

Robust Adaptive Control of a Robotic Manipulator with Unknown Dead Zone and Friction Torques

Ibrahim F. Jasim and Najah F. Jasim

Abstract— The problem of controlling a two link robotic manipulator, consisting of a rotating and a prismatic links, is addressed. The actuations of both links are assumed to have unknown dead zone nonlinearities and friction torques modeled by LuGre friction model. Because of the existence of the unknown dead zone and friction torque at the actuations, unknown parameters and unmeasured states would appear to be part of the overall system dynamics that need for estimation. Unmeasured states observer, unknown parameters estimators, and robust adaptive control laws have been derived such that closed loop global stability is achieved. Simulation results have been performed to show the efficacy of the suggested approach.

Keywords— Adaptive Robust Control, Dead Zone, Friction Torques, Robotic Manipulators.

I. INTRODUCTION

DEAD zone and friction torque are inevitable in many motion control systems. Robotic manipulators are among the many motion control systems in which actuations are mostly subjected to both dead zone and friction torques. Many methodologies were developed to present efficient control schemes for robotic manipulators; quadratic optimization control of robots was suggested in [1], adaptive control output feedback was presented in [2], robust adaptive neural controller was suggested in [3], robust adaptive fuzzy controller was suggested in [4], and a lot of other efficient controllers were suggested to control robotic manipulators. A common feature to the approaches above is the lack of consideration of actuations dead zone and friction torques.

The consideration of dead zone in nonlinear control systems was firstly pioneered by Tao and Kokotovic when they designed a dead zone inverse for the unknown dead zone [5, 6]. Other researchers suggested more powerful schemes to deal with unknown dead zones in different nonlinear control systems [7-14]. Till now, no serious scheme suggested to solve the possible existence of unknown dead zone in robotic manipulators. Moreover, considering a friction torque,

modeled by LuGre friction model, in a robotic manipulator would result in a MIMO nonlinear system containing the coupling of unmeasured states and unknown parameters and till now this issue is not addressed in the literature, even though the dead zone free SISO nonlinear systems case was addressed in [15].

Depending on the results obtained in [16], we suggest, in this paper, a stable robust adaptive control strategy for a robotic manipulator consisting of a rotating and a prismatic links. The actuations of both links are subjected to unknown dead zone nonlinearities and friction torques modeled by LuGre friction model [17]. The suggested strategy involves the design of estimators for the unknown parameters, resulted from friction torques and unknown dead zone, and observers for the unmeasured states, resulted from the friction torques. Then stable robust adaptive controller is designed for each actuation such that all closed loop signals are bounded.

The main contributions of this paper are:

1. Overcoming the control problems resulted from the existence of friction torques and unknown dead zone nonlinearities at the robot actuations.
2. Designing robust adaptive controllers, unknown parameters estimators, and unmeasured states observers for a MIMO nonlinear system (robot in this case) that contain the coupling of the unmeasured states and unknown parameters, with unknown dead zones exist at the system actuations.

The rest of the paper is organized as follows. Problem statement is explained in section 2 to describe the robotic manipulator to be controlled. In section 3, we give the dead zone and plant assumptions that would be considered throughout the paper. The main theorem of this paper is suggested in section 4. Simulation results and concluding remarks are given in sections 5 and 6 respectively.

II. PROBLEM STATEMENT

Consider the two link manipulator shown in Fig. 1 that consists of a rotating link driven by T and a prismatic link driven by F.

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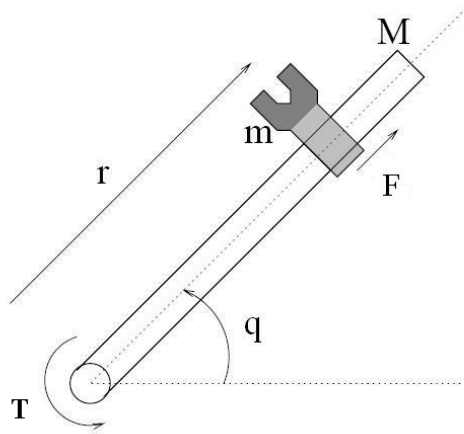


