Application of Cubic Spline Interpolation to Walking Patterns of Biped Robot

Aye Aye Thant, and Khaing Khaing Aye

***Abstract*—**This paper presents cubic spline interpolation based trajectory planning method which is aiming to achieve smooth biped robot walking trajectory. We first characterize the bipedal walking cycle and point out some major issues that need to be addressed to plan a continuous swing leg trajectory by using the concept of cubic polynomial and cubic spline interpolation. By applying these interpolations, the walking trajectory will be smooth and continuous. This paper will provide application of smooth walking trajectory of biped robot.

***Keywords*—**Biped robot, bipedal walking cycle, cubic polynomial, cubic spline interpolation, smooth walking trajectory.

I. Introduction

A ***biped robot*** is two-legged robot and is expected to eventually evolve into one with a human-like body [3]. Each leg of an anthropomorphic biped robot consists of a thigh, a shank, a foot and has six degrees of freedom (DOF); three DOF in the hip joint, one in the knee joint and two in the ankle joint as shown in Fig. 1.

Biped robots have higher mobility than conventional wheeled robots, especially when moving on rough terrain, steep stairs, and in environments with obstacles. Since a biped robot tends to tip over easily, it is necessary to take stability into account when determining a walking pattern. The walk of a biped robot can be determined by controlling the hip and foot trajectories. For a biped robot to be able to walk in various ground conditions, such as on level ground, over rough terrain, and in environments which filled with obstacle, the robot must be capable of various types of foot motion and high path curvature of trajectory planning will be needed. For example, a biped robot should be able to lift its feet high enough to negotiate obstacles, or have support feet with suitable angles to match the roughness of the terrain.

Most previous literature has described foot trajectories generated by polynomial interpolation. When there are various constraints such as ground conditions and various foot motions, the order of the polynomial is too high and its computation is difficult, and the trajectory may oscillate. Another basic requirement for trajectory planning is to achieve smooth walking pattern for biped robots. In this regard, the walking trajectories should be characterized as being continuous for both first and second order derivatives. The first-order derivative continuity guarantees the smoothness of velocity while the second-order derivative continuity guarantees the smoothness of acceleration or torque [8].

Foot and hip trajectories are discussed on one walking step. Biped robot motion in 3D space, X axis points to the forward direction, Z axis points upward, and Y axis is cross product of the Z and X axis. The X-Z plane is the sagittal plane, X-Y plane is the transverse plane and Y-Z plane is frontal plane. In this research, trajectories are discussed only in the sagittal plane.

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II. Bipedal Walking Cycle

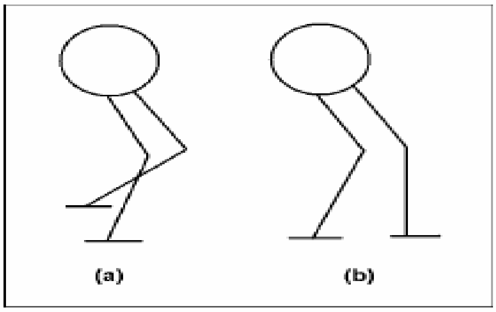


Fig. 1 Walking phases, (a) single support (b) double support

An anthropomorphic biped robot with a trunk [3] is considered. Biped walking is a periodic phenomenon. A complete walking cycle is composed of two phases. These two phases are double support phase and single support phase. During the **double support phase**, both feet are in contact with the ground. During the **single support phase**, while one foot is stationary with the ground, the other foot swings from the rear foot to the front as shown in Fig. 1.

If both foot trajectories and hip trajectory are known, all joint trajectories of the biped robot will be determined by kinematic constraints. The walking pattern can therefore be denoted by both foot trajectories and hip trajectory.

III. Mathematical Background

In this section, cubic spline interpolation will be described as mathematical background to generate walking trajectory of biped robot.

