

# Optimal Reactive Power Dispatch using Differential Evolution

C. Thammasirirat, B. Marungsri, R. Oonsivilai, and A. Oonsivilai\*

**Abstract**—This paper presents optimal reactive power dispatch (ORPD) for improvement of voltage stability. This paper uses Differential Evolution method (DE) as the approach for solving optimization issues. The objective is to obsolete voltage stability indexes (L-index) that are subjected to limits on generator voltage, bus voltage, transformer taps setting, generator reactive power and adjustable shunt capacitor. Optimal control variables results are compared between without and with optimization. As a result, the proposed method is the best for solving optimal reactive power dispatch problem. Results of tests conducted on the Wale and Hale 6-bus system.

**Keywords**—Voltage stability, Differential Evolution, L-index, Reactive power

## I. INTRODUCTION

VOLTAGE stability is an important aspect of security analyses in power system. Most power systems are affected by inadequate reactive power supply, besides reduction of voltage stability margin and equipment overloads. The problem of voltage instability is mainly considered as the inability of the network to meet the load demand imposed in terms of inadequate reactive power support or active power transmission capability or both. The dispatch of reactive powers can be effectively used to maintain acceptable voltage levels throughout the system and reduce overall real power loss in the system.

The algorithms may be classified into three groups [10] : conventional optimization methods including nonlinear programming (NLP), linear programming (LP), and mixed integer programming (MIP); intelligence searches such as simulated annealing (SA) [9], evolutionary algorithms (EAs), and Tabu search (TS) [5]; and fuzzy set application to address uncertainties in objectives and constraints. Differential Evolution (DE) [1], [8] has emerged as a popular and efficient population based meta-heuristic for solving global optimization problems. DE is an improved version of Genetic Algorithm (GA) [11] for faster optimization. The optimal reactive power dispatch in this paper will consider the voltage stability of the system. The L-index is simple to calculate

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because it utilizes information obtained from a normal load flow solution. Thus, the voltage stability L-index as the objective functions for solving the problem and improvement of voltage stability. The objective function operation under subject to limits on generator voltage, bus voltage, transformer taps setting, generator reactive power and adjustable shunt capacitor.

This paper proposes a method to use DE for solving optimal reactive power dispatch problem. The L-index results are compared with and without optimization. Simulated case studies are conducted on the Wale and Hale 6-bus system.

Organization of this paper is as following; Voltage stability index is presented in Section II. Section III presents formulation of ORPD problem. Section IV provides a brief overview of DE. In Section V, detailed simulation results and also performance analysis are given and explained. And finally, our work is concluded in Section VI.

## II. VOLTAGE STABILITY INDEX

For voltage stability bus evaluation uses L-index [8], [9], the indicator value varies in the range between zero (the no load condition) and one (voltage collapse) which corresponds to

$$I_{bus} = Y_{bus} \times V_{bus} \quad (1)$$

By segregating the load buses from generator buses, can write as

$$\begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = H \begin{bmatrix} I_L \\ V_G \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ H_3 & H_4 \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \quad (3)$$

where

$V_L, I_L$  : Voltages and Currents at the load buses

$V_G, I_G$  : Voltages and Currents at the generator buses

$H_1, H_2, H_3, H_4$  : Sub-matrices of the Hybrid matrix

$H$ , generated from  $Y_{bus}$  partial inversion.

From equation (2) and (3) can write as

$$V_L = H_1 I_L + H_2 V_G = Y_1^{-1} I_L - Y_1^{-1} Y_2 V_G \quad (4)$$

$$H_2 = -Y_1^{-1} \times Y_2 \quad (5)$$

The no load condition, currents at the load buses ( $I_L$ ) are zero, can be written as

$$V_{0j} = \sum_{i \in G} H_{2i} V_i \quad (6)$$

where

$V_{0j}$  : Voltages at bus j for no load condition

This representation can then be used to define a voltage stability indicator at the load bus, which is given by

$$L_j = \left| 1 - \frac{V_{0j}}{V_j} \right| \quad (7)$$

where

$L_j$  : L-index voltage stability indicator for bus j

$V_j$  : Voltage for bus j

The L-index approaches the numerical value 1.0, when a load bus approaches a steady state voltage collapse situation. So if the index evaluated at any bus is less than unity, the system can keep voltage stability.

