A Probabilistic Approach to Workflow Time Analyses for Business Process Management

Nguyen Khoa Viet Truong, Yongsun Choi, Ingu Kim, Sangmun Shin, and Won Joo Hwang

***Abstract*—**A number of methodologies for workflow time analysis has developed on business process modeling. Workflow structures are categorized into four basic structures in business process such as: SEQUENCE, AND, XOR and LOOP. While there are many attempts in literatures to estimate the expected waiting time of these basic structures in workflow processes, there is room for improvement. The aim of this paper is to estimate the expected waiting time for the general AND structure with arbitrary number of activity. In addition, this paper demonstrates a generalized formula for computing the expected waiting time by using a probabilistic approach.

***Keywords*—**Workflow time analysis, Probabilistic approach, Queuing theory, Business process management.

I. Introduction

TIME management and control play a vital role in business processes. One of effective tools for managing those processes is a workflow system that has received much attention researchers and practitioners recently. Workflow time modeling has emerged as a key feature of many applications of the workflow system on manufacturing industries as well as service industries because it can provide a management tool for analyzing the system more effectively.

The workflow time modeling naturally relies on the queuing theory, the mathematical study of waiting lines or queues. The queuing theory can be used to analysis mathematically several related business processes, including arriving at the queue and waiting in the queue. By mean of several performance measures including the average waiting time in the queue or the system, the expected number of waiting or receiving service, the queuing theory enables to consider the system statistically. Also, the queuing theory allow estimating the probability of encountering the system in certain situations, such as empty, full, having an available server or having to wait before being serve.

Nguyen Khoa Viet Truong is with Department of Systems Management & Engineering, Inje University. Gimhae, Gyeongnam, 621-749, South Korea. (e-mail: nkvtruong@yahoo.com).

Yongsun Choi is with Department of Systems Management Engineering, Inje University. Gimhae, Gyeongnam, 621-749, South Korea. (e-mail: yschoi@inje.ac.kr).

Ingu Kim is with Department of Systems Management & Engineering, Inje University. Gimhae, Gyeongnam, 621-749, South Korea. (e-mail: in9-min@nate.com).

Sangmun Shin is with Department of Systems Management Engineering, Inje University. Gimhae, Gyeongnam, 621-749, South Korea. (corresponding author to provide phone: 055-320-3670; fax: 055-320-3632; e-mail: sshin@inje.ac.kr).

Won-Joo Hwang is with Department of Information and Communications Engineering Inje University. Gimhae, Gyeongnam, 621-749, South Korea (e-mail: ichwang@inje.ac.kr).

In 1997, Pozewaunig et al. introduced an extension to the net-diagram technique PERT to computer internal activity deadlines in the presence of waiting of activities in [1]. After that, Eder et al. [2], [3], [4] presented a series of research of workflow time management in which a CPM-based framework and developed earliest and latest time for each activity in best and worst cases for computing activity deadlines. In [5], the authors gave an idea of the timed workflow model for distributed workflow processes. In this paper, with the calculation of flow duration on one time axis, the model can avoid time inconsistency. With a bit innovation, Son and Kim in [6] discussed a method to improve the performance of timed-constrained workflow by increasing capacities for certain activities. Also, in this period, Susan J.Chinn and Gregory R. Madey contributed an article of implementing an expert system for the engineering design using time related data [7]. One year before, Zhao and Stohr initially introduced a conceptual framework for temporal workflow management by developing predicting turnaround time and examining various potential policies for better implementation of claim handling in [8]. That idea still more exploited when Li and Yang in [9] proposed a verification method by analyzing the temporal relationship and resource constraints among concurrent workflows and Reijers and Aalst analyzed the business processes improvement in terms of lead time, service time, waiting time and resource utilization in [10]. Recently, Son et al. in [11], [12] focused on queuing network in extracting workflow critical path from well-formed schema.

Critical path is a great idea of workflow time analysis application. It is also an important tool for effective project management. The most popular algorithm for finding out critical path can be mentioned is the critical path method, a mathematically based algorithm for scheduling a set of project activities. Duk-Ho Chang and his team [13] showed initial results of determining the critical path of a workflow process by analyzing four basic types of structured: SEQUENCE, AND, XOR and LOOP. In this work, based on the queuing theory with the assumption M/M/1 queuing system for each activity, the expected waiting time of General AND structure (comprise of a number of activities) was estimated as a maximum of expected waiting time of all activities in this structure. In other word, the expected waiting time of a whole General AND structure will be the largest value among of expected waiting time of all activities in that structure.

