

An Adverse Model for Price Discrimination in the Case of Monopoly

Daniela Elena Marinescu, Ioana Manafi, Dumitru Marin

Abstract—We consider a Principal-Agent model with the Principal being a seller who does not know perfectly how much the buyer (the Agent) is willing to pay for the good. The buyer's preferences are hence his private information. The model corresponds to the nonlinear pricing problem of Maskin and Riley. We assume there are three types of Agents. The model is solved using “informational rents” as variables. In the last section we present the main characteristics of the optimal contracts in asymmetric information and some possible extensions of the model.

Keywords—Adverse selection, asymmetric information, informational rent, nonlinear pricing, optimal contract

I. INTRODUCTION

THE development of the incentives theory represents a major accomplishment of economic research in the last forty years. One major class of models corresponds to the adverse selection problems – describing situations where a Principal offers a contract to an Agent having private information about some characteristics that influence the contract's results. This private information determines the Agent's type and affects the Principal's utility (or profit). The literature on adverse selection models is ample and refers both to theoretical developments (Maskin and Riley (1984)[13], Guesnerie and Laffont (1984)[6], Rochet and Chone (1998)[15], Armstrong and Rochet (1999)[2], Jullien (2000)[7], Rochet and Stole (2002)[16], Laffont and Martimort (2002)[9], Bolton and Dewatripont (2005)[4], Kessler et al (2005)[8]) and empirical studies (nonlinear pricing, financial contracting, regulation, insurance contracts, labor contracts, optimal taxation).

The problem presented in the paper refers to the optimal nonlinear pricing of a monopoly in the case of asymmetric information. The problem of monopoly pricing – where the seller designs “quality/quantity price contracts” for each type of buyer was first studied in a one-dimensional context (Spence (1980) [19], Armstrong (1996)[1], Rochet and Chone (1998)[15]). In the last twenty years, the problem of monopoly pricing was studied in multidimensional frameworks (Saak (2007)[17], Brighi and D'Amato (2002)[5], Sibley and Srinagesh (1997)[18], Rochet and Stole (2002)[16], Matthews and Moore (1987)[14]).

A. Daniela Elena Marinescu, PhD is Associate Professor at Bucharest Academy of Economic Studies, Romania (e-mail: daniela.marinescu@csie.ase.ro, danielamarinescu@hotmail.com).

B. Ioana Manafi, PhD is Assistant Professor at Bucharest Academy of Economic Studies, Romania (e-mail: ioana.manafi@gmail.com).

C. Dumitru Marin, PhD is Professor at Bucharest Academy of Economic Studies, Romania (e-mail: dumitru.marin@csie.ase.ro, dumitrumarin@hotmail.com).

The literature on optimal nonlinear pricing is ample and evolved during thirty years of research. In one of the starting papers on nonlinear pricing, Maskin and Riley (1984) [13] study the problem of price discrimination via quantity discounts and monopoly pricing of products with different qualities; they also present an application consists in using a nonlinear price schedule to discriminate among consumers. Armstrong (1996) [1] shows that usually it is optimal for the Principal to exclude some consumers from its products in order to extract more revenue from other higher value consumers. Sibley and Srinagesh (1997) [18] analyze the multiproduct nonlinear pricing in the case when consumer tastes are characterized by more than one taste parameter. Rochet and Chone (1998) [15] show that for those consumers who participate it is usually optimal to induce a degree of bunching so that consumers with different tastes are forced to choose the same bundle of products. Armstrong and Rochet (1999) [2] provide a throughout analysis of a screening model with two dimensional private information and discrete types; they offer a complete set of solutions to their model by characterizing the optimal screening mechanisms in terms of correlation and the symmetry of types. Rochet and Stole (2002) [16] presented and solved a model of standard nonlinear pricing in which the participation constraint is modeled as a random variable, also private information of the consumer. Brighi and D'Amato (2002) [5] present a case of monopoly regulation and derive the optimal regulatory policy of a monopolist producing two goods and with two-dimensional private information about costs. They provide a complete characterization of the optimal mechanism. Bolton and Dewatripont (2005) [4] provide a useful procedure to handle with more than two types of agent in the context of monopoly pricing. In the paper we provide a complete characterization of the optimal nonlinear pricing when the buyer has some private information about his valuation of the good. Our approach is partially similar to that of Bolton and Dewatripont (2005) [4]. The difference consists in solving the seller's optimization problem using as variables the informational rents of the Agents. This approach is rather closer to that of Laffont and Martimort (2002) [9]. Next, we derive the conditions of the optimal contract when we fully separate the types of Agent (there is no bunching of types). The paper is organized as follows: the main assumptions of the model are described in Section II. Next we transform the model using a change of variables with an important economic significance (i.e. the informational rents). In Section IV we provide a fully characterization of the optimal contract (mechanism) in both situations: symmetric and asymmetric information. In the end we formulate some concluding remarks.