Cubic Spline Interpolation

Suppose that $\{(t_j, f_j)\}_{j=1}^{n}$ are n points, where $t_1 < t_2 < ... < t_n$. The function $S(t)$ is called a ***cubic spline*** if there exist $n-1$ cubic polynomials $S_j(t)$ with coefficients $a_{j,0}, a_{j,1}, a_{j,2}$ and $a_{j,3}$ that satisfy the following properties [5]:

I. $S(t) = S_j(t) = a_{j,0}(t-t_j)^3 + a_{j,1}(t-t_j)^2 + a_{j,2}(t-t_j) + a_{j,3}$

for $t \in [t_j, t_{j+1}]$ and $j = 1,2,\ldots,n-1.$

II. $S(t_j) = f_j$ for $j = 1,2,\ldots,n.$

III. $S_j(t_{j+1}) = S_{j+1}(t_{j+1})$ for $j = 1,2,\ldots,n-2.$

IV. $S'_j(t_{j+1}) = S'_{j+1}(t_{j+1})$ for $j = 1,2,\ldots,n-2.$

V. $S''_j(t_{j+1}) = S''_{j+1}(t_{j+1})$ for $j = 1,2,\ldots,n-2.$

Property I states that $S(t)$ consists of piecewise cubics. Property II states that piecewise cubics interpolate the given sets of data points. Property III and IV require that the piecewise cubics represent a smooth continuous function. Property V states that the second derivative of the resulting function is also continuous.

IV. Application of Cubic Spline Interpolation

A complete one walking step trajectory will be generated by using cubic polynomial and cubic spline interpolation. In this work, a trajectory planning algorithm to control a biped robot will be proposed. The algorithm will be constructed based on the initial and final conditions for the biped robot's position, velocity and acceleration.

A. The Proposed Trajectory Planning Algorithm

The control algorithm for one walking step trajectory is computed as the following steps:

1. The time interval for the breakpoints of one walking step is previously specified.
2. The position constraint for the breakpoints of one walking step is formulated.
3. The whole trajectory between the breakpoints of one walking step is derived by applying cubic polynomial interpolation and cubic spline interpolation.

B. Walking Trajectories for One Walking Step

In this section, the proposed trajectory planning algorithm is applied to find the complete walking trajectories on one walking step. The definition of one walking step is now given as follows:

One Walking Step: The one walking step of the biped robot is defined as to begin with the heel of the right foot leaving the ground and with the heel of the right foot making first contact with the ground.

The considered walking phase represents a single step in the sagittal plane with the right foot swinging. In order to construct complete walking trajectories for one walking step, first the time interval for one walking step is determined and that is the step 1 of the proposed algorithm.

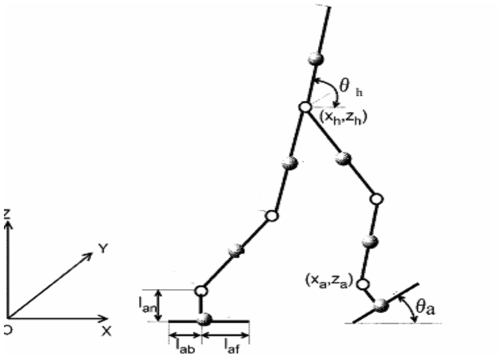


Fig. 2 Model of the biped robot

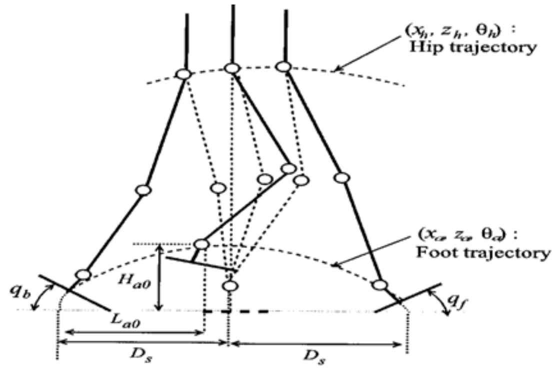


Fig. 3 Walking parameters

Algorithm step 1: **Time Interval for One Walking Step**

Assuming that the period necessary for one walking step is T_c. The time intervals for one walking step as shown in Fig. 4 are specified as follows:

1. Place (entire sole contact) $(t = 0)$
2. Deploy (heel off) $(t = T_d)$
3. Swing (lift on the air) $(t = T_m)$
4. Deploy (heel contact) $(t = T_c)$
5. Place (entire sole contact) $(t = T_c + T_d)$.

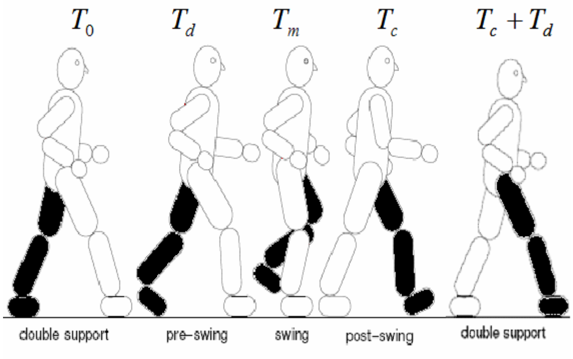


Fig. 4 Time specified for breakpoints of complete walking cycle

Algorithm step 2: **Position Constraints for Walking Trajectories**

The walking of biped robot can be determined by foot and hip trajectories as shown in Fig. 3. So, the walking trajectories are foot trajectory and hip trajectory.

