

Gravitational and Centrifugal Forces in the Nut-Kerr-Newman Space-Time

Atikur Rahman Baizid, Md. Elias Uddin Biswas, and Ahsan Habib

Abstract—Nayak et al have discussed in detail the inertial forces such as Gravitational, Coriolis-Lense-Thirring and Centrifugal forces in the Kerr-Newman Space-time in the Kerr-Newman Space-time. The main theme of this paper is to study the Gravitational and Centrifugal forces in the NUT-Kerr-Newman Space-time.

Keywords—Gravitational Forces, Centrifugal Forces, Nut-Kerr-Newman, Space time.

I. INTRODUCTION

CHAKRABARTI and Prasanna [1] have described centrifugal forces in the Kerr space-time. They have used the formalism developed by Abramowicz et al[2], which considers the forces in the quotient space orthogonal to the time like killing vector ξ^a . Nayak et al[3] have discussed in detail the inertial forces in the Kerr-Newman Space-time. In this paper, the Gravitational and Centrifugal forces in the NUT- Kerr-Newman Space-time has studied. The NUT- Kerr-Newman Space-time is the Kerr-Newman black-hole Space-time with extra magnetic mass parameter. Here, the expressions for the Gravitational and Centrifugal forces in the NUT- Kerr-Newman Space-time are obtained.

II. THE NUT- KERR-NEWMAN SPACE-TIME PROCEDURE

The NUT- Kerr-Newman Space-time can be written in the form:

$$ds^2 = \left(1 - \frac{\mu}{\rho}\right) dt^2 + 2 \frac{\mu h}{\rho} \sin^2 \theta dt d\Phi - \left(h^2 + r^2 + \frac{\mu h^2}{\rho} \sin^2 \theta\right) \sin^2 \theta d\Phi^2 - \rho^2 \left(\frac{dr^2}{\Delta} + d\theta^2\right) \quad (1)$$

Where

Atikur Rahman Baizid is a lecturer of Department of Business Administration with the Leading University, Sylhet, Bangladesh (e-mail: sustatik2@gmail.com).

Md. Elias Uddin Biswas, is a professor of Department of Mathematics with the Shah Jalal University of Science and Technology, Sylhet, Bangladesh.

Ahsan Habib is a lecturer of Department of ICT with the Metropolitan University, Sylhet, Bangladesh (e-mail: ahferoz@gmail.com, ahabib@metrouni.edu.bd).

$$\begin{aligned} \mu &= 2Mr - e^2 + 2nh \cos \theta \\ \Delta &= r^2 - 2Mr + h^2 + e^2 - n^2 \\ \rho &= r^2 + (n + h \cos \theta)^2 \end{aligned} \quad (2)$$

Where M, e, h and n are the mass, electric charge, angular momentum per unit mass and NUT(magnetic mass) parameters respectively.

The NUT- Kerr-Newman Space-time gives the following results with certain conditions:

- (i) Kerr-Newman black-hole Space-time when $n = 0$
- (ii) Kerr black-hole Space-time $n = e = 0$
- (iii) Reissner-Nordstrom black-hole Space-time if $n = h = 0$
- (iv) Schwarzschild black-hole Space-time when $n = h = e = 0$
- (v) NUT-Kerr Space-time when $e = 0$
- (vi) NUT Space-time when $e = h = 0$

So the NUT- Kerr-Newman Space-time includes all the black-hole space-times which are asymptotically flat are observed.

III. INERTIAL FORCES

The particle 4-velocity is decomposed as

$$\mu^i = \gamma(n^i + v \tau^i) \quad (3)$$

In the above, n^i is a globally hyper surface orthogonal time like unit vector τ^i is the unit vector orthogonal to it along which the spatial 3-velocity 'v' of the particle is aligned and γ is the normalization factor that makes $u^i u_i = 1$. Then the forces acting on the particle can be written as
Gravitational Force

$$G_k = \phi_{,k} \quad (4)$$

Centrifugal force

$$Z_k = -(\gamma v)^2 \bar{\tau}^j \bar{\nabla}_j \bar{\tau}_k \quad (5)$$

Where

$$\phi_{,k} = -n^j n_{k;j} \quad (6)$$

Here $\bar{\tau}^j$ is the unit vector along τ^j in the conformal space orthogonal to n^j with the metric

$$\bar{h}_{jk} = e^{-2\Phi} (g_{jk} - n_j n_k) \quad (7)$$

One can show that the covariant derivatives in the two spaces are related by

$$\bar{\tau}^j \bar{\nabla}_j \bar{\tau}_k = \tau^j \nabla_j \tau_k - \tau^j \tau_k \nabla_j \phi - \nabla_k \phi \quad (8)$$

IV. GRAVITATIONAL AND CENTRIFUGAL FORCES IN THE NUT-KERR-NEWMAN SPACE-TIME

Inertial forces in the NUT-Kerr-Newman space-time for circular orbits with fixed but arbitrary values of r , θ and ω can be computed by using the formalism summarized in the previous section. The forces have the following expressions:

Gravitational force

$$G_k = \frac{1}{\Delta} (0, g_1, g_2, 0) \quad (9)$$

Where

$$g_1 = (r - M) - \frac{\Delta \left[r - \left(\frac{\lambda}{\rho^2} \right) \{ h^2 - (n + h \cos \theta)^2 \} \right]}{\left[(r^2 + h^2 + n^2) + \left(\frac{\mu}{\rho} \right) \{ h^2 - (n + h \cos \theta)^2 \} \right]}$$

$$g_2 = - \frac{\mu \Delta (r^2 + h^2 + n^2) (n + h \cos \theta) \sqrt{ \{ h^2 - (n + h \cos \theta)^2 \} }}{\rho \{ (r^2 + h^2 + n^2) + \mu h \sin^2 \theta \}}$$

$$\lambda = M \{ r^2 - (n + h \cos \theta)^2 \} - (e^2 - n^2) r \quad (10)$$

Centrifugal force

$$Z_k = \frac{W^2}{A} (0, z_1, z_2, 0) \quad (11)$$

Where

$$W = \omega - \frac{\mu h}{\left\{ \rho (r^2 + h^2 + n^2) + \mu (n + h \cos \theta)^2 \right\}}$$

$$A = 1 - \omega^2 \sin^2 \theta (r^2 + h^2 + n^2) - \frac{\mu}{\rho} \left\{ 1 - \omega \left(h - \frac{(n + h \cos \theta)^2}{h} \right) \right\}$$

$$z_1 = \frac{\sin^2 \theta}{\Delta} \left[(r - M) \left\{ (r^2 + h^2 + n^2) + \left(\frac{\mu}{\rho} \right) \{ h^2 - (n + h \cos \theta)^2 \} \right\} - 2\Delta \left\{ r - \left(\frac{\mu}{\rho^2} \right) \{ h^2 - (n + h \cos \theta)^2 \} \right\} \right]$$

$$z_2 = - \sin \theta \cos \theta \left\{ \frac{(r^2 + h^2 + n^2) + \left(\frac{\mu}{\rho} \right) \{ h^2 - (n + h \cos \theta)^2 \}}{2(r^2 + h^2 + n^2) + \rho} \right\} \quad (12)$$

For the equatorial plane, take $\theta = \frac{\pi}{2}$ so that the expressions of the inertial forces in NUT-Kerr-Newman space-time become

Gravitational force

$$G_k = \frac{1}{\Delta} (0, g_1, g_2, 0) \quad (13)$$

Where

$$g_1 = (r - M) - \frac{\Delta \left[r - \left(\frac{\lambda}{(r^2 + n^2)^2} \right) (h^2 - n^2) \right]}{\left[(r^2 + h^2 + n^2) + \left(\frac{\mu}{r^2 + n^2} \right) (h^2 - n^2) \right]} \quad (14)$$

$$g_2 = - \frac{\mu \Delta n (r^2 + h^2 + n^2) \sqrt{ (h^2 - n^2) }}{(r^2 + n^2) \left\{ (r^2 + h^2 + n^2) + \mu h \right\}}$$

$$\lambda = M (r^2 - n^2) - (e^2 - n^2) r$$

Centrifugal force

$$Z_k = \frac{W^2}{A} (0, z_1, 0, 0) \quad (15)$$

Where

$$W = \omega - \frac{(2Mr - e^2) h}{\left\{ (r^2 + h^2 + n^2) (r^2 + n^2) + (2Mr - e^2) n^2 \right\}}$$

$$A = 1 - \omega^2 (r^2 + h^2 + n^2) - \frac{(2Mr - e^2)}{r^2 + n^2} \left\{ 1 - \omega \left(h - \frac{n^2}{h} \right) \right\}$$

$$z_1 = \frac{1}{\Delta} \left[(r - M) \left\{ (r^2 + h^2 + n^2) + \left(\frac{2Mr - e^2}{r^2 + n^2} \right) (h^2 - n^2) \right\} - 2\Delta \left\{ r - \left(\frac{2Mr - e^2}{(r^2 + n^2)^2} \right) (h^2 - n^2) \right\} \right] \quad (16)$$

On the axis of symmetry take $\theta = 0$. So that the expressions of the inertial forces in the NUT-Kerr-Newman Space-time become

Gravitational force

$$G_k = \frac{1}{\Delta} (0, g_1, g_2, 0) \quad (17)$$

Where

$$g_1 = (r - M) - \frac{\Delta \left[r + \left(\frac{\lambda}{\rho^2} \right) (n^2 + 2nh) \right]}{\left[(r^2 + h^2 + n^2) - \left(\frac{\mu}{\rho} \right) (n^2 + 2nh) \right]}$$

$$g_2 = - \frac{i \mu \Delta (r^2 + h^2 + n^2) (n + h) \sqrt{ n^2 + 2nh }}{\rho \{ (r^2 + h^2 + n^2) + \mu h \sin^2 \theta \}}$$

$$\lambda = M \{ r^2 - (n + h)^2 \} - (e^2 - n^2) r \quad (18)$$

Centrifugal force

$$Z_k = 0 \quad (19)$$

V. GRAVITATIONAL AND CENTRIFUGAL FORCES IN THE KERR-NEWMAN SPACE-TIME

(25)