II. THE MODEL

A. The main assumptions

We study the problem of a monopoly pricing, when a firm designs quantity-price bundles for each buyer's type and the buyers self select the bundle they wish to consume. We assume the Principal (the seller being a monopolist) produces a good at a marginal cost (let c be this cost) and he has no fixed costs (without loosing generality). The utility function is:

$$V = t - cq \quad (1)$$

where t represents the transfer received from the Agent. The seller is therefore risk neutral.

The preferences of the agent are described by the utility function:

$$U(q, t) = \theta u(q) - t \quad (2)$$

with $u' > 0, u'' < 0, u(0) = 0$.

Let q be the quantity consumed and t the payment (the transfer) to the Principal.

We assume the differences in consumers' tastes are captured by the parameter θ , representing the valuation of the good. This valuation is private information for the buyer. The parameter belongs to the set: $\theta \in \Theta = \{\underline{\theta}, \hat{\theta}, \bar{\theta}\}$, where $\hat{\theta} - \underline{\theta} = \Delta\theta$ and $\bar{\theta} - \hat{\theta} = \Delta\theta + x$. The valuation corresponds to one of these three values with the respective probabilities: $\underline{v}, \hat{v}, \bar{v}$ with: $\underline{v} + \hat{v} + \bar{v} = 1$.

B. The set of incentive feasible contracts

As in the standard Principal-Agent models, we assume that the Principal has all the bargaining power, such there is no negotiation between the parties. The Agent accepts or rejects the Principal's offer.

The economic variables of the problem considered are the quantity q and the transfer t . A selling procedure (a contract between the Principal and the Agent) is then a schedule of pairs (t, q) which the seller offers to the buyers.

The set of incentive feasible contracts satisfies the following constraints:

- participation constraints:

$$\underline{\theta}u(\underline{q}) - \underline{t} \geq 0 \quad (3)$$

$$\hat{\theta}u(\hat{q}) - \hat{t} \geq 0 \quad (4)$$

$$\bar{\theta}u(\bar{q}) - \bar{t} \geq 0 \quad (5)$$

- incentive compatibility constraints:

$$\underline{\theta}u(\underline{q}) - \underline{t} \geq \underline{\theta}u(\hat{q}) - \hat{t} \quad (\text{local upward constraint}) \quad (6)$$

$$\underline{\theta}u(\underline{q}) - \underline{t} \geq \underline{\theta}u(\bar{q}) - \bar{t} \quad (\text{global upward constraint}) \quad (7)$$

$$\hat{\theta}u(\hat{q}) - \hat{t} \geq \hat{\theta}u(\bar{q}) - \bar{t} \quad (\text{local upward constraint}) \quad (8)$$

$$\hat{\theta}u(\hat{q}) - \hat{t} \geq \hat{\theta}u(\underline{q}) - \underline{t} \quad (\text{local downward constraint}) \quad (9)$$

$$\bar{\theta}u(\bar{q}) - \bar{t} \geq \bar{\theta}u(\hat{q}) - \hat{t} \quad (\text{local downward constraint}) \quad (10)$$

$$\bar{\theta}u(\bar{q}) - \bar{t} \geq \bar{\theta}u(\underline{q}) - \underline{t} \quad (\text{global downward constraint}) \quad (11)$$

C. The optimization problem of the Principal

The Principal objective is to maximize his expected profit obtained from the contractual relation, when the Agent consumes the good produced by the Principal:

$$\max_{(\underline{t}, \underline{q}), (\hat{t}, \hat{q}), (\bar{t}, \bar{q})} \left\{ \underline{v}(\underline{t} - c\underline{q}) + \hat{v}(\hat{t} - c\hat{q}) + \bar{v}(\bar{t} - c\bar{q}) \right\} \quad (12)$$

s.t. all participation and incentive compatibility constraints.