1) Position Constraints for Foot Trajectories

Each foot trajectory can be denoted by a vector

$$X_a = [x_a(t), z_a(t), \theta_a(t)]^T,$$

where $(x_a(t), z_a(t))$ is the coordinate of the ankle position, and $\theta_a(t)$ denotes the angle of the foot as shown in Fig. 2 [3].

Assuming that entire sole surface of the right foot is in contact with the ground at $t = 0$ and $t = T_c + T_d$. Over rough terrain or in environments with obstacles, it is necessary to lift the swing foot high enough to negotiate obstacles. Letting (L_{ao}, H_{ao}) be the position of the highest point of the swing foot , D_s is the length of one step, T_m is the time when the right foot is at its highest point, T_d is the interval of the double-support phase, L_{an} is the height of the foot, L_{af} is the length from the ankle joint to the toe, L_{ab} is the length from the ankle joint to the heel as shown in Fig. 2, $h_{gs}(k)$ and $h_{ge}(k)$ are the heights of the ground surface which is under the support foot and $q_{gs}(k)$ and $q_{ge}(k)$ are the angles of the ground surface under the support foot. Letting q_b and q_f be the designated angles of the right foot as it leaves and lands on the ground respectively as shown in Fig. 3.

The position constraint for breakpoints of one walking step on various ground conditions are described by following:

$$x_a(t) = \begin{cases} 0, & t = 0 \\ L_{an}\sin q_b + L_{af}(1-\cos q_b), & t = T_d \\ L_{ao,} & t = T_m \\ 2D_s - L_{an}\sin q_f - L_{ab}(1-\cos q_f), & t = T_c \\ 2D_s, & t = T_c + T_d \end{cases} \quad (1)$$

$$z_a(t) = \begin{cases} h_{gs}(k) + L_{an}, & t = 0 \\ h_{gs}(k) + L_{af}\sin(q_b) + L_{an}\cos(q_b), & t = T_d \\ H_{ao}, & t = T_m \\ h_{ge}(k) + L_{ab}\sin(q_f) + L_{an}\cos(q_f), & t = T_c \\ h_{ge}(k) + L_{an}, & t = T_c + T_d \end{cases} \quad (2)$$

$$\theta_a(t) = \begin{cases} q_{gs}(k), & t = 0 \\ q_b, & t = T_d \\ q_f, & t = T_c \\ q_{ge}(k), & t = T_c + T_d \end{cases} \quad (3)$$

All the parameters defined in Eq. (1), Eq. (2) and Eq. (3) can be classified foot parameters and ground parameters. D_s, l_{an}, l_{ab}, l_{af}, q_b and q_f are foot parameters and $q_{gs}(k)$, $q_{ge}(k)$, $h_{gs}(k)$, $h_{ge}(k)$, L_{ao} and H_{ao} are ground parameters.

The different foot trajectories for various grounds can be easily produced, by varying the values of ground parameters according to the ground conditions. From this step 2 of the proposed algorithm, walking trajectories on various grounds can be constructed as the followings.

1) Walking on Level Ground

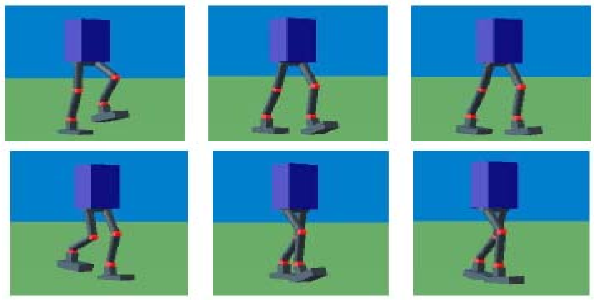


Fig.5 Walking of Biped Robot on Level Ground

If biped robot walks on level ground, the foot trajectories can be obtained by setting $q_{gs}(k) = q_{ge}(k) = h_{gs}(k) = h_{ge}(k) = 0$ as shown in Fig. 5.