### III. FORMULATION OF ORPD PROBLEM

The objective of ORPD is to identify the reactive power control variables, which minimizes of L-index value. This is mathematically stated as follows [9]

$$\text{Minimize } F = \max(L_j; j = 1, 2, \dots, n) \quad (8)$$

where

$n$  : number of buses.

The reactive power optimization problem is subject to the following constraints

*Equality Constraints:*

These constraints represent load flow equation such as

$$P_i = V_i \sum_{j \in i} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij})$$

$$Q_i = V_i \sum_{j \in i} V_j (G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) \quad (9)$$

*Inequality Constraints:*

These constraints represent the system operating constraints.

$$V_G^{\min} \leq V_G \leq V_G^{\max}$$

$$V_L^{\min} \leq V_L \leq V_L^{\max}$$

$$Q_G^{\min} \leq Q_G \leq Q_G^{\max}$$

$$Q_C^{\min} \leq Q_C \leq Q_C^{\max}$$

$$t^{\min} \leq t \leq t^{\max} \quad (10)$$

where

$V_G, V_L$  : Generator bus voltages and load bus voltages respective.

$Q_G, Q_C$  : Reactive power generated of generator and capacitor respective.

$t$  : Transformer taps

With the inclusion of penalty function, the new objective function then becomes,

$$\text{Minimize } F = \max(L_j; j = 1, 2, \dots, n) + K \sum_{i=1}^{N_G} (V_G - V_G^{\lim})^2 +$$

$$K \sum_{i=1}^{N_L} (V_L - V_L^{\lim})^2 + K \sum_{i=1}^{N_G} (Q_G - Q_G^{\lim})^2 +$$

$$K \sum_{i=1}^{N_C} (Q_C - Q_C^{\lim})^2 + K \sum_{i=1}^{N_t} (t - t^{\lim})^2 \quad (11)$$

Where  $K$  is the penalty factors

In the above objective function  $V_G^{\lim}, V_L^{\lim}, Q_G^{\lim}, Q_C^{\lim}$  and  $t^{\lim}$  are defined as;

$$V_G^{\lim} = \begin{cases} V_G^{\min}; V_G < V_G^{\min} \\ V_G^{\max}; V_G > V_G^{\max} \end{cases}$$

$$V_L^{\lim} = \begin{cases} V_L^{\min}; V_L < V_L^{\min} \\ V_L^{\max}; V_L > V_L^{\max} \end{cases}$$

$$Q_G^{\lim} = \begin{cases} Q_G^{\min}; Q_G < Q_G^{\min} \\ Q_G^{\max}; Q_G > Q_G^{\max} \end{cases} \quad (12)$$

$$Q_C^{\lim} = \begin{cases} Q_C^{\min}; Q_C < Q_C^{\min} \\ Q_C^{\max}; Q_C > Q_C^{\max} \end{cases}$$

$$t^{\lim} = \begin{cases} t^{\min}; t < t^{\min} \\ t^{\max}; t > t^{\max} \end{cases}$$

### IV. OVERVIEW OF DIFFERENTIAL EVOLUTION

Differential evolution (DE) [1], [8] was developed by Storn and Price in 1996. This method is based on stochastic optimization algorithm. It is an improved version of Genetic Algorithm (GA). DE is very simple to implement and relies only on variables with a floating point representation.

DE has a specialized nomenclature that describes the adopted configuration. This takes the form of DE/x/y/z, where x represents the solution to be perturbed (such a random or best). The y signifies the number of difference vectors used in the perturbation of x, where a difference vectors is the difference between two randomly selected although distinct members of the population. Finally, z signifies the recombination operator [6], [7] performed such as bin for binomial and exp for exponential.

The optimization process is carried out with four basic operations.

- Initialization: creation of a population of individuals.
- Mutation (and migration in multi-population versions): random change of the vector  $x$  component (genes). It can be single-point mutation, inversion, translocation, deletion, etc.
- Crossover (Recombination): merging the genetic information of two or more parent individuals for producing one or more descendants.
- Selection: choice of the best individuals for the next cycle.