However, the estimation of the expected waiting time for the general AND structure as a maximum of expected waiting time for all activities in that this structure stated above may have a number of limitations. Indeed, if we consider the expected waiting time of each activity is a random variable, also the expected waiting time of the general AND structure is a random variable that is a maximum function of all expected waiting time of activities.

Based on the maximum function of random variables, the expected waiting time of the general AND structure were estimated probabilistically in this paper. Solving this problem with a stochastic point of view can provide a better understanding of problem intrinsically. The results of this work are to build a general formula for calculating the expected waiting time of the general AND structure which is considered a random variable. This value is the result of maximum function of n random variables of activities in it.

The remainder of this paper is organized as follows. Section II introduced workflow queuing model on which the model developed in section III based. Section IV is illustration and discussion of results gained in section III. The last section is intended for conclusions.

II. Workflow Queuing Model

A workflow is a network of activities in which developed queuing theory can be efficiently used to analyze. Networks of queues are systems of many linked activities which contain an arbitrary, but finite, a number of queues.

One of the basic assumption of queuing theory is that all activities in the system are considered as M/M/1 model (further information on M/M/1 model can be referred to [14]). As a result, a workflow can be modeled as an M/M/1 queuing network. The illustration of M/M/1 model can be showed in Fig. 1 in which the arrival rate is of Poisson distribution with mean λ and the service rate is of exponential distribution with mean $1/\mu$ [13]. Also, M/M/1 model itself bases on following basic assumptions:

- The activation of each activity is followed M/M/1 model, with (first in-first out) FIFO dispatching principle or queuing discipline. This also mean that workflow instances are served in the same order as they arrive at queue.
- The queue size of an activity is sufficiently large to accommodate a large number of workflow instances.

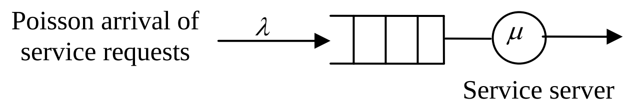

Fig. 1 M/M/1 model of each activity

For each activity i^{th}, the waiting time (including queuing time and service time) has the probability density function (PDF) as following [14]:

$$w_{X_i}(x_i) = \mu_i(1-\rho_i)e^{-\mu_i(1-\rho_i)x_i} = (\mu_i - \lambda_i)e^{-(\mu_i-\lambda_i)x_i} \quad (1)$$

where

$\rho_i = \dfrac{\lambda_i}{\mu_i}$: Utilization rate of activity i^{th}. For stable condition, it must be satisfied following condition:

$$\rho_i \le 1 \text{ or } \lambda_i \le \mu_i \quad (2)$$

The expectation of waiting time of activity i^{th} is:

$$E(X_i) = \frac{1}{\mu_i(1-\rho_i)} = \frac{1}{\mu_i - \lambda_i} \quad (3)$$

The simple result of (3) can be explained that the expectation of waiting time of each activity depends on the deviation of rate of service and the rate of arrival. The larger this deviation is, the longer customers have to wait before being served. This result will be utilized for developing model for general AND structure in the next section.

III. Model development

A. Conventional Approach of AND Structure Analysis

Given a following general AND structure that comprise of n activities linked as parallel branches from task A to task B (Fig. 2). Each activity X_i has the same arrival rate λ but different service rate μ_i. An instance leaving task A must do all of activities from X_1 to X_n before doing task B. Our goal is to estimate the average waiting time of this structure.

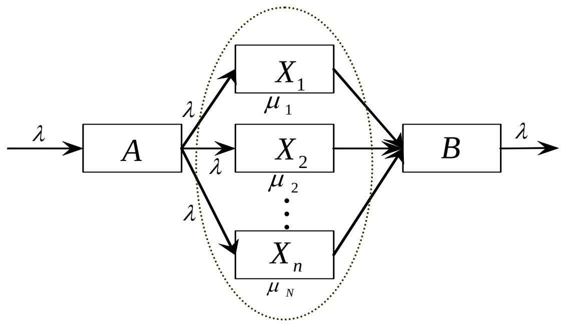

Fig. 2 A general AND structure in workflow

Conventionally, the expected waiting time of the whole structure T can be obtained by two steps: first, estimate the expected waiting time of activities individually and second, choose the largest value among them as (4).

$$T = \max_i\left(\frac{1}{\mu_i - \lambda}\right) \quad (4)$$

where $\dfrac{1}{\mu_i - \lambda}$ is expected waiting time of activity i^{th}.