To find the inertial forces in Kerr-Newman space-time, substitute in the above expressions $n = 0$ and then find the following expressions:

Gravitational force

$$G_k = \frac{1}{\Delta}(0, g_1, g_2, 0) \quad (20)$$

Where

$$g_1 = \left[\frac{\Delta \left[r - \left(\frac{\lambda}{\rho^2} \right) h^2 \sin^2 \theta \right]}{(r-m) \left[(r^2 + h^2) + \left(\frac{\mu}{\rho} \right) h^2 \sin^2 \theta \right]} \right]$$

$$g_2 = -\frac{\mu \Delta a^2 (r^2 + h^2) \sin \theta \cos \theta}{\rho \left\{ (r^2 + h^2) + \mu h \sin^2 \theta \right\}}$$

$$\lambda = M(r^2 - h^2 \cos^2 \theta) - e^2 r \quad (21)$$

Centrifugal force

$$Z_k = \frac{W}{A}(0, z_1, z_2, 0) \quad (22)$$

Where

$$W = \omega - \frac{\mu h}{\left\{ \rho(r^2 + h^2) + \mu h^2 \cos \theta \right\}}$$

$$A = 1 - \omega^2 \sin^2 \theta (r^2 + h^2) - \frac{\mu}{\rho} \left\{ 1 - \omega h \sin^2 \theta \right\}^2$$

$$z_1 = \frac{\sin^2 \theta}{\Delta} \left[\frac{(r-M) \left\{ (r^2 + h^2) + \left(\frac{\mu}{\rho} \right) h^2 \sin^2 \theta \right\} - 2\Delta \left\{ r^2 - \left(\frac{\mu}{\rho^2} \right) h^2 \sin^2 \theta \right\}}{\Delta \left\{ r^2 - \left(\frac{\mu}{\rho^2} \right) h^2 \sin^2 \theta \right\}} \right]$$

$$z_1 = -\sin \theta \cos \theta \left\{ (r^2 + h^2) + \left(\frac{\mu}{\rho^2} \right) h^2 \sin^2 \theta (2(r^2 + h^2) + \rho) \right\} \quad (23)$$

For the equatorial plane, take $\theta = \frac{\pi}{2}$ so that the expressions of the inertial forces in Kerr-Newman space-time become:

Gravitational force

$$G_k = \frac{(r-M) \left\{ (r^2 + h^2) r^2 + (2Mr - e^2) h^2 \right\} - \frac{\Delta}{r} \left\{ r^4 - (Mr - e^2) h \right\}}{\left[\Delta \left\{ (r^2 + h^2) r^2 + (2Mr - e^2) h^2 \right\} \right]} (0, 1, 0, 0) \quad (24)$$

Centrifugal force

$$Z_k = \frac{W}{Ar^2 \Delta} \left[\frac{(r-M)r \left\{ (r^2 + h^2) r^2 + (2Mr - e^2) h^2 \right\}}{-2\Delta \left\{ r^4 - (Mr - e^2) h \right\}} \right] (0, 1, 0, 0)$$

Where

$$W = \omega - \frac{(2Mr - e^2) h}{\left\{ (r^2 + h^2) r^2 + (2Mr - e^2) h^2 \right\}}$$

$$A = 1 - \omega^2 (r^2 + h^2) - \frac{(2Mr - e^2)}{r^2} (1 - \omega h)^2 \quad (26)$$

$$\Delta = r^2 - 2Mr + h^2 + e^2$$

On the axis of symmetry take $\theta = 0$.so that the expressions of the inertial forces in the NUT-Kerr-Newman Space-time become:

Gravitational force

$$G_k = \frac{r-m}{r^2 - 2Mr + h^2 + e^2} (0, 1, 0, 0) \quad (27)$$

Centrifugal force

$$Z_k = 0 \quad (28)$$

VI. CONCLUSION

It has derived the expressions for the general relativistic analogues of inertial forces such as Gravitational and Centrifugal forces in the NUT-Kerr-Newman Space-time. For circular orbits the r components and z components of the two forces are active. On the equatorial plane $\left(\theta = \frac{\pi}{2} \right)$ r components and z components of the two forces are active where as on the axis of symmetry $(\theta = 0)$ the gravitational force is non-zero but the Centrifugal force is zero. When $n = 0$, and get the known result for the case of the Kerr-Newman Space-time which is obtained by Neyak et al [3]. In this paper it is observed that the expressions of the Gravitational and Centrifugal forces for the NUT-Kerr-Newman Space-time becomes to the expressions for the Kerr-Newman Space-time when $n = 0$. Due to this observation, it is claimed that this study encompasses the known result of Neyak et al [3] in the context of Kerr-Newman Space-time.

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