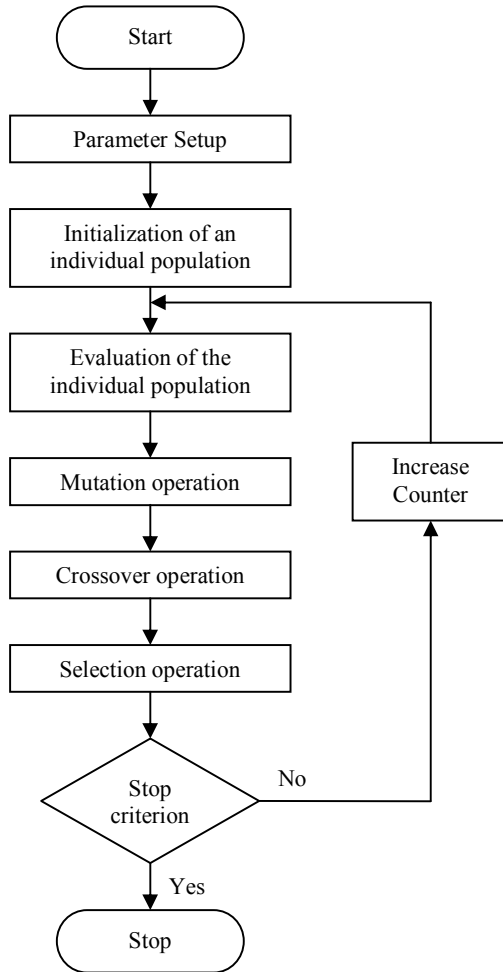


Fig. 1 The flowchart of differential evolution

**Algorithm**

*Definition:*  $x_i^{-g}$  are the parameters for the individual  $i$  ( $i=1, \dots, P$ ) in the generation  $g$  ( $g=1, \dots, G$  max)

*Mutation:*

$i, a, b, c$  are mutually different indexes of individuals  
 $x_i^{-g}$  is the target vector  
 $\vec{d}_i = x_a^{-g} + F(x_b^{-g} - x_c^{-g})$  is the donor vector  
 $F$  is the scaling (weighting) factor

*Crossover:*

Construct a trial vector

$$t_{n,i} = \begin{cases} d_{n,i} \\ X_{n,i}^g \end{cases}$$

rand < CR or n=rand(D)+1  
 Otherwise

$C_r$  is the crossover

Here rand generates the random numbers in the interval [0, 1) and rand(D) generates integer numbers in the interval 0, 1, ..., D-1.

*Selection:*

$$x_i^{-g+1} = \begin{cases} t_i \\ x_i \end{cases}$$

$f(t_i) < x_i$   
 Otherwise

*Variants of mutation:*

$$\vec{d}_i = x_a^{-g} + F(x_b^{-g} - x_c^{-g}) \tag{13}$$

$$\vec{d}_i = x_a^{-g} + K(x_{best}^{-g} - x_i^{-g}) + F(x_b^{-g} - x_c^{-g}) \tag{14}$$

$$\vec{d}_i = x_i^{-g} + K(x_a^{-g} - x_i^{-g}) + F(x_b^{-g} - x_c^{-g}) \tag{15}$$

Here  $K$  is the combination factor. For  $K=1$  Eq. (15) reduce to Eq. (13). Eq. (14) has the following limits:

$$\vec{d}_i = x_{best}^{-g} + F(x_b^{-g} - x_c^{-g})K = 1 \tag{16A}$$

$$\vec{d}_i = x_i^{-g} + F(x_b^{-g} - x_c^{-g})K = 0 \tag{16B}$$

*Acceleration:*

In the case when the mutation and crossover operations do not further improve the best fitness, a steepest descent method is applied to push the best individual towards a better point:

$$x_{best}^{-new} = x_{best}^{-g} - \rho \nabla f(x) \Big|_{x_{best}}$$

The gradient can be estimated numerically.

V. SIMULATION RESULTS

The demonstrated Wale and Hale 6-bus test system [9] show in Fig. 2, which consist of 2 generator buses, 3 load buses, and 7 transmission lines of which 2 branches, (3-4) and (5-6), are with the tap-setting transformer. The line data and bus data are given in Table I and Table II, respectively.

