Marangoni Instability in a Fluid Layer with Insoluble Surfactant

Ainon Syazana Ab. Hamid, Seripah Awang Kechil and Ahmad Sukri Abd. Aziz

***Abstract*—**The Marangoni convective instability in a horizontal fluid layer with the insoluble surfactant and nondeformable free surface is investigated. The surface tension at the free surface is linearly dependent on the temperature and concentration gradients. At the bottom surface, the temperature conditions of uniform temperature and uniform heat flux are considered. By linear stability theory, the exact analytical solutions for the steady Marangoni convection are derived and the marginal curves are plotted. The effects of surfactant or elasticity number, Lewis number and Biot number on the marginal Marangoni instability are assessed. The surfactant concentration gradients and the heat transfer mechanism at the free surface have stabilizing effects while the Lewis number destabilizes fluid system. The fluid system with uniform temperature condition at the bottom boundary is more stable than the fluid layer that is subjected to uniform heat flux at the bottom boundary.

***Keywords*—**Analytical solutions, Marangoni Instability, Nondeformable free surface, Surfactant.

I. Introduction

CONVECTION driven by surface tension effects is called as Marangoni-Bénard convection or simply Marangoni convection. The Marangoni convection can be observed in industrial and technological processes such as the crystal growth production, welding and semi-conductor manufacturing. The convective instability in a horizontal fluid layer heated from below and cooled from above was first studied experimentally by Bénard [1] and theoretically by Rayleigh [2] and Pearson [3]. Marangoni convection usually can affect the quality of the products due to striations, dendrites and bubbles that occur during the manufacturing process.

The Marangoni instability problems due to temperature-dependent surface tension have been investigated for steady and oscillatory convection by [4] and [5]. Numerous studies on the effects of physical factors are considered such as the effect of feedback control [6]–[7], internal heat generation [8], variable viscosity [9] and porous layer [10]. Another important factor is the influence of surface-active agents on thermocapillary convection where the surface tension is dependent on the concentration gradients [11]. Mikishev and Nepomnyashchy [11] studied the long-wavelength Marangoni convection in a liquid layer with insoluble surfactant using perturbation method.

Ainon Syazana Ab. Hamid, Seripah Awang Kechil and Ahmad Sukri Abd. Aziz are with the Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Shah Alam, 40450 Selangor, Malaysia (phone: +603-55435429; fax: +603-55435501; e-mail: ainonsyazana@gmail.com;
e-mail: seripah_awangkechil@salam.uitm.edu.my).

In this paper, we shall investigate the Marangoni convection in a fluid layer with nondeformable surface in the presence of insoluble surfactant and subject to uniform temperature and uniform heat flux at the bottom boundary. We will find the exact analytical solutions for the steady Marangoni convection for the bottom conditions of uniform temperature and uniform heat flux.

II. Problem Formulation

Consider a horizontal layer of fluid with thickness d, bounded below by a rigid wall plate and above by a flat free surface subject to a transverse temperature gradient. Two-dimensional Cartesian coordinates x and z are introduced with $z = 0$ coincides with the plate surface and z-axis is directed vertically upward. The two-dimensional consideration is sufficient for the development of the linear stability theory because of the rotational symmetry [11].

The surface tension σ, is assumed to depend linearly on both temperature T and surfactant concentration Γ,

$$\sigma = \sigma_0 - \sigma_1 T - \sigma_2 \Gamma, \tag{1}$$

where σ_0 is reference value of surface tension, $\sigma_1 = -\partial\sigma/\partial T$ and $\sigma_2 = -\partial\sigma/\partial\Gamma$. Heat is transmitted from the free surface to the atmosphere by Newton's law of cooling

$$\lambda \frac{\partial T}{\partial \mathbf{n}} + qT = 0, \tag{2}$$

where λ is the fluid's thermal conductivity, $\mathbf{n}$ is a normal unit vector to the surface and q is the rate of heat transfer at the free surface.