2) Walking on Rough Terrain

If biped robot walks on rough terrain, the foot trajectories can be obtained by varying the values of $q_{gs}(k)$, $q_{ge}(k)$,

$h_{gs}(k)$ and $h_{ge}(k)$ according to its ground conditions. For example, $q_{gs} = 0.2\ rad$ for uneven terrain as shown in Fig. 6.

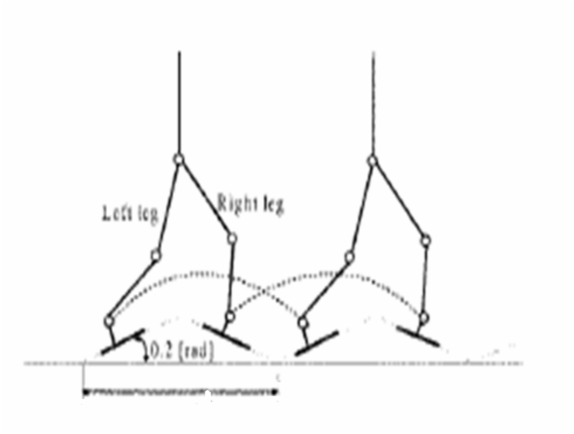


Fig. 6 Walking of biped robot on rough terrain

3) Walking on climbing stairs

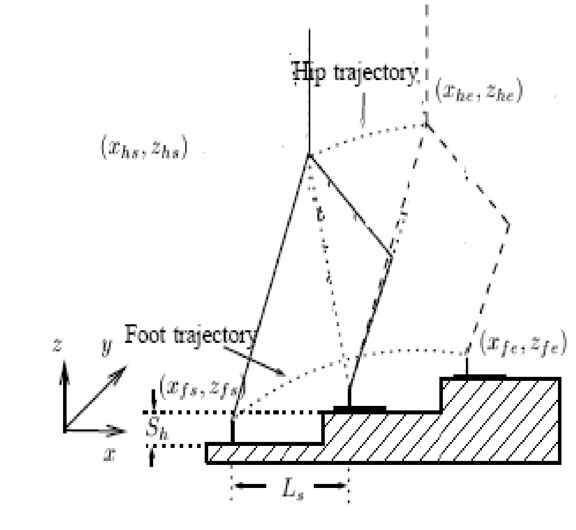


Fig.7 Walking of biped robot on climbing stairs

If biped robot walks on climbing stairs, the foot trajectories can be obtained by varying the values of start and end positions. For example,

$$x_{fe} = x_{fs} + 2L_s,$$
$$z_{fe} = z_{fs} + 2S_h,$$

where (x_{fs}, x_{fe}) and (z_{fs}, z_{fe}) are initial and final position of one walking cycle and L_s is step length and S_h is stair height as shown in Fig. 7.

4) Walking over obstacles

If biped robot walks on the ground that filled with obstacles, the foot trajectories can be obtained by varying L_{ao} and H_{ao} according to the obstacles as shown in Fig. 8.

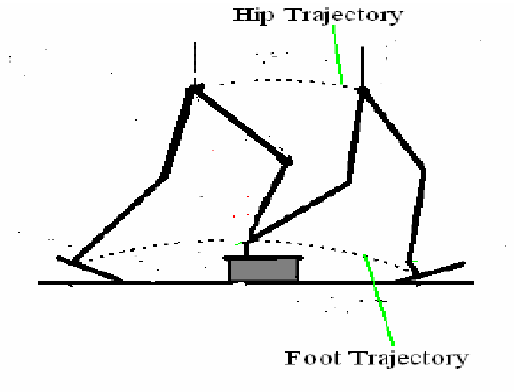


Fig. 8 Walking of biped robot over obstacles

2) Position Constraints for Hip Trajectory

The hip trajectory can be denoted by a vector

$$X_h = [x_h(t), z_h(t), \theta_h(t)]^T,$$

where $(x_h(t), z_h(t))$ denotes the coordinate of the hip position and $\theta_h(t)$ denotes the angle of the hip as shown in Fig. 2 [3].

A complete walking process is composed of three phases: a starting phase in which the walking speed varies from zero to a desired constant velocity, a steady phase with a desired constant velocity, an ending phase in which the walking speed varies from a desired constant velocity to zero.