Fig.1 A two link manipulator

Suppose that both actuators T and F suffer from friction torques and dead zone nonlinearities. Then the dynamics of the robotic manipulator would be [18, 19]:

$$\ddot{q} = \frac{1}{(mr^2 + ML^2/3)} \cdot (-2mr\dot{r}\dot{q} + T - f_{f1}) \quad (1)$$

$$\ddot{r} = r\dot{q}^2 + \frac{1}{m}(F - f_{f2})$$

Where q is the angular displacement of the robot arm, r is the position of the robot hand, M is the mass of the robot arm, m is the mass of the robot hand, T and F are the robot arm and hand actuation torques respectively, f_{f1} and f_{f2} are the robot arm and hand friction torques respectively. Suppose that both T and F are subjected to dead zone nonlinearities. Now, if we use LuGre friction model for the friction torques and system modification used in [15] then the overall system dynamics becomes:

$$\ddot{q} = \frac{1}{(mr^2 + \frac{ML^2}{3})} \cdot \begin{pmatrix} -2mr\dot{r}\dot{q} + D_1(v_1(t)) \\ + f_1(\dot{q})\Theta_1 + Z_1 G_{Z1}(\dot{q})\Theta_1 \end{pmatrix} \quad (2)$$

$$\dot{Z}_1 = a_{z1}(\dot{q}) + B_{z1}(\dot{q})Z_1$$

$$\ddot{r} = r\dot{q}^2 + \frac{1}{m}(D_2(v_2(t)) + f_2(\dot{r})\Theta_2 + Z_2 G_{Z2}(\dot{r})\Theta_2)$$

$$\dot{Z}_2 = a_{z2}(\dot{r}) + B_{z2}(\dot{r})Z_2$$

Where:

$$\Theta_1 = \begin{bmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{13} \end{bmatrix}, \Theta_2 = \begin{bmatrix} \theta_{21} \\ \theta_{22} \\ \theta_{23} \end{bmatrix}, Z_1 = \begin{bmatrix} z_1 \\ z_1 \\ z \end{bmatrix}, Z_2 = \begin{bmatrix} z_2 \\ z_2 \\ z_2 \end{bmatrix}$$

$$f_1(\dot{q}) = \begin{bmatrix} 0 \\ \dot{q} \\ \dot{q} \end{bmatrix}, f_2(\dot{r}) = \begin{bmatrix} 0 \\ \dot{r} \\ \dot{r} \end{bmatrix}, a_{z1}(\dot{q}) = \begin{bmatrix} \dot{q} \\ \dot{q} \\ \dot{q} \end{bmatrix}, a_{z2}(\dot{r}) = \begin{bmatrix} \dot{r} \\ \dot{r} \\ \dot{r} \end{bmatrix}$$

$$G_{Z1}(\dot{q}) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \sigma_1 \frac{|\dot{q}|}{g_1(\dot{q})} & 0 \\ 0 & 0 & 0 \end{bmatrix}, G_{Z2}(\dot{r}) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \sigma_2 \frac{|\dot{r}|}{g_2(\dot{r})} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_1(\dot{q}) = \begin{bmatrix} \sigma_1 \frac{|\dot{q}|}{g_1(\dot{q})} & 0 & 0 \\ 0 & \sigma_1 \frac{|\dot{q}|}{g_1(\dot{q})} & 0 \\ 0 & 0 & \sigma_1 \frac{|\dot{q}|}{g_1(\dot{q})} \end{bmatrix}$$

$$B_2(\dot{r}) = \begin{bmatrix} \sigma_2 \frac{|\dot{r}|}{g_2(\dot{r})} & 0 & 0 \\ 0 & \sigma_2 \frac{|\dot{r}|}{g_2(\dot{r})} & 0 \\ 0 & 0 & \sigma_2 \frac{|\dot{r}|}{g_2(\dot{r})} \end{bmatrix}$$

$$g(x)_i = F_{ci} + (F_{si} - F_{ci})e^{-\left(\frac{\dot{x}_i}{\omega_{si}}\right)^2}$$

$$i = 1, 2$$

F_{ci} , F_{si} , and ω_{si} (Form more details on LuGre friction model see [17]). $D_i(v_i(t))$ and $v_i(t)$ are the output and input of the ith dead zoned actuation.