The system is governed by the equations of conservation of mass, momentum and energy given by

$$\nabla \cdot \mathbf{v} = 0, \tag{3}$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \nu \nabla^2 \mathbf{v}, \tag{4}$$

$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)\, T = \chi \nabla^2 T, \tag{5}$$

where $\mathbf{v} = (u, w)$, T is the temperature, ρ density, p pressure, ν kinematic viscosity, χ thermal diffusivity, ∇ gradient vector and t is the time.

By using the linear stability theory and the introduction of infinitesimal disturbances and scaling for length, time, velocity, temperature and pressure as $d, d^2/\chi, \chi/d, \beta d$ and

$\rho\nu\chi/d^2$, respectively, the linearized non-dimensional equations (3)–(5), in z-component of the velocity w and eliminating the pressure, p, are

$$\Pr^{-1}\frac{d}{dt}\nabla^2 w = w_{xxxx} + 2w_{xxzz} + w_{zzzz}, \tag{6}$$

$$\frac{dT}{dt} + \nabla^2 T = w. \tag{7}$$

The linearized boundary conditions are,

a) at the rigid plate, $z = 0$,

$$w = w_z = 0, \tag{8}$$

$$T = 0 \text{ (uniform temperature) or} \tag{9}$$

$$T_z = 0 \text{ (uniform heat flux).} \tag{10}$$

b) at the free nondeformable surface, $z = 1$,

$$w = 0, \tag{11}$$

$$T_z + Bi \cdot T = 0, \tag{12}$$

$$\gamma_t - w_z = L\gamma_{xx}, \tag{13}$$

$$w_{xx} - w_{zz} + MT_{xx} + N\gamma_{xx} = 0, \tag{14}$$

where γ is the perturbation function with surfactant concentration, $\Pr = \nu/(\rho\chi)$ Prandtl number, $Bi = qd/\lambda$ Biot number, $L = D_0/\chi$ Lewis number, $M = \sigma_1\beta d^2/\eta\chi$ Marangoni number and $N = \sigma_2 d\,\Gamma_0/\eta\chi$ is the elasticity number. D_0 is the surface diffusivity and Γ_0 is the concentration of surfactant in the absence of convection.

At the equilibrium state where the fluid is at rest, the velocity vector and temperature are

$$\mathbf{v} = 0, \tag{15}$$

$$T_{eq} = -z + \frac{1+Bi}{Bi}. \tag{16}$$

In the analysis of normal modes, the velocity, temperature and concentration are in the form of

$$(w, T, \gamma) = (W, \theta, \phi)\exp(ikx + rt), \tag{17}$$

where k and r are the dimensionless wave number and growth rate, respectively. $W(z)$, $\theta(z)$ and $\phi(z)$ are the vertical, temperature and concentration amplitudes.

The linearized dimensionless momentum and heat transfer equations (6) and (7) as in [11],

$$\Pr^{-1} r\left(D^2 - k^2\right)W = \left(D^4 - 2k^2D^2 + k^4\right)W, \tag{18}$$

$$\left(D^2 - k^2\right)\theta = r\theta - w, \tag{19}$$

subject to boundary conditions (8)–(10) at the rigid plate, $z = 0$, become

$$W = DW = 0, \tag{20}$$

$$\theta = 0 \text{ (uniform temperature) or} \tag{21}$$

$$D\theta = 0 \text{ (uniform heat flux),} \tag{22}$$

and the boundary conditions (11)–(14) at the upper free surface, $z = 1$, are

$$W = 0, \tag{23}$$

$$D\theta + Bi\theta = 0 \tag{24}$$

$$r\phi - DW + k^2 L\phi = 0, \tag{25}$$

$$D^2W + k^2W + k^2(M\theta + N\phi) = 0. \tag{26}$$

where $D = \dfrac{\mathrm{d}}{\mathrm{d}z}$.