B. Proposed approach of AND structure analysis

Actually, the problem does not look like simply as it appeared. If we consider that the waiting time of each activity i^{th} is a random variable X_i, the expected waiting time of the whole AND structure is also a random variable Z :

$$Z = \max\left(X_1, X_2, ..., X_n\right) \tag{5}$$

It is clear that the expected waiting time of the general AND structure is the expectation of random variable Z , not the largest value among individual waiting time of all activities. So, the expectation of Z need to be estimated statistically:

$$E(Z) = E\left[\max\left(X_1, X_2, ..., X_n\right)\right] \tag{6}$$

For the sake of simplicity, at first, the problem will be solved in the case of two variables: $Z = \max(X, Y)$. The maximum function for two variables can be expressed simply as follows:

$$Z = \max(X, Y) = \begin{cases} X, & X > Y \\ Y, & X \le Y \end{cases} \tag{7}$$

The cumulative density function (CDF) $F_Z(z)$ of Z can be described by below formulas:

$$\begin{aligned} F_Z(z) &= P(X \le z, Y \le z) \\ &= F_{XY}(z, z) \end{aligned} \tag{8}$$

and

$$\begin{aligned} F_Z(z) &= P\left(\max(X, Y) \le z\right) \\ &= P\left[\left(X \le z, X > Y\right) \cup \left(Y \le z, X \le Y\right)\right] \\ &= P\left(X \le z, X > Y\right) + P\left(Y \le z, X \le Y\right) \end{aligned} \tag{9}$$

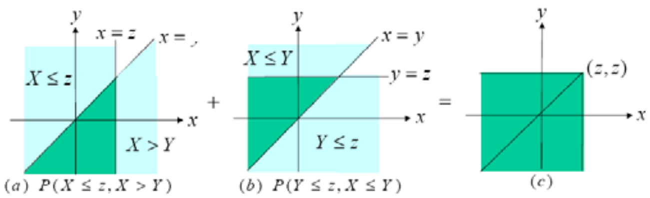

Fig. 3 (a)-(b) show the regions satisfying the corresponding inequalities in each term above; (c) represents the total region

The Fig. 3 shows the relationship of two variables X and Y in the maximum function on which the CDF $F_Z(z)$ of Z based. Assume that X and Y are independent, the CDF $F_Z(z)$ will be:

$$F_Z(z) = F_X(x) F_Y(y) \tag{10}$$

Once the CDF of Z was obtained, the PDF of Z can be derived as:

$$f_Z(z) = F_X(z) f_Y(z) + F_Y(z) f_X(z) \tag{11}$$

With the assumptions of queuing theory based on M/M/1 model as discussed in previous section, we can consider that X and Y have PDF and CDF as follows:

$$f_X(x) = \gamma_1 e^{-\gamma_1 x} \text{ and } F_X(x) = 1 - e^{-\gamma_1 x} \tag{12}$$

$$f_Y(y) = \gamma_2 e^{-\gamma_2 y} \text{ and } F_Y(y) = 1 - e^{-\gamma_2 y} \tag{13}$$

The PDF of Z can be calculated as follows:

$$\begin{aligned} f_Z(z) &= \left(1 - e^{-\gamma_1 z}\right)\gamma_2 e^{-\gamma_2 z} + \left(1 - e^{-\gamma_2 y}\right)\gamma_1 e^{-\gamma_1 z} \\ &= \gamma_1 e^{-\gamma_1 z} + \gamma_2 e^{-\gamma_2 z} - \left(\gamma_1 + \gamma_2\right) e^{-(\gamma_1 + \gamma_2) z} \end{aligned} \tag{14}$$

and the expectation of Z is

$$E_2(Z) = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} - \frac{1}{\gamma_1 + \gamma_2} \tag{15}$$

In the case of three variables with parameters $\lambda_1, \lambda_2, \lambda_3$, similarly, the expectation of Z can be calculated as

$$\begin{aligned} E_3(Z) = &\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} - \frac{1}{\gamma_1 + \gamma_2} - \frac{1}{\gamma_2 + \gamma_3} \\ &- \frac{1}{\gamma_1 + \gamma_3} + \frac{1}{\gamma_1 + \gamma_2 + \gamma_3} \end{aligned} \tag{16}$$

Generalize for N variables:

$$E_N(Z) = \sum_{k=1}^{N} \left((-1)^{k+1} \sum_{1 \le t_1 < t_2 < ... < t_k \le N}^{N} \frac{1}{\gamma_{t_1} + \gamma_{t_2} + ... + \gamma_{t_k}} \right) \tag{17}$$

in which k and t_k denote the activity k^{th} and t_k^{th}, respectively.