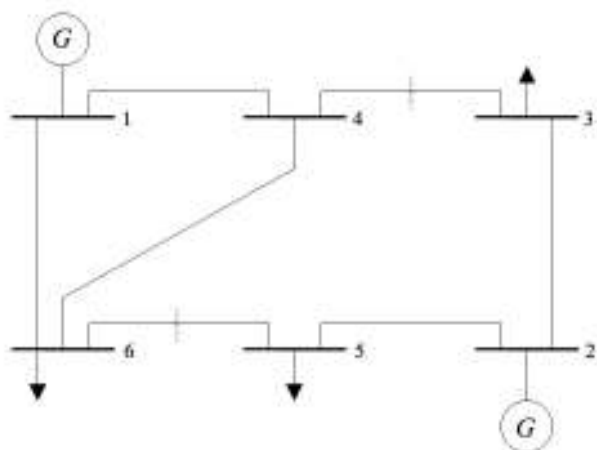


Fig. 2 Wale and Hale 6-bus system

TABLE I  
LINE DATA ON 100 MVA BASE

| Line | From | to | R (pu) | X (pu) | Taps setting |
|------|------|----|--------|--------|--------------|
| 1    | 1    | 6  | 0.123  | 0.518  | -            |
| 2    | 1    | 4  | 0.080  | 0.370  | -            |
| 3    | 4    | 6  | 0.097  | 0.407  | -            |
| 4    | 6    | 5  | 0.000  | 0.300  | 1.025        |
| 5    | 5    | 2  | 0.282  | 0.640  | -            |
| 6    | 2    | 3  | 0.723  | 1.050  | -            |
| 7    | 4    | 3  | 0.000  | 0.133  | 1.100        |

TABLE II  
BUS DATA ON 100 MVA BASE

| Bus | V (pu) | P <sub>G</sub> (pu) | P <sub>L</sub> (pu) | Q <sub>L</sub> (pu) |
|-----|--------|---------------------|---------------------|---------------------|
| 1   | 1.05   | -                   | -                   | -                   |
| 2   | 1.10   | 0.50                | 0.00                | 0.00                |
| 3   | 1.00   | -                   | 0.55                | 0.13                |
| 4   | 1.00   | -                   | 0.00                | 0.00                |
| 5   | 1.00   | -                   | 0.30                | 0.18                |
| 6   | 1.00   | -                   | 0.50                | 0.05                |

The determination of the two weakest buses is based on minimum voltage stability of load buses for reactive power compensation with capacitor. The voltage stability analysis at each bus uses L-index, the L-index are calculated using power flow solution by Newton-Raphson algorithm.

TABLE III  
THE L-INDEX VALUES OF NORMAL CONDITION

| Bus | L-index      |
|-----|--------------|
| 3   | <b>0.288</b> |
| 4   | 0.211        |
| 5   | <b>0.278</b> |
| 6   | 0.258        |

The results of L-index values are given in TABLE III, the system without compensation devices. It can be seen that

values of the L-index at bus 3 and bus 5 are maximum, indicating they are the weakest buses. Consequently, the candidate buses for capacitor installment are 3 and 5. The power loss of system is 11.63 MW.

In this paper proposes two different cases are comparative values of L-index, case 1 without optimization and case 2 with optimization.

A. Case 1: Without optimization

In this case, the line data and bus data are taken from Table I and Table II, respectively. The test results of L-index are in normal condition, the candidate buses for capacitor installment are 3 and 5. The voltage stability analysis at each bus uses L-index. The power flow solution derived from Newton-Raphson algorithm.

B. Case 2: With optimization

In this case, the same line data and bus data as in case 1 are lagging with capacitor. The optimal reactive power dispatch using differential evolution method with capacitor under base load condition for 100% load level. The parameters of differential evolution uses for optimal reactive power dispatch are given in Table IV and limits of variables shown in Table V.

TABLE IV  
PARAMETERS OF DIFFERENTIAL EVOLUTION

| Parameter            | Value         |
|----------------------|---------------|
| Strategy             | DE/rand/1/bin |
| Number of population | 20            |
| Maximum generation   | 1000          |
| Mutation factor (F)  | 1             |
| Crossover rate (CR)  | 0.8           |

TABLE V  
LIMITS OF VARIABLES

| Variables                   |                  | Low Limits | High Limits |
|-----------------------------|------------------|------------|-------------|
| Generator Voltages (pu)     | V <sub>G1</sub>  | 1.00       | 1.10        |
|                             | V <sub>G2</sub>  | 1.10       | 1.15        |
| Voltages at load Buses (pu) | V <sub>L3</sub>  | 0.90       | 1.00        |
|                             | V <sub>L4</sub>  | 0.90       | 1.00        |
|                             | V <sub>L5</sub>  | 0.90       | 1.00        |
|                             | V <sub>L6</sub>  | 0.90       | 1.00        |
|                             |                  |            | 0.90        |
| Transformer Taps            | t <sub>5-6</sub> | 0.90       | 1.10        |
|                             | t <sub>3-4</sub> | 0.90       | 1.10        |
| Shunt Capacitors (MVAR)     | Q <sub>C3</sub>  | 0.00       | 5.50        |
|                             | Q <sub>C5</sub>  | 0.00       | 5.50        |
| Generator MVAR              | Q <sub>G1</sub>  | -20.00     | 100.00      |
|                             | Q <sub>G2</sub>  | -20.00     | 100.00      |

The optimal values of the control variable and power loss obtained are presented in Table VI. The minimum power losses in case 2 obtained is 8.89 MW which is smaller than the result obtained in case 1.