The dead zone model $D(v(t))$ can be described by the equation below:

$$D(v(t)) = \begin{cases} m_1(v(t) - b) & \text{for } v(t) \geq b_r \\ 0 & \text{for } b_l < v(t) < b_r \\ m_2(v(t) - b) & \text{for } v(t) \leq b_l \end{cases} \quad (3)$$

We can also describe the dead zone by the graph shown in Fig.2.

Where m_1 , m_2 , b_r , and b_l are the dead zone constants. The objective of the paper is to design robust adaptive control laws $v_1(t)$ and $v_2(t)$, parameter update laws for $\Theta_{1,2}$, and observers for $Z_{1,2}$ such that desirable tracking performance is achieved and all closed loop signals are bounded.

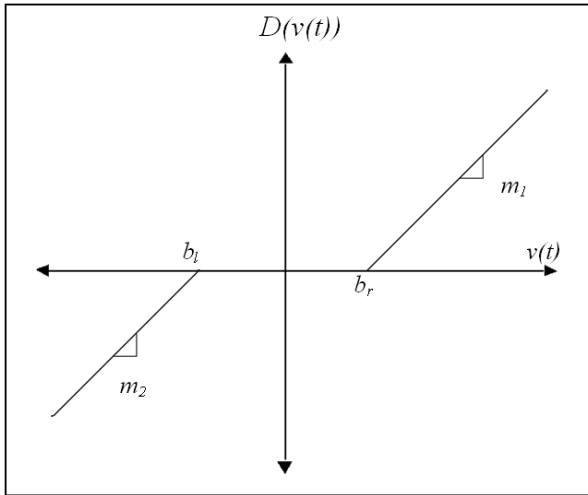


Fig.2 Dead zone model

III. DEAD ZONE AND PLANT ASSUMPTIONS

The dead zone features of the control problem investigated throughout this paper are:

- A1. The dead zone outputs $D_i(v_i(t))$ are not available for measurement ($i=1,2$).
- A2. The dead zone slopes in positive and negative regions are the same, i.e. $m_{ri}=m_{li}=m_i$ ($i=1,2$).
- A3. The dead zone parameters b_{ri} , b_{li} , and m_i are unknown but their signs are known, i.e. $b_{ri}>0$, $b_{li}<0$, $m_i>0$ ($i=1,2$).
- A4. The dead zone parameters b_{ri} , b_{li} , and m_i are bounded, i.e. there exist known constants $b_{ri\ min}$, $b_{ri\ max}$, $b_{li\ min}$, $b_{li\ max}$, $m_{i\ min}$, $m_{i\ max}$ such that $b_{ri} \in [b_{ri\ min}, b_{ri\ max}]$, $b_{li} \in [b_{li\ min}, b_{li\ max}]$, and $m_i \in [m_{i\ min}, m_{i\ max}]$ ($i=1,2$).

Remark. Assumption (A1) is common in many practical systems such as servomotors and servovalves. If $D_i(v_i(t))$ are available for measurement then the dead zone problem of this paper becomes trivial and easy to be solved. Assumption (A2) is generally adopted in the literature [8, 10] and reasonable for dead zones that are found in many practical systems. Similarly for assumptions (A3) and (A4) they are physically satisfied in many real plants.

As a result assumptions (A1) through (A4) are reasonable for dead zones that are found in many practical systems.

We can rewrite the dead zone model given by (3) to be composed of a linear part and a bounded function of $v_i(t)$:

$$D_i(v_i(t)) = m_i v_i(t) + d_i(v_i(t)) \quad (4)$$

m_i is the general slope of the dead zone, and $d_i(v_i(t))$ can be modeled as:

$$d_i(v_i(t)) = \begin{cases} -m_i b_{ri} & \text{for } v_i(t) \geq b_{ri} \\ -m_i v_i(t) & \text{for } b_{li} < v_i(t) < b_{ri} \\ -m_i b_{li} & \text{for } v_i(t) \leq b_{li} \end{cases}$$

From assumptions (A2) and (A4), one can easily conclude that $d_i(v_i(t))$ is bounded, and satisfies:

$$d_i(v_i(t)) \leq \rho_i$$

Where ρ_i is the upper bound and can be chosen as:

$$\rho_i = \max\{m_{i\ max} b_{ri\ max}, -m_{i\ max} b_{li\ min}\} \quad (5)$$

For the robotic manipulator and frictions, the following assumption should be satisfied.