This is a very beautiful formula for estimating the expected waiting time of a general AND structure, one of the most complex basic structure in workflow modeling.

IV. Illustration and Discussions

Once the generalized formula was obtained, several specific cases of result can be illustrated. Let $\mu_i - \lambda_i$ equal to γ_i , the expected waiting time for the case of two activities X_1 and X_2 that expressed in the result (15), will become

$$E_2(Z) = \frac{1}{\mu_1 - \lambda_1} + \frac{1}{\mu_2 - \lambda_2} - \frac{1}{\mu_1 + \mu_2 - \lambda_1 - \lambda_2}. \tag{18}$$

It is easily recognized that the expected waiting time for the case of two activities X_1 and X_2 depends on all of parameters of two activities not just parameters of one of two activities as (4) determined.

If three activities X_1, X_2 and X_3 are considered, the result [14] will become:

$$E_3(Z) = \frac{1}{\mu_1-\lambda_1} + \frac{1}{\mu_2-\lambda_2} + \frac{1}{\mu_3-\lambda_3} - \frac{1}{\mu_1+\mu_2-\lambda_1-\lambda_2} - \frac{1}{\mu_2+\mu_3-\lambda_2-\lambda_3} - \frac{1}{\mu_1+\mu_3-\lambda_1-\lambda_3} + \frac{1}{\mu_1+\mu_2+\mu_3-\lambda_1-\lambda_2-\lambda_3} \quad (19)$$

Generalize for N activities, the result (17) will become:

$$E_N(Z) = \sum_{k=1}^{N}\left((-1)^{k+1} \sum_{1\le t_1<t_2<\ldots<t_k\le N}^{N} \frac{1}{\left(\mu_{t_1}+\mu_{t_2}+\ldots+\mu_{t_k}\right)-\left(\lambda_{t_1}+\lambda_{t_2}+\ldots+\lambda_{t_k}\right)}\right) \quad (20)$$

So far, the results were derived basely on the general assumption that the arrival rates are different from each activity. Considering the case in which the arrival rates are the same with all activities $\lambda_i = \lambda$, the results can be simplified as

$$E_2(Z) = \frac{1}{\mu_1-\lambda} + \frac{1}{\mu_2-\lambda} - \frac{1}{\mu_1+\mu_2-2\lambda} \quad (21)$$

$$E_3(Z) = \frac{1}{\mu_1-\lambda} + \frac{1}{\mu_2-\lambda} + \frac{1}{\mu_3-\lambda} - \frac{1}{\mu_1+\mu_2-2\lambda} - \frac{1}{\mu_2+\mu_3-2\lambda} - \frac{1}{\mu_1+\mu_3-2\lambda} + \frac{1}{\mu_1+\mu_2+\mu_3-3\lambda}. \quad (22)$$

In general case, the following formula can be yielded:

$$E_N(Z) = \sum_{k=1}^{N}\left((-1)^{k+1} \sum_{1\le t_1<t_2<\ldots<t_k\le N}^{N} \frac{1}{\left(\mu_{t_1}+\mu_{t_2}+\ldots+\mu_{t_k}\right)-t_k\lambda}\right). \quad (23)$$

If considering the more strict assumption that all activities have the same service rate $\mu_i = \mu$, a simpler form of results can be achieved as follow:

$$E_2(Z) = \frac{2}{\mu-\lambda} - \frac{1}{2(\mu-\lambda)} = \frac{3}{2(\mu-\lambda)} \quad (24)$$

$$E_3(Z) = \frac{3}{\mu-\lambda} - \frac{3}{2(\mu-\lambda)} + \frac{1}{3(\mu-\lambda)} = \frac{11}{6(\mu-\lambda)} \quad (25)$$

and

$$E_N(Z) = \sum_{k=1}^{N}\left((-1)^{k+1} \sum_{1\le t_1<t_2<\ldots<t_k\le N}^{N} \frac{1}{t_k(\mu-\lambda)}\right) \quad (26)$$

This result can be expressed by an alternative form:

$$E_N(Z) = \frac{\binom{N}{1}}{\mu-\lambda} - \frac{\binom{N}{2}}{2(\mu-\lambda)} + \ldots + \frac{(-1)^{k+1}\binom{N}{k}}{k(\mu-\lambda)} + \ldots + \frac{(-1)^{N+1}\binom{N}{N}}{N(\mu-\lambda)}. \quad (27)$$

The (27) can be simplified as

$$E_N(Z) = \sum_{k=1}^{N} \frac{(-1)^{k+1}\binom{N}{k}}{k(\mu-\lambda)}. \quad (28)$$