III. THE MODEL TRANSFORMED USING THE VARIABLES INFORMATIONAL RENTS-QUANTITIES

Using the approach suggested by Laffont and Martimort (2002), we define the *informational rent* for each type of consumer (Agent) as:

$$\underline{U} = \underline{\theta}u(\underline{q}) - \underline{t} \quad (13)$$

$$\hat{U} = \hat{\theta}u(\hat{q}) - \hat{t} \quad (14)$$

and:

$$\bar{U} = \bar{\theta}u(\bar{q}) - \bar{t}. \quad (15)$$

Using these new variables, we can rewrite the constraints as follows:

i) now the participation constraints become sign constraints:

$$\underline{U} \geq 0 \quad (16)$$

$$\hat{U} \geq 0 \quad (17)$$

$$\bar{U} \geq 0 \quad (18)$$

ii) the incentive constraints are:

$$\underline{U} \geq \hat{U} - \Delta\theta u(\hat{q}) \quad (19)$$

$$\underline{U} \geq \bar{U} - 2(\Delta\theta + x)u(\bar{q}) \quad (20)$$

$$\hat{U} \geq \bar{U} - (\Delta\theta + x)u(\bar{q}) \quad (21)$$

$$\hat{U} \geq \underline{U} + \Delta\theta u(\underline{q}) \quad (22)$$

$$\bar{U} \geq \hat{U} + (\Delta\theta + x)u(\hat{q}) \quad (23)$$

$$\bar{U} \geq \underline{U} + (2\Delta\theta + x)u(\underline{q}) \quad (24)$$

And with this change of variables, the objective function is:

$$\max_{\underline{U}, \hat{U}, \bar{U}, \underline{q}, \hat{q}, \bar{q}} \left\{ \underline{v}[\underline{\theta}u(\underline{q}) - c\underline{q}] + \hat{v}[\hat{\theta}u(\hat{q}) - c\hat{q}] + \bar{v}[\bar{\theta}u(\bar{q}) - c\bar{q}] - (\underline{v}\underline{U} + \hat{v}\hat{U} + \bar{v}\bar{U}) \right\} \quad (25)$$

IV. SOLVING THE PROBLEM

A. First best production and transfer levels

First we suppose there is no asymmetric information between the seller and the buyer. The optimal contract is represented by the optimal solution of the following optimization problem:

$$(\max)_{t, q} \{t - cq\}$$

(P₁) s.t.

$$\theta u(q) - t \geq 0$$

$$q \geq 0, t \geq 0$$

The first order conditions for the above problem (details in Appendix) are:

$$\begin{cases} c = \theta u'(q^*) \\ t^* = \theta u(q^*) \end{cases} \quad (26)$$

Some remarks:

1. The above conditions show that, in the situation of symmetric information, the Principal produces a quantity such that his marginal cost of production is equal to the Agent's marginal utility and the Agent pays to the Principal a transfer equal to his valuation of the good.
2. For each type of Agent, the optimal quantity and transfer levels are:

$$\text{- for the type } \underline{\theta}: \begin{cases} c = \underline{\theta} u'(q^*) \\ t^* = \underline{\theta} u(q^*) \end{cases}$$

$$\text{- for the type } \hat{\theta}: \begin{cases} c = \hat{\theta} u'(q^*) \\ t^* = \hat{\theta} u(q^*) \end{cases}$$

$$\text{- for the type } \bar{\theta}: \begin{cases} c = \bar{\theta} u'(q^*) \\ t^* = \bar{\theta} u(q^*) \end{cases}$$

3. We have $\underline{\theta} < \hat{\theta} < \bar{\theta}$ and then $\frac{c}{\underline{\theta}} < \frac{c}{\hat{\theta}} < \frac{c}{\bar{\theta}}$. Hence:

$$u'(q^*) > u'(\hat{q}^*) > u'(\bar{q}^*) \quad (27)$$

It follows immediately:

$$q^* < \hat{q}^* < \bar{q}^* \quad (28)$$

Therefore, the buyer with the highest valuation has the highest consumption level.

B. Second best production and transfer levels

Now, we suppose the good valuation represent the Agent's private information. In this situation, the Principal offers a menu of contracts $\{(t, q), (\hat{t}, \hat{q}), (\bar{t}, \bar{q})\}$, hoping that each Agent selects the contract designed for his type.