- A5. The sign of each parameter, θ_{ij} , $i=1, 2, 3$, $j=1, 2$, in the parameter vectors Θ_j , $j=1,2$ is known, and θ_{ij} is bounded.

IV. THE CONTROLLER AND OBSERVER DESIGN

Based on the dead zone model, properties, and system dynamics described in (2), we shall present the main theorem of this paper. However, to simplify the derivation we need to define the following parameters:

$$\phi_i = \frac{1}{m_i} \quad (i=1, 2)$$

and

$$\Psi_i = \phi_i \cdot \Theta_i \quad (i=1, 2)$$

From assumptions, (A3), (A4), and (A5) one can easily see that both ϕ and Ψ are bounded and their signs are known.

For the desired states trajectory, the following assumption should be achieved:

(A6) The desired trajectories $q_d = [q_d, \dot{q}_d]$ and $r_d = [r, \dot{r}]$ are continuous, bounded and available for measurement.

To achieve the stated control objective, filtered tracking errors are defined as:

$$\begin{aligned} s_1(t) &= \left(\frac{d}{dt} + \lambda_1 \right) \tilde{q}(t) \\ s_2(t) &= \left(\frac{d}{dt} + \lambda_2 \right) \tilde{r}(t) \end{aligned} \quad \text{with } \lambda_{1,2} > 0 \quad (6)$$

We can rewrite (6) as:

$$\begin{aligned} s_1(t) &= \Lambda_1^T \tilde{q}(t) \quad \text{where } \Lambda_1^T = [\lambda_1, 1] \\ s_2(t) &= \Lambda_2^T \tilde{r}(t) \quad \text{where } \Lambda_2^T = [\lambda_2, 1] \\ \tilde{q}(t) &= q(t) - q_d(t) \quad \text{and} \quad \tilde{r}(t) = r(t) - r_d(t) \end{aligned}$$

From the equations above and by defining $\Lambda_{vi}^T = [0, \lambda_i]$ we can rewrite equation (6) as:

$$\begin{aligned} \dot{s}_1(t) &= \Lambda_{v1}^T \tilde{q}(t) + \ddot{\tilde{q}}(t) \\ &= \Lambda_{v1}^T \tilde{q}(t) + \frac{1}{\left(mr^2 + \frac{ML^2}{3} \right)} (-2mrr\dot{q}) \\ &\quad + m_1 v_1(t) + d_1(v_1(t)) + f_1(\dot{q}) \Theta_1 \\ &\quad + Z_1 G_{Z1}(\dot{q}) \Theta_1 - \ddot{q}_d(t) \end{aligned} \quad (7)$$

$$\begin{aligned}\dot{s}_2(t) &= \Lambda_{v_2}^T \tilde{\mathbf{r}}(t) + \ddot{\tilde{\mathbf{r}}}(t) \\ &= \Lambda_{v_2}^T \tilde{\mathbf{r}}(t) + r\dot{q}^2 + \frac{1}{m}(m_2 v_2(t) + d_2(v_2(t))) \\ &\quad + f_2(\dot{r})\Theta_2 + Z_2 G_{Z_2}(\dot{r})\Theta_2 - \ddot{r}_d(t)\end{aligned}\quad (8)$$

Note: It has been shown that the filtered error described by (6) has the following properties: (i) the equation $s_i(t)=0$ defines a time-varying hyperplane in \mathbf{R}^2 , on which the tracking error vectors $\tilde{\mathbf{q}}(t)$ and $\tilde{\mathbf{r}}(t)$ decay exponentially to zero, (ii) if $\tilde{\mathbf{q}}(t)=0$ and $\tilde{\mathbf{r}}(t)=0$ and $|s_i(t)| \leq \varepsilon_i$ with constant ε_i ,