By eliminating ϕ in (25) and (26), we obtain

$$D^2W + k^2\left(W^2 + M\theta + \frac{N(DW)}{r + k^2L}\right) = 0. \tag{27}$$

Here, we remark that Mikishev and Nepomnyashchy [11] obtained an approximate solution by solving the system (18)–(20) and (23)–(26) for the uniform heat flux condition (22) using asymptotic approximation of perturbation method for the long-wavelength instabilities. In this paper, we consider both uniform temperature (21) and uniform heat flux (22) conditions and solve the system (18)–(24) and (27) by finding the exact analytical solutions for steady convection at all wave number k. We will also assess the effects of the physical parameters on the onset of steady convection.

III. Steady Marangoni Solutions

The onset of convection is determined by the Marangoni number, M, and the growth rate r. At $r = 0$, the stability is at the marginal state in which disturbances neither amplified nor damped. The closed form analytical expressions can be obtained for the marginal stability curves for the onset of steady Marangoni convection by setting the growth rate, $r = 0$.

From (18), the general solution for $W(z)$ is simply

$$W(z) = (C_1 + C_2 z)[\cosh(kz) + \sinh(kz)] + (C_3 + C_4 z)[\cosh(kz) - \sinh(kz)], \tag{28}$$

where $C_i, i = 1,2,3,4$ are constants. From (20) and (23), we obtain,

$$C_2 = C_1(k\coth k - k - 1), \tag{29}$$

$$C_3 = -C_1, \tag{30}$$

$$C_4 = C_1(1 + k - k\coth k), \tag{31}$$

and

$$W = 2C_1\left[(z - 1 - kz\coth k)\sinh(kz) + kz\cosh(kz)\right], \tag{32}$$

where C_1 is an arbitrary constant.

The general solution for the temperature from (19) is

$$\theta(z) = C_5[\cosh(kz) + \sinh(kz)] + C_6[\cosh(kz) - \sinh(kz)] + \theta_p(z), \tag{33}$$

with $\theta_p(z)$ is the particular solution given by

$$\theta_p(z) = \frac{1}{8k^3}\{B_1[\cosh(kz) + \sinh(kz)] + B_2[\cosh(kz) - \sinh(kz)]\} \tag{34}$$

where

$$B_1 = 2C_1(k - 2k^2 z) + C_2(2kz - 2k^2 z^2 - 1), \tag{35}$$

$$B_2 = 2C_3(2k^2 z + k) + C_4(1 + 2kz - 2k^2 z^2). \tag{36}$$

A. Uniform Temperature Condition, $\theta = 0$

From boundary conditions (21) and (27), we obtain the expression for Marangoni number, *M*,

$$M = \frac{4}{LQ}\Big[N\cosh^2 k(k\sinh k + Bi\cosh k) + (2k^2 L - k^3 N - 2k^3 L - kN)\sinh k + (2BiLk\sinh k - k^2 BLN - 2k^2 L - 2k^2 BiL - BiN)\cosh k\Big] \tag{37}$$

where $Q = \cosh^3 k - (k^3 + 2k)\sinh k + (k^2 - 1)\cosh k$.

B. Uniform Heat Flux Temperature Condition, $D\theta = 0$

The solution for the Marangoni number, *M*, obtained from boundary conditions (22) and (27),

$$M = \frac{4}{LQ}\Big\{(2k^2 L - k^3 N - 2k^3 L - kN)\sinh k - (2k^2 L + Nk^2 Bi + 2k^2 LBi + NBi)\cosh k + \cosh^2 k\,[(kN + 2kBiL)\sinh k + BiN\cosh k]\Big\} \tag{38}$$

where $Q = \cosh^3 k - (k^3 + 2k)\sinh k + (k^2 - 1)\cosh k$.

The expression for Marangoni number, *M* in (37) and (38) will be used to plot the marginal curves for steady Marangoni convection.