Solving the problem we use some important results given in the following *propositions*:

Proposition 1. If the set of incentive feasible solutions is nonempty, then the *implementability condition* (IC) $q \leq \hat{q} \leq \bar{q}$ holds.

Proof. We use the constraints (1) and (4):

$$\underline{U} \geq \hat{U} - \Delta\theta u(\hat{q}) \quad (29)$$

$$\hat{U} \geq \underline{U} + \Delta\theta u(q) \quad (30)$$

Adding these two constraints, we get:

$$0 \geq \Delta\theta [u(q) - u(\hat{q})] \text{ or } u(\hat{q}) \geq u(q).$$

The utility function $u(\cdot)$ is strictly increasing, so that we have:

$$\hat{q} \geq q \quad (31)$$

Similarly, using the constraints (3) and (5):

$$\hat{U} \geq \bar{U} - (\Delta\theta + x)u(\bar{q}) \quad (32)$$

$$\bar{U} \geq \hat{U} + (\Delta\theta + x)u(\hat{q}) \quad (33)$$

and adding these two constraints we get:

$$0 \geq (\Delta\theta + x)[u(\hat{q}) - u(\bar{q})] \text{ or } u(\bar{q}) \geq u(\hat{q})$$

This implies:

$$\bar{q} \geq \hat{q} \quad (34)$$

From the conditions (31) and (34) it follows that $q \leq \hat{q} \leq \bar{q}$. This is exactly the *implementability condition*.

The meaning of this proposition is somewhat straightforward. The buyer with the highest evaluation has the incentive to consume more than the other types of potential buyers.

Proposition 2. The constraint (20) and (24) are not relevant for the optimization program. The incentive compatibility constraint (7) is implied by the constraints (19) and (21). The incentive compatibility constraint (24) is implied by the constraints (22) and (23).

Proof:

We use the constraints (1) and (3) as follows:

$$\begin{aligned} \underline{U} &\geq \hat{U} - \Delta\theta u(\hat{q}) \geq \bar{U} - (\Delta\theta + x)u(\bar{q}) - \Delta\theta u(\hat{q}) \\ &\geq \bar{U} - (2\Delta\theta + x)u(\bar{q}) \end{aligned}$$

In the above inequality we used the implementability condition: $\hat{q} \leq \bar{q}$. We get $\underline{U} \geq \bar{U} - (2\Delta\theta + x)u(\bar{q})$ and this corresponds to the constraint (2).

Similarly, we can show that the constraints (9) and (10) imply the constraint (11). We use the result of the above proposition to reduce the number of relevant constraints in the Principal's optimization problem.

Proposition 3. The only relevant participation constraint is the one corresponding to the type with the least valuation. (The participation constraints $\hat{U} \geq 0$ and $\bar{U} \geq 0$ are implied by the participation constraint $\underline{U} \geq 0$ and the incentive compatibility constraints).

It is easy to proof this proposition using only the participation and incentive constraints. We leave to the reader to check this result.

Some remarks:

The types with the higher valuation do not pretend that they have lower valuation for the good produced by the Principal. Therefore, we can neglect for a while the constraints (19) and (21) and we check (after solving the problem) that these constraints are satisfied at the optimum.

With all these remarks, the only relevant constraints are (22) and (23) and the participation constraint: $\underline{U} \geq 0$.

The optimization program becomes:

(P₂)

$$\max_{\underline{U}, \bar{U}, \underline{q}, \hat{q}, \bar{q}} \left\{ \underline{v} \left[\underline{\theta} u(\underline{q}) - c \underline{q} \right] + \hat{v} \left[\hat{\theta} u(\hat{q}) - c \hat{q} \right] + \bar{v} \left[\bar{\theta} u(\bar{q}) - c \bar{q} \right] - \left(\underline{v} \underline{U} + \hat{v} \hat{U} + \bar{v} \bar{U} \right) \right\}$$

s.t.

$$\hat{U} \geq \underline{U} + \Delta \theta u(\underline{q})$$

$$\bar{U} \geq \hat{U} + (\Delta \theta + x) u(\hat{q})$$

and

$$\underline{q} \leq \hat{q} \leq \bar{q} \quad (IC)$$

The next *proposition* considerably reduces the optimization problem.

Proposition 4. The constraints (22), (23) and (16) are binding at the optimum.

Proof. The constraint $\bar{U} \geq 0$ is binding at the optimum.