Letting x_{sd} and x_{ed} denote distances along the x-axis from the hip to the ankle of the support foot at the start and end of the single-support phase, respectively as shown in Fig. 9. $x_h(t)$ can be described by the double support phase and the single support phase, during one-step cycle. We get the following equation

$$x_h(t) = \begin{cases} x_{ed}, & t = 0 \\ D_s - x_{sd}, & t = T_d. \\ D_s + x_{ed}, & t = T_c \end{cases} \quad (4)$$

Hip motion $x_h(t)$ hardly affects the position of the ZMP. By defining different values for x_{sd} and x_{ed} to vary within a fixed range, in particular

$$\begin{cases} 0.0 < x_{sd} < 0.5D_s \\ 0.0 < x_{ed} < 0.5D_s \end{cases}$$

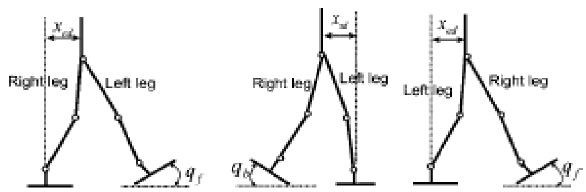


Fig. 9 Walking cycle

Based on the trajectory of $x_h(t)$ and (4) and the ZMP, a smooth trajectory with the largest stability margin can be formulated as follows:

$$\begin{aligned} &\max \quad \mathrm{d}_{\mathrm{zmp}}(x_{sd}, x_{ed}) \\ &x_{sd} \in (0, 0.5D_s), x_{ed} \in (0, 0.5D_s) \end{aligned} \tag{5}$$

where $\mathrm{d}_{\mathrm{zmp}}(x_{sd}, x_{ed})$ denotes the stability margin.

Hip motion $z_h(t)$ to be constant, or to vary within a fixed range. Assuming that $H_{h\max}$ be the hip highest position at the middle of the single-support phase, and $H_{h\min}$ be the hip lowest position at the middle of the double- support phase during one walking step, $z_h(t)$ has the following constraints:

$$z_h(t) = \begin{cases} \mathrm{H_{hmin}}, & t = 0.5\mathrm{T_d} \\ \mathrm{H_{hmax}}, & t = 0.5(\mathrm{T_c} - \mathrm{T_d}) \\ \mathrm{H_{hmin}}, & t = \mathrm{T_c} + 0.5\mathrm{T_d} \end{cases} \tag{6}$$

From the view point of the stability, hip motion parameter $\theta_h(t)$ is constant when there is no waist joint; in particular $\theta_h(t) = 0.5\pi$ rad on level ground.

Algorithm step 3: The Whole Trajectory between the Break Points of One Walking Step

In one walking step trajectory, the path is described in terms of number of points greater than two. The points are to be satisfied more densely in those segments of the paths where obstacles have to be avoided or a high path curvature is expected. Therefore, the problem is to generate a trajectory when *N* points, termed *path points*, are specified and have to be walked at certain instants of time. For each joint variable there are *N* constraints, and then one might want to use (*N-1*) order polynomial. This choice has many disadvantages.

These drawbacks can be overcome if a suitable number of lower order interpolating polynomials, continuous at the path points, are considered in place of single higher order polynomial.

The interpolating polynomial of lowest order is the **cubic polynomial**, since it allows imposing continuity of velocities at the path points. With reference to the single joint variable, a function $q(t)$ is sought, formed by a sequence of *N-1* cubic polynomials $S_j(t)$ for $j = 1,..., N-1$, continuous with continuous first derivatives. The function $q(t)$ attains the values q_j for $t = t_j$ (*j=1,…,N*), and $q_1 = q_i$, $q_N = q_f$, $t_N = t_f$; the q_j represents the path points describing the desired trajectory at $t = t_j$.

To do this, the desired velocity at each via points is needed to specify. There are several ways in which the desired velocity at via points must be satisfied.

1) *The user specifies arbitrary values of velocities at the path points*.

2) *The system automatically chooses the velocities at the path points by applying a certain criterion*.

3) *The system automatically chooses the velocities at the path points to cause the acceleration shall be continuous*.

Based on the above three concepts, three methods can be determined, namely, method 1 for concept 1, method 2 for concept 2 and method 3 for concept 3 respectively.

Method 1: Cubic polynomial interpolations with velocity constraints at path points

This solution requires the user to be able to specify the desired velocity at each path points; the solution does not possess any novelty with respect to the above concepts.