then $\tilde{\mathbf{q}}(t) \in \Omega_{\varepsilon_1} = \left\{ \begin{array}{l} \tilde{\mathbf{q}}(t) \leq 2^{j-1} \lambda_1^{j-2} \varepsilon_1, j=1,2 \\ \tilde{\mathbf{q}}_j \end{array} \right\}$ and

$$\tilde{\mathbf{r}}(t) \in \Omega_{\varepsilon_2} = \left\{ \begin{array}{l} \tilde{\mathbf{r}}(t) \leq 2^{j-1} \lambda_2^{j-2} \varepsilon_2, j=1,2 \\ \tilde{\mathbf{r}}_j \end{array} \right\} \text{ for } \forall t \geq 0,$$

and (iii) if $\tilde{\mathbf{q}}(0) \neq 0$, $\tilde{\mathbf{r}}(0) \neq 0$ and $|s_i(t)| \leq \varepsilon_i$, then $\tilde{\mathbf{q}}(t)$ and $\tilde{\mathbf{r}}(t)$ will converge to Ω_{ε_1} and Ω_{ε_2} respectively within a time-constant $(2-1)/\lambda_i$ [20, 21].

It is important to mention that rather than deriving the adaptive laws depending on the filtered error $s_i(t)$, a tuning error $s_{\varepsilon i}$ is introduced as follows:

$$s_{\varepsilon i} = s_i - \varepsilon_i \text{sat}\left(\frac{s_i}{\varepsilon_i}\right) \quad (9)$$

Where ε_i is an arbitrary positive constant and $\text{sat}(\cdot)$ is the saturation function defined as:

$$\text{sat}(c) = \begin{cases} 1 & \text{for } c \geq 1 \\ c & \text{for } -1 < c < 1 \\ -1 & \text{for } c \leq -1 \end{cases} \quad (10)$$

Theorem: For the robot described by (2) with unknown dead zones, modeled by (4), exist at the robot inputs, the following control laws (11), parameters estimation algorithm (12) and (13), and observers (14)

$$\begin{aligned}v_1(t) &= -k_{d1}s_1(t) + \frac{\hat{\phi}_1}{b_1(\mathbf{q}, \mathbf{r})} u_{fd1} - f_1^T(\dot{q})\hat{\Psi}_1 \\ &\quad - \hat{Z}_1^T G_{Z_1}(\dot{q})\hat{\Psi}_1 - k_1^* s_{\varepsilon 1} \text{sat}\left(\frac{s_1}{s_{\varepsilon 1}}\right)\end{aligned}\quad (11.a)$$

$$\begin{aligned}v_2(t) &= -k_{d2}s_2(t) + \frac{\hat{\phi}_2}{b_2(\mathbf{q}, \mathbf{r})} u_{fd2} - f_2^T(\dot{r})\hat{\Psi}_2 \\ &\quad - \hat{Z}_2^T G_{Z_2}(\dot{r})\hat{\Psi}_2 - k_2^* s_{\varepsilon 2} \text{sat}\left(\frac{s_2}{s_{\varepsilon 2}}\right)\end{aligned}\quad (11.b)$$

$$\dot{\hat{\Psi}}_1 = -\Gamma_1^{-1} [s_{\varepsilon 1} f_1(\dot{q}) + s_{\varepsilon 1} \hat{Z}_1^T G_{Z_1}(\dot{q})] \quad (12.a)$$

$$\dot{\hat{\Psi}}_2 = -\Gamma_2^{-1} [s_{\varepsilon 2} f_2(\dot{r}) + s_{\varepsilon 2} \hat{Z}_2^T G_{Z_2}(\dot{r})] \quad (12.b)$$

$$\dot{\hat{\phi}}_1 = \eta_1 s_{\varepsilon 1} \frac{u_{fd1}}{b_1(\mathbf{q}, \mathbf{r})} \quad (13.a)$$

$$\dot{\hat{\phi}}_2 = \eta_2 s_{\varepsilon 2} \frac{u_{fd2}}{b_2(\mathbf{q}, \mathbf{r})} \quad (13.b)$$

$$\dot{\hat{Z}}_1 = a_1(\dot{q}) + B_1(\dot{q})\hat{Z}_1 + s_{\varepsilon 1} P_{Z_2}^{-1} \text{sgn}(\Psi_1) G_{Z_1}(\dot{q}) \quad (14.a)$$