Otherwise, we could reduce all the informational rents \underline{U} , \hat{U} and \bar{U} by a small positive number ε and the Principal's optimal profit would increase by ε and this contradicts the definition of the optimal solution.

The constraint $\hat{U} \geq (\Delta \theta + x) u(\underline{q})$ is binding at the optimum. Otherwise, we could reduce \hat{U} and \bar{U} by ε and the Principal's profit would increase by $\varepsilon(\hat{v} + \bar{v})$.

Similarly, the constraint (10) is binding; otherwise we could reduce the informational rent \bar{U} by ε , and the Principal would gain a profit surplus $\varepsilon \bar{v}$.

Therefore, it is optimal:

$$\underline{U} = 0, \hat{U} = \Delta \theta u(\underline{q}) \text{ and}$$

$$\bar{U} = \Delta \theta [u(\hat{q}) + u(\underline{q})] + x u(\hat{q})$$

With this new result, the program we have to solve is reduced to the following optimization problem:

$$\begin{aligned} (\max)_{\underline{q}, \hat{q}, \bar{q}} H(\cdot) = & \left\{ \underline{v} \left[\underline{\theta} u(\underline{q}) - c \underline{q} \right] + \hat{v} \left[\hat{\theta} u(\hat{q}) - c \hat{q} \right] + \right. \\ & \left. + \bar{v} \left[\bar{\theta} u(\bar{q}) - c \bar{q} \right] - \hat{v} \Delta \theta u(\underline{q}) - \bar{v} x u(\hat{q}) - \right. \\ (P_3) \quad & \left. - \bar{v} \Delta \theta \left[u(\hat{q}) + u(\underline{q}) \right] \right\} \end{aligned}$$

s.t.

$$\underline{q} \leq \hat{q} \leq \bar{q} \quad (IC)$$

We solve the problem as if it was an optimization problem without constraints and then we state the conditions for the optimal solution to satisfy the implementability condition (see the Appendix for details).

The first order conditions written for the above optimization problem yield to:

$$c = \bar{\theta} u'(\bar{q}) \text{ and so } \bar{q}^{SB} = \bar{q}^* \quad (35)$$

$$c = u'(\hat{q}) \left[\hat{\theta} - \frac{\bar{v}}{\hat{v}} (\Delta \theta + x) \right] \text{ and so: } \hat{q}^{SB} < \hat{q}^* \quad (36)$$

$$c = u'(\underline{q}) \left[\underline{\theta} - \frac{\hat{v} + \bar{v}}{\underline{v}} \Delta \theta \right] \text{ and so: } \underline{q}^{SB} < \underline{q}^* \quad (37)$$

After deriving the solution of the unconstrained problem, we must check if this solution satisfies also the implementability condition.

We have already proved that $\hat{q}^{SB} < \hat{q}^* < \bar{q}^* = \bar{q}^{SB}$. The optimal solution satisfies the right side of the implementability condition. We are interested now in deriving the conditions for the optimal solution to satisfy the left side of that inequality, i.e. $\underline{q}^{SB} \leq \hat{q}^{SB}$.

Proposition 5. The optimal solution in asymmetric information satisfies the implementability condition if:

$$\frac{x}{\Delta \theta} < \frac{\hat{v} - \underline{v} \bar{v}}{\underline{v} \bar{v}} \quad (38)$$

If it is not true, then the program leads to another optimal solution, corresponding to some *bunching of types*.

Proof. From the first order conditions (35)-(37):

$$c = u'(\hat{q}^{SB}) \cdot \left[\hat{\theta} - \frac{\bar{v}}{\hat{v}} (\Delta \theta + x) \right] = u'(\underline{q}^{SB}) \cdot \left[\underline{\theta} - \frac{\hat{v} + \bar{v}}{\underline{v}} \Delta \theta \right]$$

Rearranging the terms, we get:

$$\frac{u'(\underline{q}^{SB})}{u'(\hat{q}^{SB})} = \frac{\hat{\theta} - \frac{\bar{v}}{\hat{v}} (\Delta \theta + x)}{\underline{\theta} - \frac{\hat{v} + \bar{v}}{\underline{v}} \Delta \theta} \quad (39)$$

The implementability condition is satisfied, if $\underline{q}^{SB} < \hat{q}^{SB}$.

This yields to: $u'(\underline{q}^{SB}) > u'(\hat{q}^{SB})$.