Since $\gamma = \mu - \lambda$, the (28) can be transformed into the following

$$E_N = \sum_{k=1}^{N} \frac{(-1)^{k+1}\binom{N}{k}}{k\gamma}. \quad (29)$$

The (28) (or (29)) represent generalized formula. This result was illustrated by some succeeding charts in Fig. 4 as below:

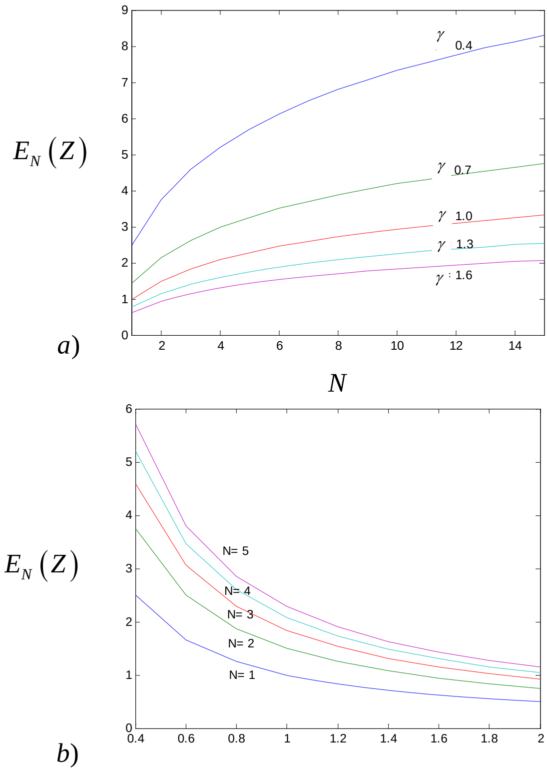

Fig. 4 Expected waiting time $E_N(Z)$ as a function of: a) number of activities N and; b) $\gamma = \mu - \lambda$

The Fig. 4a depicts the relationship between expected waiting time $E_N(Z)$ and the number of activities N while fixing the deviation of rate of service and the rate of arrival $\gamma = \mu - \lambda$. The lager the number of activities is, the longer the expected waiting time will be. On the other hand, the Fig. 4a demonstrates the relationship between expected waiting time $E_N(Z)$ and the deviation of rate of service and the rate of arrival γ while positioning the value of *N*. The lager the difference between service rate and arrival rate is, the shorter the expected waiting time will be.

V. Conclusion

With a stochastic point of view, the expected waiting time of basic AND structures of workflow systems were analyzed probabilistically. Firstly, we established a beautiful general formula for computing the expected waiting time of AND structure. Secondly, special cases of these results were considered. Finally, an illustrated figure was shown the relationship between the expected waiting time and the number of activities in AND structure as well as the deviation from the rates of service and arrival. The results of this paper can reduce remarkably the time consumption in simulation when looking for the critical path of business processes.

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Nguyen Khoa Viet Truong is a PhD student at InJe University, South Korea. His major is Industrial Engineering. He belongs to Quality Design Laboratory and interested in research involved to Quality Design, Process Design and Workflow Modeling.

Yongsun Choi is a Professor in the Department of Systems Management & Engineering. He holds his Ph.D. in Industrial Engineering from Korea Advanced Institute and Technology (KAIST). His main research areas are in process workflow system modeling/analysis and pharmaceutical quality by design. He has been with Stanford University, Tokyo University, and University of Arizona as a Visiting Scholar.

Ingu Kim is also a Master student at InJe University, South Korea with major of Industrial Engineering. His research interests are Quality Design and Business Process Modeling.

Sangmun Shin is an assistant professor and the chair in the Department of Systems Management & Engineering and the director of Quality Design Laboratory at Inje University, South Korea. He holds his M.S. and Ph.D. degrees in Industrial Engineering from Clemson University, USA. His research interests include information management, quality engineering, robust process design, and multi-objective optimization. He received a career development research project from *Korea Research Foundation* (KRF). He currently serves on the editorial board of *International Journal of Quality Engineering and Technology* and *International Journal of Experimental Design and Process Optimization*. He is a member of IIE and a member of *Alpha Pi Mu*.

Won-Joo Hwang received the Ph.D Degree in Information Systems Engineering from Osaka University Japan in 2002. He received his B.S. and M.S. degree in Computer Engineering from Pusan National University, Pusan, Republic of Korea, in 1998 and 2000. Since September 2002, he has been a associate professor at Inje University, Gyeongnam, Republic of Korea. His research interests are in network optimization and future Internet. Mr. Hwang is a member of the IEICE and IEEE.