The system of equations allowing the computation of the coefficients of the *N-1* cubic polynomials interpolating the *N* path points is obtained by imposing the following conditions on the generic polynomials $S_j(t)$ interpolating q_j and q_{j+1} for *j=1,…,N-1*:

$$\begin{aligned} S_j(t_j) &= q_j \\ S_j(t_{j+1}) &= q_{j+1} \\ \dot{S}_j(t_j) &= \dot{q}_j \\ \dot{S}_j(t_{j+1}) &= \dot{q}_{j+1} \end{aligned} . \tag{7}$$

The result is $N-1$ system of four equations in the four unknown coefficients of the generic polynomial; these can be solved one independently of the other. The initial and final velocities of the trajectory are typically set to zero i.e., $\dot{q}_1 = \dot{q}_N = 0$, and continuity of velocity at the path points is ensured by setting

$$\dot{S}_j(t_{j+1}) = \dot{S}_{j+1}(t_{j+1}) \tag{8}$$

for $j = 1,..., N-2$. In this method, the resulting discontinuity on the acceleration, since only continuity of velocity is guaranteed. Therefore a convenient system should include either method 2 or 3.

Method 2: Cubic polynomial interpolations with computed velocities at path points

In this case, the velocity at a path point has to be computed according to a certain criterion. Imagine the path points connected with straight line segments. If the slope of these line changes sign at via points, choose zero velocity; if the slope of these line does not change sign, choose the average of the two slopes as the via velocity. In this way, from specification of the desired via points alone, the system can choose the velocity at each points. By interpolating path points with linear segments, the relative velocities can be computed according to the following rules:

$$\dot{q}_1 = 0$$

$$\dot{q}_j = \begin{cases} o & \text{sgn}(v_j) \neq \text{sgn}(v_{j+1}) \\ \frac{1}{2}(v_j + v_{j+1}) & \text{sgn}(v_j) = \text{sgn}(v_{j+1}) \end{cases} \quad (9)$$

$$\dot{q}_n = 0,$$

where $v_j = \dfrac{(q_j - q_{j-1})}{(t_j - t_{j-1})}$ (10)

gives the slope of the segment in the time interval $\lfloor t_{j-1}, t_j \rfloor$. With the above settings the determination of the interpolating polynomials is reduced to the previous case.

Method 3: Cubic spline interpolations with Continuous accelerations at Path Points

Both the above two solutions do not ensure continuity of acceleration at the path points. This system chooses velocities in such a way that acceleration is continuous at via points. To do this, a new approach is needed. In this kind of spline, the two velocity constraints are replaced at the connection of two cubics with two constraints that velocity and acceleration be continuous. The following equations have then to be satisfied:

$$\begin{aligned} S_{j-1}(t_j) &= q_j \\ S_{j-1}(t_j) &= S_j(t_j) \\ \dot{S}_{j-1}(t_j) &= \dot{S}_j(t_j) \\ \ddot{S}_{j-1}(t_j) &= \ddot{S}_j(t_j) \end{aligned} \quad . \quad (11)$$

Since the entire sole surface of the right foot is in contact with the ground at $t = 0$ and $t = T_c + T_d$, The following derivative must be satisfied:

$$\begin{matrix} \dot{x}_a(0) = 0 \\ \dot{x}_a(T_c + T_d) = 0 \end{matrix}, \begin{matrix} \dot{z}_a(0) = 0 \\ \dot{z}_a(T_c + T_d) = 0 \end{matrix}, \begin{matrix} \dot{\theta}_a(0) = 0 \\ \dot{\theta}_a(T_c + T_d) = 0 \end{matrix} . \quad (12)$$

$$\begin{matrix} \ddot{x}_a(0) = 0 \\ \ddot{x}_a(T_c + T_d) = 0 \end{matrix}, \begin{matrix} \ddot{z}_a(0) = 0 \\ \ddot{z}_a(T_c + T_d) = 0 \end{matrix}, \begin{matrix} \ddot{\theta}_a(0) = 0 \\ \ddot{\theta}_a(T_c + T_d) = 0 \end{matrix} . \quad (13)$$

$$\begin{matrix} \dot{x}_h(0) = 0 \\ \dot{x}_h(T_c) = 0 \end{matrix}, \begin{matrix} \dot{z}_h(0.5T_d) = 0 \\ \dot{z}_h(o.5(T_c + T_d)) = 0 \end{matrix} . \quad (14)$$

$$\begin{matrix} \ddot{x}_h(0) = 0 \\ \ddot{x}_h(T_c) = 0 \end{matrix}, \begin{matrix} \ddot{z}_h(0.5T_d) = 0 \\ \ddot{z}_h(o.5(T_c + T_d)) = 0 \end{matrix} . \quad (15)$$

Due to the concept from Eq. (12) to Eq. (15), natural cubic spline interpolation [5] is applied to get smooth trajectories. By using proposed the values of walking parameters as shown in Table. 1 and applying the proposed three methods, we can construct mathematical model for walking trajectory of biped robot.