The left-hand side of the fraction from (39) satisfies the above condition, i.e.:

$$\frac{u'(\underline{q}^{SB})}{u'(\hat{q}^{SB})} > 1 \quad (40)$$

if and only if:

$$\hat{\theta} - \frac{\bar{v}}{\hat{v}} (\Delta \theta + x) > \underline{\theta} - \frac{\hat{v} + \bar{v}}{\underline{v}} \Delta \theta \quad (41)$$

or

$$\hat{\theta} - \underline{\theta} - \frac{\bar{v}}{\hat{v}} \Delta \theta + \frac{\hat{v} + \bar{v}}{\underline{v}} \Delta \theta > x \frac{\bar{v}}{\hat{v}} \quad (42)$$

and this can be written as:

$$\frac{x}{\Delta \theta} < \frac{\hat{v}}{\underline{v}} \left(\frac{1}{\underline{v}} - \frac{\bar{v}}{\hat{v}} \right) \text{ or } \frac{x}{\Delta \theta} < \frac{\hat{v} - \underline{v} \bar{v}}{\underline{v} \bar{v}} \quad (43)$$

V. CONCLUSION

We can now summarize the characteristics of the optimal contract in asymmetric information:

Theorem. In the above adverse selection model with three types of Agents (assuming that the condition from Proposition 5 is satisfied), the optimal contract entails:

A. The Agent with the highest valuation has an efficient level of consumption (the same as in symmetric information situation), given by:

$$c = \bar{\theta}u'(\bar{q}) \text{ with } \bar{q}^{SB} = \bar{q}^*$$

B. The Agents with the lower valuations have inefficient levels of consumption with respect to the first best consumption levels. The second best consumption levels are given by:

$$c = u'(\hat{q}) \left[\hat{\theta} - \frac{\bar{v}}{\hat{v}} (\Delta\theta + x) \right] \text{ with } \hat{q}^{SB} < \hat{q}^*$$

and

$$c = u'(\underline{q}) \left[\underline{\theta} - \frac{\hat{v} + \bar{v}}{\underline{v}} \Delta\theta \right] \text{ with } \underline{q}^{SB} < \underline{q}^*$$

C. The optimal informational rents are:

$$\underline{U}^{SB} = 0 \text{ (the Agent with the type } \underline{\theta} \text{ gets no informational rent);}$$

$$\hat{U}^{SB} = \Delta\theta u(\underline{q}^{SB}) \text{ (the Agent with type } \hat{\theta} \text{ gets a positive informational rent);}$$

$$\bar{U}^{SB} = \Delta\theta u(\underline{q}^{SB}) + (\Delta\theta + x)u(\hat{q}^{SB}) \text{ (the Agent with type } \bar{\theta} \text{ gets the highest informational rent).}$$

D. The optimal transfer levels received by the Principal are:

$$\underline{t}^{SB} = \underline{\theta}u(\underline{q}^{SB}) \quad (44)$$

(if the Agent has the least efficient valuation he will pay to the Principal a transfer equal to his utility valuation of the good).

$$\hat{t}^{SB} = \hat{\theta}u(\hat{q}^{SB}) - \Delta\theta u(\underline{q}^{SB}) \quad (45)$$

$$\bar{t}^{SB} = \bar{\theta}u(\bar{q}^*) - \Delta\theta u(\underline{q}^{SB}) - (\Delta\theta + x)u(\hat{q}^{SB}) \quad (46)$$

In this paper we have analyzed the optimal nonlinear pricing problem for a monopolist facing with more than two types of buyers. We provided a full characterization of the optimal nonlinear pricing in the case when the buyer's valuation was his private information. In this situation, the best the Principal can do is to offer a menu of price-quantity schedule, hoping that each type of buyer chooses the contract designed for him.

It seems worthwhile to apply these techniques to other areas of mechanism design such as optimal regulation of a multiproduct firm with unknown costs, optimal design of financial contracts and optimal design of labor contracts or public utilities regulation.