IV. Results and Discussions

In this section, the graphical results for the steady marginal curves which separate the regions of stable and unstable modes show the effects of the physical parameters and temperature boundary conditions on the critical Marangoni number, M_c. The critical value, M_c takes the value of the global minimum of the marginal curves which determine the onset of convection.

We noted that by setting *N* = 0.03, *L* = 0.1 and *Bi* = 1 in the case of uniform heat flux, we recover the results of Mikishev and Nepomnyashchy [11] for the nondeformable free surface of Marangoni problem.

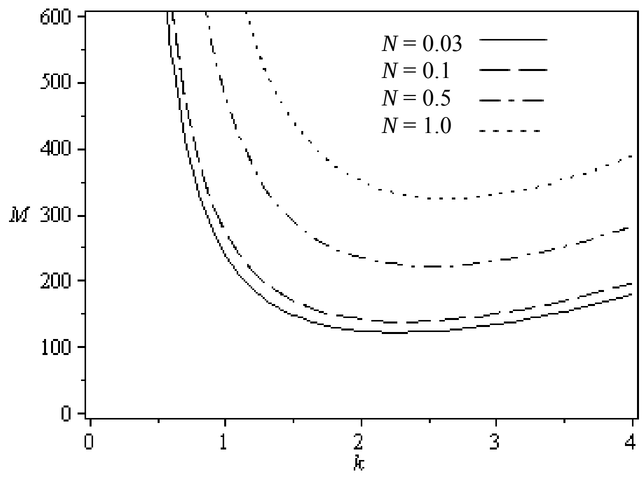

Fig. 1 (a) Marginal stability curves for *L*= 0.1, *Bi*=1, and various values of elasticity parameter *N* for uniform temperature $\theta = 0$.

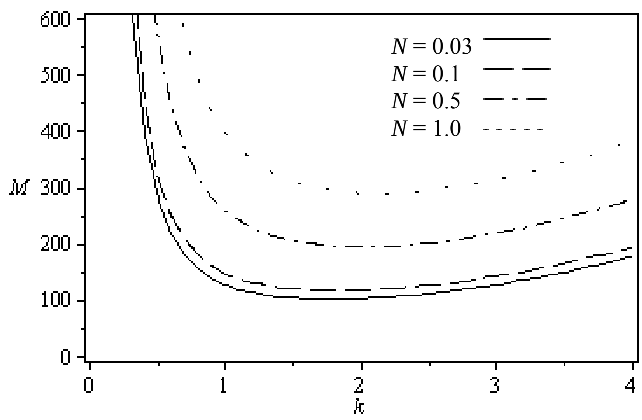

Fig. 1(b) Marginal stability curves for *L* = 0.1, *Bi* = 1, and various values of elasticity parameter *N* for uniform heat flux $D\theta = 0$.

TABLE I

Critical Wave Number k_c and Marangoni Number M_c for Various Elasticity Number *N*

N	Uniform temperature $\theta = 0$		Uniform Heat Flux $D\theta = 0$	
	k_c	M_c	k_c	M_c
0.03	2.2718	122.591	1.7721	102.3081
0.1	2.3245	137.548	1.8231	116.1398
0.5	2.5153	221.053	2.0109	193.6357
1.0	2.6384	323.373	2.1339	288.8640

Figs. 1 – 3 show the effects of elasticity number *N*, Biot number *Bi* and Lewis number *L* on the onset of steady Marangoni convection for the case of uniform temperature and uniform heat flux. Tables I – III show the numerical values of the critical wave number.

Fig. 1 shows the marginal curves for Lewis number L = 0.1, Biot number Bi = 1 and several values of elasticity number N for the case of uniform temperature (Fig. 1(a)) and uniform heat flux (Fig. 1(b)). The increasing value of elasticity parameter N increases the critical Marangoni number, M_c . It also can be seen in Table I that the elasticity parameter N has an increasing effect on the critical wave number k_c and Marangoni number M_c. The values of M_c for the case of uniform temperature are greater than the values when the temperature condition is a uniform heat flux.