$$\dot{\hat{Z}}_2 = a_2(\dot{r}) + B_2(\dot{r})\hat{Z}_2 + s_{\varepsilon 2} P_{Z_2}^{-1} \text{sgn}(\Psi_2) G_{Z_2}(\dot{r}) \quad (14.b)$$

ensure that all closed loop signals are bounded and the state vectors $\mathbf{q}(t)$ and $\mathbf{r}(t)$ converge to $\Omega_{\varepsilon 1}$ and $\Omega_{\varepsilon 2}$ for $\forall t \geq t_0$.

Where k_{d1} , k_{d2} , η_1 , η_2 are positive constants, $\Gamma_1 = \Gamma_1^{-1}$, $\Gamma_2 = \Gamma_2^{-1}$, P_{Z_1} and P_{Z_2} are diagonal positive definite matrices, and $k_i^* \geq \rho_i / m_{i\min}$ ($i=1, 2$).

Where $u_{fd1} = \ddot{q}_d(t) - \Lambda_{v_1}^T \tilde{\mathbf{q}}(t) + 2mrr\dot{q}$

$$u_{fd2} = \ddot{r}_d(t) - \Lambda_{v_2}^T \tilde{\mathbf{r}}(t)$$

$$b_1(\mathbf{q}, \mathbf{r}) = \frac{1}{(mr^2 + ML^2/3)}$$

$$b_2(\mathbf{q}, \mathbf{r}) = \frac{1}{m}$$

Proof:

Using the control laws (11.a) and (11.b) in (7) and (8) respectively, then we can rewrite the filtered errors by:

$$\begin{aligned}\dot{s}_1(t) &= \Lambda_{v_1}^T \tilde{\mathbf{q}}(t) + b_1(\mathbf{q}, \mathbf{r}) \left[\frac{1}{\phi_1} \left(-k_{d1}s_1(t) + \frac{\hat{\phi}_1}{b_1(\mathbf{q}, \mathbf{r})} u_{fd1} \right. \right. \\ &\quad \left. \left. - f_1^T(\dot{q})\hat{\Psi}_1 - \hat{Z}_1 G_{Z_1}(\dot{q})\hat{\Psi}_1 - k_1^* s_{\varepsilon 1} \text{sat}\left(\frac{s_1}{s_{\varepsilon 1}}\right) \right) \right] \\ &\quad + d_1(v_1(t)) + f_1^T(\dot{q})\Theta_1 + Z_1^T G(\dot{q})\Theta_1 - \dot{q}_d^{(2)}\end{aligned}\quad (14.a)$$

$$\begin{aligned}\dot{s}_2(t) &= \Lambda_{v_2}^T \tilde{\mathbf{r}}(t) + b_2(\mathbf{q}, \mathbf{r}) \left[\frac{1}{\phi_2} \left(-k_{d2}s_2(t) + \frac{\hat{\phi}_2}{b_2(\mathbf{q}, \mathbf{r})} u_{fd2} \right. \right. \\ &\quad \left. \left. - f_2^T(\dot{r})\hat{\Psi}_2 - \hat{Z}_2 G_{Z_2}(\dot{r})\hat{\Psi}_2 - k_2^* s_{\varepsilon 2} \text{sat}\left(\frac{s_2}{s_{\varepsilon 2}}\right) \right) \right] \\ &\quad + d_2(v_2(t)) + f_2^T(\dot{r})\Theta_2 + Z_2^T G(\dot{r})\Theta_2 - \dot{r}_d^{(2)}\end{aligned}\quad (14.b)$$

(11.a) Consider the Lyapunov candidate:

$$V = \frac{1}{2} \sum_{i=1}^2 \left[\frac{\phi_i}{b_i(\mathbf{q}, \mathbf{r})} s_{\varepsilon i}^2 + \tilde{\Psi}_i^T \Gamma_i \tilde{\Psi}_i + \frac{1}{\eta_i} \tilde{\phi}_i^2 + \tilde{Z}_i^T \Lambda_{|\Psi_i|} \tilde{Z}_i \right] \quad (15)$$