TABLE I
PROPOSED PARAMETER FOR ONE WALKING STEP

Parameter	Value
T_d	0.15s
T_m	0.5s
T_c	0.9s
L_{an}	7cm
L_{ab}	8cm
L_{af}	8cm
L_{ao}	24cm
H_{ao}	12cm
D_s	25cm
q_b	0.5rad
q_f	0.5rad
x_{sd}	10 cm
x_{ed}	11 cm
$H_{h\min}$	62 cm
$H_{h\max}$	63 cm

For method 1, we let the velocity constraint for via points as the following equation.

$$\dot{x}_a(t) = \begin{cases} v_{xd} & t = T_d \\ v_{xm} & t = T_m \\ v_{xc} & t = T_c \\ v_{xcd} & t = T_c + T_d \end{cases}$$

$$\dot{z}_a(t) = \begin{cases} v_{zd} & t = T_d \\ v_{zm} & t = T_m \\ v_{zc} & t = T_c \\ v_{zcd} & t = T_c + T_d \end{cases}$$

$$\dot{\theta}_a(t) = \begin{cases} v_{\theta d} & t = T_d \\ v_{\theta m} & t = T_m \\ v_{\theta c} & t = T_c \\ v_{\theta cd} & t = T_c + T_d \end{cases}$$

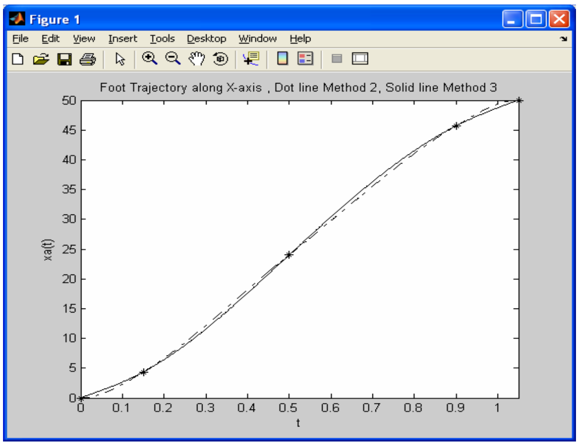
Finally, we get the numerical solutions of complete one walking step foot and hip trajectories for method 1, 2 and 3.

V. One Walking Trajectory by MATLAB Program

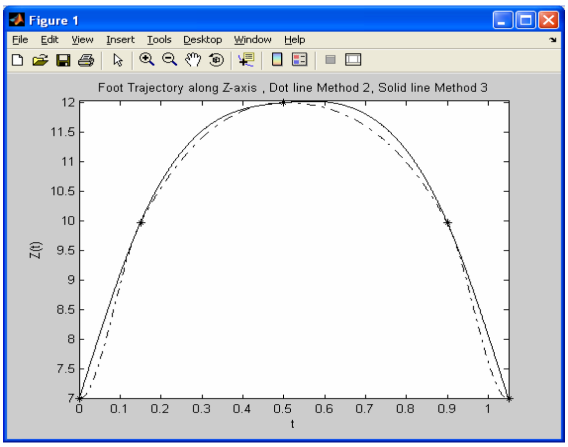
We now show all these cubic polynomial and cubic spline interpolation given by method 2 and 3 by drawing graphs in Matlab program. So that we can check all the results we obtained are correct. But we cannot be able to draw a graph by method 1 as it includes unknown parameters. In the following figure, we mark “star” to show the breakpoints of one walking step trajectory.

A. Demonstration of Foot Trajectory by Applying Matlab

In Fig. 11 (a), (b) and (c), the two result foot trajectories along x axis, z axis and along angle are demonstrated. Cubic polynomial curve is demonstrated by dot line and cubic spline interpolation curve is demonstrated by solid line. Both trajectories of cubic polynomial and cubic spline interpolation are continuous. The trajectory of cubic spline interpolation is smoother than the trajectory of cubic polynomial.