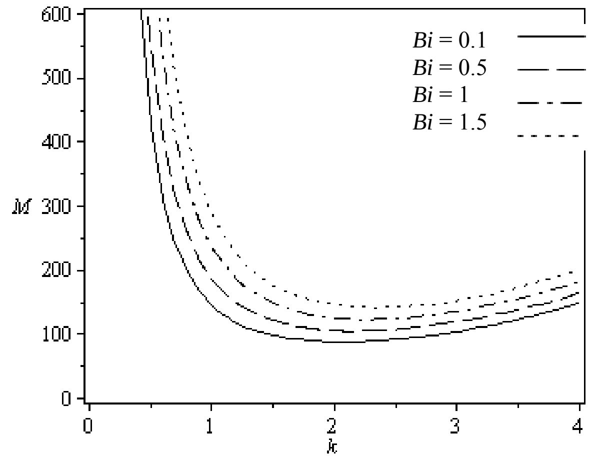

Fig. 2 (a) Marginal stability curves for L = 0.1, N = 0.03, and various values of Biot number Bi for uniform temperature $\theta = 0$.

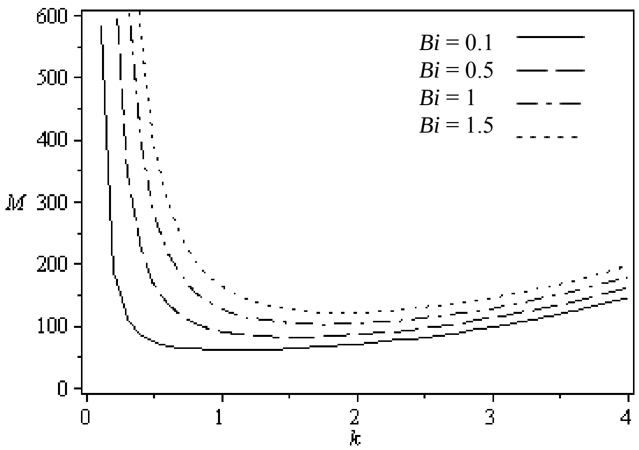

Fig. 2 (b) Marginal stability curves for L = 0.1, N = 0.03, and various values of Biot number Bi for uniform heat flux $D\theta = 0$.

The effect of the Biot number, Bi when L = 0.1, N = 0.03 and for the case of both temperature conditions is illustrated in Fig. 2. It can be seen that higher Biot number corresponds to higher critical Marangoni number M_c. Numerical values in Table II show that increasing Bi also increases the values of k_c and M_c. The values for the critical parameters for the case of uniform temperature are higher than the case of uniform heat flux.

TABLE II

CRITICAL WAVE NUMBER k_c AND MARANGONI NUMBER M_c FOR VARIOUS BIOT NUMBER Bi

Bi	Uniform temperature $\theta = 0$		Uniform Heat Flux $D\theta = 0$	
	k_c	M_c	k_c	M_c
0.1	2.0504	88.304	1.0764	62.2008
0.5	2.1662	103.857	1.5392	82.1722
1.0	2.2718	122.592	1.7721	102.3081
1.5	2.3515	140.849	1.9127	120.8946