APPENDIX

A. First best production and transfer levels

The optimization problem is:

$$(\max)_{t,q} \{t - cq\}$$

s.t.

$$\theta u(q) - t \geq 0$$

$$q \geq 0, t \geq 0$$

Let λ be the multiplier assigned to the participation constraint. The Lagrange function is:

$$L(t, q, \lambda) = t - cq + \lambda [\theta u(q) - t] \quad (47)$$

The first order conditions are:

$$\frac{\partial L}{\partial t} \leq 0, \quad t \geq 0 \text{ and } t \cdot \frac{\partial L}{\partial t} = 0 \text{ or } (1 - \lambda) \leq 0, \quad t \geq 0$$

and $t(1 - \lambda) = 0$

$$\frac{\partial L}{\partial q} \leq 0, \quad q \geq 0 \text{ and } q \cdot \frac{\partial L}{\partial q} = 0 \text{ or } -c + \theta u'(q) \leq 0,$$

$$q \geq 0, \quad q[-c + \lambda q u'(q)] = 0$$

$$\frac{\partial L}{\partial \lambda} \geq 0, \quad \lambda \geq 0 \text{ and } \lambda \cdot \frac{\partial L}{\partial \lambda} = 0 \text{ or } \theta u(q) - t \geq 0,$$

$$\lambda \geq 0, \quad \lambda [\theta u(q) - t] = 0$$

If $\lambda = 0$ then $1 \leq 0$ in the first order condition. It follows that $\lambda \neq 0$; therefore, the participation constraint is binding at the optimum. The Agent's optimal transfer to the Principal is given by $t^* = \theta u(q^*)$ with $t^* > 0$. And $\lambda = 1$.

Assuming the good is produced ($q^* > 0$), we find immediately: $c = \theta u'(q^*)$.

B. The optimal second best solution

The optimization problem is:

$$(\max)_{\underline{q}, \hat{q}, \bar{q}} H(\cdot) = \left\{ \underline{v} [\underline{\theta} u(\underline{q}) - c \underline{q}] + \hat{v} [\hat{\theta} u(\hat{q}) - c \hat{q}] + \bar{v} [\bar{\theta} u(\bar{q}) - c \bar{q}] - \hat{v} \Delta\theta u(\underline{q}) - \bar{v} x u(\hat{q}) - \bar{v} \Delta\theta [u(\hat{q}) + u(\underline{q})] \right\}$$

s.t.

$$\underline{q} \leq \hat{q} \leq \bar{q} \quad (IC)$$

The first order conditions for the unconstrained optimization problem are:

$$\frac{\partial H(\cdot)}{\partial \bar{q}} = 0 \Rightarrow \bar{v} [\bar{\theta} u'(\bar{q}) - c] = 0 \quad (48)$$

or $c = \bar{\theta} u'(\bar{q})$ and so $\bar{q}^{SB} = \bar{q}^*$.

$$\frac{\partial H(\cdot)}{\partial \hat{q}} = 0 \Rightarrow \hat{v} [\hat{\theta} u'(\hat{q}) - c] - \bar{v} (\Delta\theta + x) u'(\hat{q}) = 0 \quad (49)$$

$$\text{or } c = u'(\hat{q}) \left[\hat{\theta} - \frac{\bar{v}}{\hat{v}} (\Delta\theta + x) \right]$$

$$\frac{\partial H(\cdot)}{\partial \underline{q}} = 0 \Rightarrow \underline{v} [\underline{\theta} u'(\underline{q}) - c] - \hat{v} \Delta \theta u'(\underline{q}) - \bar{v} \Delta \theta u'(\underline{q}) = 0 \quad (50)$$

$$\text{or } c = u'(\underline{q}) \left[\underline{\theta} - \frac{\hat{v} + \bar{v}}{\underline{v}} \Delta \theta \right].$$

From (48) it follows:

$$\bar{q}^{SB} = q^* \quad (51)$$

so there is no distortion of the quantity consumed by the highest valuation type of Agent.

The equation (49) can be rewritten as:

$$\hat{v} [\hat{\theta} u'(\hat{q}) - c] = \bar{v} (\Delta\theta + x) u'(\hat{q}) \quad (52)$$

Using the result from symmetric information situation,

$c = \hat{\theta} u'(\hat{q}^*)$, the above relation yields to:

$$\hat{v} [\hat{\theta} u'(\hat{q}^{SB}) - \hat{\theta} u'(\hat{q}^*)] = \bar{v} (\Delta\theta + x) u'(\hat{q}^{SB}) \quad (53)$$

Or

$$\hat{v} \theta [u'(\hat{q}^{SB}) - u'(\hat{q}^*)] = \bar{v} (\Delta\theta + x) u'(\hat{q}^{SB}) > 0 \quad (54)$$