(a)



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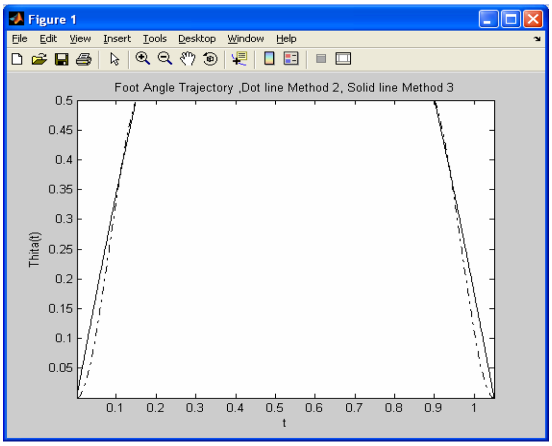


(c)

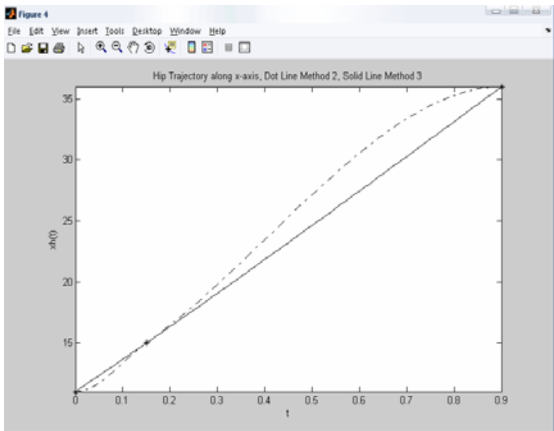
Fig. 11 Graph for One Walking Step Trajectory

(a) $x_a(t)$, (b) $z_a(t)$, (c) $\theta_a(t)$

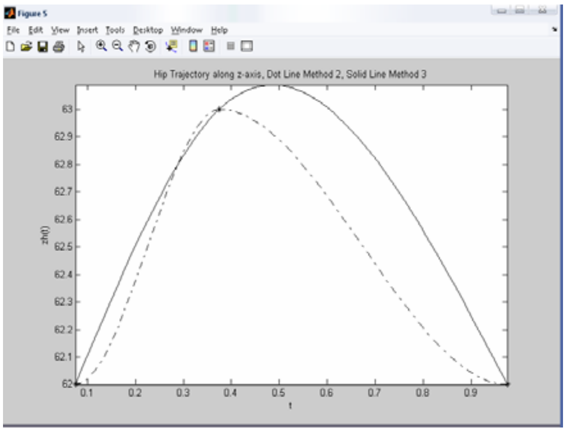
B. Demonstration of Hip Trajectory by Applying Matlab

In this section, the hip trajectory along x axis and z axis are demonstrated and the result trajectories by method 2 and method 3 are compared.

The two result trajectories of hip trajectory along x axis and z axis are demonstrated respectively in Fig.12 (a) and (b). Both trajectories of cubic polynomial curve and cubic spline interpolation curve are continuous. The cubic spline interpolation curve is smoother than the cubic polynomial curve. In Fig.12 (b), it can be clearly seen that the solid curve drawn by method 3 is symmetric and seem to be smooth due to the properties of cubic spline interpolation.



(a)



(b)

Fig. 12 Graph for One Walking Step Trajectory

(a) $x_h(t)$, (b) $z_h(t)$

VI. Conclusion and Discussion

In this paper, mathematical modeling for walking trajectory of a biped robot has been described. As mathematical background of this paper, cubic polynomial and cubic spline interpolation have been described. And then, theoretical element of walking trajectory has been discussed. Finally, the goal of this research has been approached.

The goal of this research is to generate trajectory planning of biped robot for one walking step. So, trajectory planning algorithm has been proposed. The proposed algorithm has the following major contributions:

1) Applying cubic polynomial by method 1, the resulting discontinuity on the acceleration, since only continuity of velocity is guaranteed. But this solution requires the user to be able to specify the desired velocity at each path points. The convenient system should include either method 2 or 3.

2) Applying cubic polynomial by method 2, the suitable continuous velocities for break points of one walking step can be chosen by certain criterion but do not ensure continuity of acceleration at the path points.

3) Applying cubic spline interpolation by method 3, acceleration and velocity continuous at break points of one walking step and this spline method automatically chosen the value of its continuous acceleration by its spline properties.

The work carried out in this paper has shown that dynamically stable, physically feasible and naturally looking walking phase can be generated by mathematical modeling using cubic polynomial, cubic spline interpolation. By applying three methods, we observed that cubic spline interpolation is the best method for trajectory planning. Future work will be considered the other method for trajectory planning of biped robot.

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