Table VII shows a comparative list of results using both voltage stability evaluation of L-index without and with optimization. It can be seen that value of L-index at load buses are reduced, therefore, the voltage stability of the system is enhanced and improved.

TABLE VI

CONTROL VARIABLES AND SYSTEM LOSSES RESULTS

| Control Variables       |           | Without | With  |
|-------------------------|-----------|---------|-------|
| Generator Voltages (pu) | $V_{G1}$  | 1.050   | 1.089 |
|                         | $V_{G2}$  | 1.100   | 1.150 |
| Transformer Taps        | $t_{5-6}$ | 1.025   | 0.972 |
|                         | $t_{3-4}$ | 1.100   | 0.991 |
| Shunt Capacitors (MVAR) | $Q_{C3}$  | 5.500   | 5.500 |
|                         | $Q_{C5}$  | 5.500   | 5.500 |
| System Losses (MW)      |           | 10.60   | 8.89  |

TABLE VII

COMPARE OF L-INDEX WITHOUT AND WITH OPTIMIZATION

| Case    | L-index      |       |       |       |
|---------|--------------|-------|-------|-------|
|         | bus 3        | bus 4 | bus 5 | bus 6 |
| Without | <b>0.266</b> | 0.197 | 0.256 | 0.246 |
| With    | <b>0.233</b> | 0.181 | 0.229 | 0.223 |

The L-index values at load buses versus load factor without and with optimization are shown in Fig. 3 and Fig. 4, respectively. The voltage stability distance from collapse has increased.

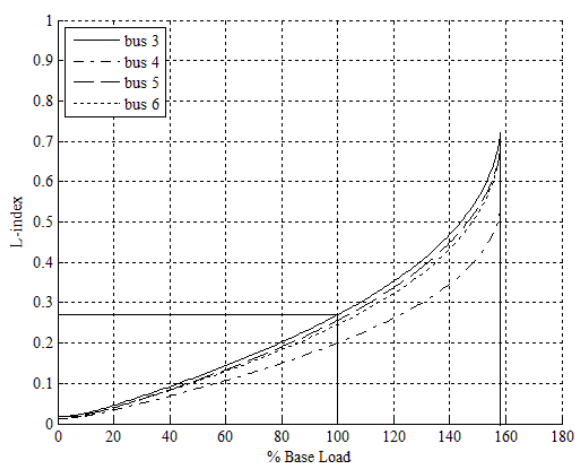


Fig. 3 L-index values at load buses for without optimization

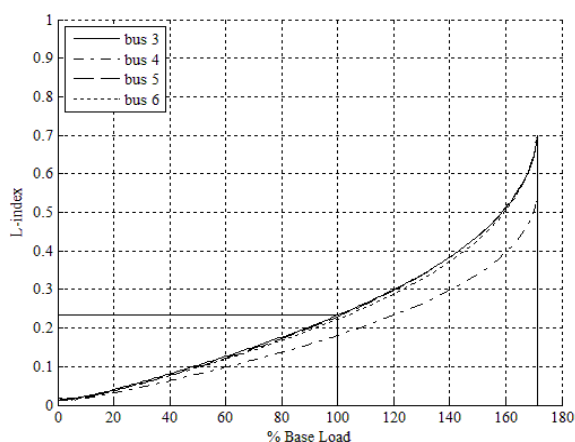


Fig. 4 L-index values at load buses for with optimization

## VI. CONCLUSION

This paper presents optimal reactive power dispatch for improvement of voltage stability using differential evolution (DE) and is applied to Wale and Hale 6-bus system. The results show that the DE-based reactive power dispatch algorithm is able to improve voltage stability condition along with reduced loss in the system. Also, it is found that the results of the DE-based algorithm are always better than that obtained without optimization.

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