The right hand side of the above relation being positive, we have therefore:

$$u'(\hat{q}^{SB}) > u'(\hat{q}^*) \quad (55)$$

and this yields to (using the monotonicity property of the function $u'(\cdot)$):

$$\hat{q}^{SB} < \hat{q}^*. \quad (56)$$

We can rewrite the equation (40) as:

$$\underline{v} [\underline{\theta} u'(\underline{q}) - c] = (\hat{v} + \bar{v}) \Delta \theta u'(\underline{q}) \quad (57)$$

or

$$\underline{v} \theta [u'(\underline{q}^{SB}) - u'(\underline{q}^*)] = (\hat{v} + \bar{v}) \Delta \theta u'(\underline{q}^{SB}) \quad (58)$$

The right hand side of the above relation being positive, it follows:

$$u'(\underline{q}^{SB}) - u'(\underline{q}^*) > 0 \text{ or } \underline{q}^{SB} < \underline{q}^*. \quad (59)$$

ACKNOWLEDGMENT

This work was cofinanced from the European Social Fund through Sectoral Operational Programme Human Resources Development 2007-2013, project number POSDRU/1.5/59184 "Performance and excellence in postdoctoral research in Romanian economics science domain".

REFERENCES

[1] Armstrong, M. (1996). Multiproduct Nonlinear Pricing. *Econometrica*, 64 (1): 51-75
 [2] Armstrong, M. and Rochet, J.C. (1999). Multidimensional Screening: A User's Guide. *European Economic Review*, 43: 959-979
 [3] Baron, D. and Myerson, R. (1982). Regulating a Monopolist with Unknown Costs. *Econometrica*, 50 (4): 911-930
 [4] Bolton, P. and Dewatripont, M. (2005). *Contract Theory*. MIT Press. Cambridge, Massachusetts, London, England

[5] Brighi, L. and D'Amato, M. (2002). Two Dimensional Screening: A Case of Monopoly Regulation. *Research in Economics*, 56(3): 251-264
 [6] Guesnerie, R. and Laffont, J.J. (1984). A Complete Solution to a Class of Principal-Agent Problem with an Application to the Control of a Self-Managed Firm. *Journal of Public Economics*, 25: 329-369
 [7] Jullien, B. (2000). Participation Constraints in Adverse Selection Models. *Journal of Economic Theory*, 93 (1): 1-47
 [8] Kessler, A.S., Lulfesmann, C. and Schmitz, P.W. (2005). Endogenous Punishments in Agency with Verifiable ExPost Information. *International Economic Review*, 46(4): 1207-1231
 [9] Laffont J.J. and Martimort D., (2002). *The Theory of Incentives. The Principal-Agent Model*. Princeton University Press. Princeton and Oxford
 [10] Laffont, J.J. and Tirole, J. (1986). Using Cost Observation to Regulate Firms. *Journal of Political Economy*, 94 (3): 614-641
 [11] Lewis, T.R. and Sappington, D.M. (1988). Regulating a Monopolist with Unknown Demand and Cost Functions. *RAND Journal of Economics*, 19 (3): 438-457
 [12] Malin, E. and Martimort, D. (2001). Les limites a la discrimination par le prix. *Annales d'Economie et de Statistique*, 62: 209-249
 [13] Maskin, E. and Riley, J. (1984). Monopoly with Incomplete Information. *RAND Journal of Economics*, 15 (2): 171-196
 [14] Matthews, S. and Moore, J. (1987). Monopoly Provision of Quality and Warranties: An Exploration in the Theory of Multidimensional Screening. *Econometrica*, 55: 441-467
 [15] Rochet, J.C. and Chone, P. (1998). Ironing, Sweeping and Multidimensional Screening. *Econometrica*, 66 (4): 783-826
 [16] Rochet, J.C. and Stole, L.A. (2002). Nonlinear Pricing with Random Participation. *The Review of Economic Studies*, 69 (1): 277-311
 [17] Saak, A.E. (2008). The Value of Private Information in Monopoly. *The Journal of Industrial Economics*, 56 (1): 169-189
 [18] Sibley, D.S. and Srinagesh, P. (1997). Multiproduct Nonlinear Pricing with Multiple Taste Characteristics. *RAND Journal of Economics*, 28 (4): 684-707
 [19] Spence, A.M. (1980). Multi Product Quantity-Dependent Prices and Profitability Constraints. *Review of Economic Studies*, 47(5): 821-841