f_A(F) = f(F)$ is a wg-closed set of Y. Hence f_A is wg-closed. Also f_A is continuous.

Theorem 3.10: If $f : X \rightarrow Y$ is wg-closed and $A = f^{-1}(B)$ for some closed set B of Y then $f_A : A \rightarrow Y$ is wg-closed .

Proof: Let F be a closed set in A. Then there is a closed set H in X such that $F = A \cap H$. Then $f_A(F) = f(A \cap H) = f(H) \cap f(B)$. Since f is wg-closed $f(H)$ is wg-closed in Y. so $f(H) \cap B$ is wg-closed in Y. Since the intersection of a wg-closed and a closed set is a wg-closed set. Hence f_A is wg-closed.

Remark 3.11: If B is not closed in Y then the above theorem is not hold from the following example.

Example 3.12: Take $B = \{a, b\}$. Then $A = f^{-1}(B) = \{a, b\}$ and $\{a\}$ is closed in A but$f_A(\{a\}) = \{a\}$ is not wg-closed in $Y.\{a\}$ is also not wg-closed in B.

4. Normal and Regularity

In this section we introduce the new class of wg-regular and studied some of its properties.

Theorem 4.1: If $f : X \rightarrow Y$is continuous , wg-closed map from a normal space X onto a space Y then Y is normal.

Proof: Let A, B be disjoint closed sets in Y. Then $f^{-1}(A), f^{-1}(B)$ are disjoint closed sets of X. Since X is normal there are disjoint open sets U, Vin X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is wg-closed by theorem 3.4, there are wg-open sets G, H in Y such that $A \subset G, B \subset H$ and $f^{-1}(G) \subset U$and $f^{-1}(H) \subset V$. Since U, V are disjoint $intG, intH$ are disjoint open sets. Since G is wg-open, A is closed and $A \subset G, A \subset intG$. similarly $B \subset intH$. Hence Y is normal.

Theorem 4.2: If $f : X \rightarrow Y$ is an open continuous wg-closed surjection, where X is regular then$Y$ is regular.

Proof: Let U be an open set containing a point P in Y. Let X be a point of X such that$f(X) = P$. Since X is regular and f is continuous there is an open set U such that $x \in V \subset cl(V) \subset f^{-1}(V)$. Hence $P \in f(V) \subset f(Cl(V)) \subset U$. Since f is wg-closed $f(Cl(V))$ is wg-closed set contained in the open set U. It follows that$cl(f(Cl(V)) \subset U$ and hence$p \in f(V() \subset cl(f(V)) \subset U$and $f(V)$ is open. Since f is open. Hence Y is regular.

Remark 4.3: The normality is preserved under regular closed, continuous and surjective.

Example 4.4: In the example 3.12. It is shown that f is wg-closed $\{a, b\}$ is a regular closed set in $(X, \tau_1)$and it is not closed in (X, τ_2). Hence f is not regular closed.

Example 4.5 Let T_1 be the countable complement topology on the real line R and T_2 be the usual topology on R and $f : (R, T_1) \rightarrow (R, T_2)$ be the identity map. Then f is regular closed by the remark immediately after the above example. But f is not wg-closed. For if$A = \{1/n, n \in N\}$ then A is closed in (R, T_1) and $f(A) = A$ is not wg-closed as $f(A) \subset (0, 2)$ and $(0, 2)$ is open in (R, T_2). But $clf(A) \subset (0, 2)$.

Theorem 4.6: If A is wg-closed set of a space X then $IndA \leq IndX$

Proof: It suffices to show that if $IndX \leq n$ and A is wg-closed set of X then $IndA \leq n$. We prove this theorem by induction. The result holds trivially for n=1. Assume that for every wg-closed set A of X ind $X \leq n - 1 \Rightarrow Ind \leq n - 1$.

Let X be space with $Ind \leq n$. Let A be a wg-closed set of X. Let E be a closed set of A and G be an open set of A such that $E \subset G$. Then there exist a closed set F of X and an open set H of X such that $E = A \cap F$ and $G = A \cap H$. Since E is closed in A and A is wg-closed. Since $IndX \leq n$, there is an open set V of X such that $clE \subset V \subset H$and $Indbd(V) \leq n - 1$. Then $V \cap A$ is an open set of A such that $E \subset V \cap A \subset G$ and $bd_A(V \cap A) \subset bd(V)$. Now $bd_A(V \cap A)$ is a wg-closed set of $bd(V)$. By induction hypothesis and $Indbd_A(V \cap A) \leq n-1$. Hence $IndA \leq n$.

Theorem 4.7: If A is a wg-closed set of a space X then dime $A \leq dimX$.

Proof If $dimX = 0$ then $dimA \leq 0 = dimX$. Hence $dimA \leq dimX$.
If $dimX \leq 0$ then $dimX = n$, where n is an integer greater than or equal to -1. If $n = -1 dimX = -1$ which implies that$X = \phi$ and hence $A = \phi$ and $dimA = -1 = dimX$ and thus $dimA \leq dimX$.
Next suppose $dimX = n$ where $n \geq -1$ and let A be a wg-closed set of X. Let $\{u_1, u_2, u_3, ...u_k\}$ be a finite open cover of A. Then for$i = 1, 2, 3, ...K$there exist open sets.V_1 of X such that $u_1 = A \cap V_1$. Since A is wg-closed and $\bigcup_k^{i=1} v_i$ is an open set containing $A, clA \subset \bigcup_{i=1}^K pv_i$Since $cl(A)$ is a closed set,$dimcl(A) \leq n$ so the finite open cover $\{clA \cap v_i, i = 1, 2, 3, ..k\}cl(A)$ has a refinement $cl(A) \cap w_i, i = 1, 2, 3, ..k$ or order at most $n + 1$, where each w_1 is open in X and $clAw_1 \subset clA \cap V_1$

for each i. Then $\{A \cap w_i) : i = 1, 2, ...\}$ is an open cover of A refining $\{u_i, i = 1, 2, 3, ...k\}$ and of order not exceeding $n + 1$. Hence $dimA \leq n$ which implies that $dimA \dim X$.

Theorem 4.8: If A is a wg-closed set of a space X then $DindA \leq DindX$.

Proof Let X be a space such that $DindX = n$ and A be a wg-closed set of X. By using the notations of the above thoerem, $clA \subset \bigcup V_i$. Since clA is a closed set,$DindA \leq n$. Hence for every open cover $V_i \cap clA, i = 1, 2, 3...k$ there is a disjoint family $W_i, J = 1, 2, 3, ...k$ of open sets clA refining $V_i \cap clA, i = 1, 2, 3, ...k$ and such that $Dind(clA - \bigcup_{j=1}^{k} W_j) \leq n - 1$. But $A - \bigcup_{j=1}^{k} W_j \subset clA - \bigcup_{j=1}^{k} W_j$and $A - \bigcup_{j=1}^{k} W_j = A \cap (clA - \bigcup_{j=1}^{k} w_j)$is a wg-closed set of clA as the intersection of wg-closed set and closed set is a wg-closed set. By induction hypothesis $Dind(A - \bigcup_{j=1}^{k} W_j) \leq n - 1$. Also $W_j \cap A, j = 1, 2, 3...k$is a disjoint family of open sets of A refining $u_1, U_2, ...U_k$. Thus $DindA \leq n$and the theorem is proved.

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