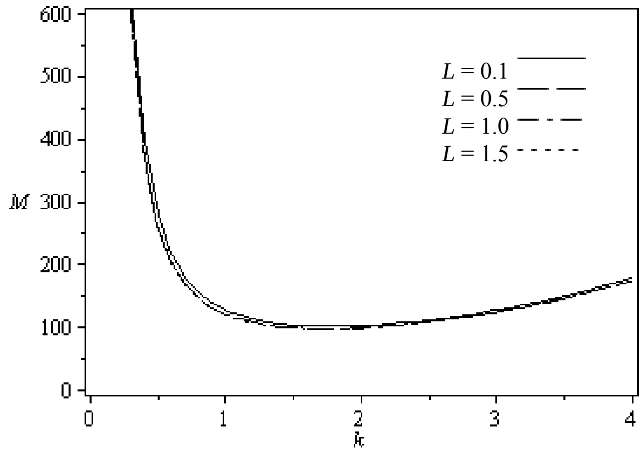

Fig. 3 (a) Marginal stability for N = 0.03, Bi = 1, and various values of Lewis number L for uniform temperature $\theta = 0$.

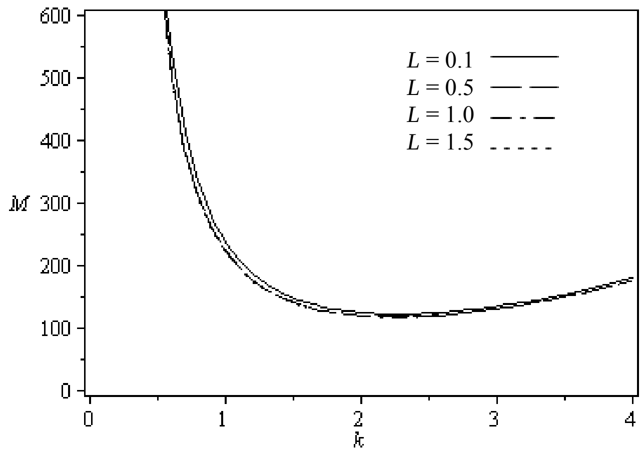

Fig. 3 (b) Marginal stability curves N = 0.03, Bi = 1, and various values of Lewis number L for uniform heat flux $D\theta = 0$.

As shown in Fig. 3, with N = 0.03, Bi = 1 the marginal stability curves shifts to the lower region as the value of the Lewis number L increases for both temperature conditions. Table III shows increasing value of L decreases the values of

k_c and M_c. Similar result can be observed that the critical values for the case of uniform temperature are higher that the values for the case of uniform heat flux.

TABLE III
CRITICAL WAVE NUMBER k_c AND MARANGONI NUMBER M_C FOR VARIOUS LEWIS NUMBER L

L	Uniform temperature $\theta = 0$		Uniform Heat Flux $D\theta = 0$	
	k_c	M_c	k_c	M_c
0.1	2.2718	122.592	1.7721	102.3081
0.5	2.2515	117.423	1.7525	97.5343
1.0	2.2489	116.775	1.7499	96.9363
1.5	2.2479	116.559	1.7491	96.7369

The elasticity parameter N and Biot number Bi have increasing effects on the critical wave number k_c and Marangoni number M_c. However, the Lewis number L decreases the critical wave number k_c and Marangoni number M_c. Therefore, the elasticity parameter N and Biot number Bi stabilize the fluid system but the Lewis number L destabilizes the fluid system. The surfactant and heat transfer mechanism at the free surface act as stabilizer. However, the convective fluid layer with simultaneous heat and mass transfer characterized by the increasing Lewis number is less stable. The uniform temperature condition gives more stability to the fluid layer than the condition of uniform heat flux.

V. CONCLUSION

The Marangoni convective instability in a horizontal fluid layer with the insoluble surfactant and nondeformable free surface subject to uniform temperature and uniform heat flux has been investigated. Insoluble surfactant has a stabilizing effect on the fluid layer. The Biot number in fluid layer also stabilizes the fluid system because of increased Biot number means that more heat is allowed to escape to the gas phases and therefore, the fluid layer becomes stable. The Lewis number destabilizes the fluid system. The fluid layer with uniform temperature boundary condition is more stable than the one with the condition of uniform heat flux. Therefore temperature setting can be used to delay or promote the onset of steady Marangoni instability in fluid layer with the insoluble surfactant.

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