Where $\Lambda_{|\Psi_i|}$ are diagonal matrices that the i th diagonal

element is the absolute value of the i th element in the parameter vector Ψ_I , that

$$\Lambda_{|\Psi_I|} = \text{diag}(|\psi_{I1}| \mathbf{I}_m, \dots, |\psi_{Ii}| \mathbf{I}_m, \dots, |\psi_{Ip}| \mathbf{I}_m, \mathbf{I}_m)$$

$$\therefore \dot{V} = \sum_{I=1}^2 \left[\frac{\phi_I}{b_I(\mathbf{q}, \mathbf{r})} s_{\epsilon I} \dot{s}_{\epsilon I} + \dot{\Psi}_I^T \Gamma_I \tilde{\Psi}_I + \frac{1}{\eta_I} \tilde{\phi}_I \dot{\hat{\phi}} + \tilde{Z}_I^T \Lambda_{|\Psi_I|} \dot{\tilde{Z}}_I \right] \quad (16)$$

Now, using the fact $s_{\epsilon I} \dot{s}_{\epsilon I} = s_{\epsilon I} \dot{s}_I$ and (14), equation (16) can be rewritten as:

$$\dot{V} = \sum_{I=1}^2 \left[\frac{\phi_I}{b_I(\mathbf{q}, \mathbf{r})} s_{\epsilon I} \dot{s}_I + \dot{\Psi}_I^T \Gamma_I \tilde{\Psi}_I + \frac{1}{\eta_I} \tilde{\phi}_I \dot{\hat{\phi}} + \tilde{Z}_I^T \Lambda_{|\Psi_I|} \dot{\tilde{Z}}_I \right] \quad (17)$$

Using the observers described in (14), we can easily obtain:

$$\dot{\tilde{Z}}_1 = B_1(\dot{q}) \tilde{Z}_1 + s_{\epsilon 1} P_{Z1}^{-1} \text{sgn}(\Psi_1) G_{Z1}(\dot{q}) \quad (18.a)$$

$$\dot{\tilde{Z}}_2 = B_2(\dot{r}) \tilde{Z}_2 + s_{\epsilon 2} P_{Z2}^{-1} \text{sgn}(\Psi_2) G_{Z2}(\dot{r}) \quad (18.b)$$

Substituting (12), (13), (14) and (18) in (17), using the fact that $\Lambda_{|\Psi_I|} \text{sgn}(\Psi_I) = \Psi_I$, and after mathematical manipulations we obtain:

$$\dot{V} = \sum_{I=1}^2 \left[-k_{dI} s_{\epsilon I} s_I - k_I^* s_{\epsilon I} \text{sat} \left(\frac{s_I}{s_{\epsilon I}} \right) + \phi_I s_{\epsilon I} d_I(v_I(t)) + \tilde{Z}_I^T \Lambda_{|\Psi_I|} P_Z B_I(x) \tilde{Z} \right]$$

$$x = \begin{cases} \dot{q} & \text{for } I=1 \\ \dot{r} & \text{for } I=2 \end{cases}$$

$$\dot{V} = \sum_{I=1}^2 \left[-k_{dI} s_{\epsilon I} s_I - k_I^* s_{\epsilon I} \text{sat} \left(\frac{s_I}{s_{\epsilon I}} \right) + \phi_I s_{\epsilon I} d_I(v_I(t)) + \frac{1}{2} \tilde{Z}_I^T \Lambda_{|\Psi_I|} (P_{ZI} B_I(x) + B_I^T(x) P_Z) \tilde{Z}_I \right]$$

$$\dot{V} \leq \sum_{I=1}^2 \left[-k_{dI} s_{\epsilon I} s_I - k_I^* s_{\epsilon I} \text{sat} \left(\frac{s_I}{s_{\epsilon I}} \right) + \phi_I s_{\epsilon I} d_I(v_I(t)) - \frac{1}{2} \tilde{Z}_I^T \Lambda_{|\Psi_I|} \mathbf{Q}_{ZI} \tilde{Z}_I \right]$$

\mathbf{Q}_{ZI} are positive definite matrices.

$$\therefore \dot{V} \leq \sum_{I=1}^2 \left[-k_{dI} s_{\epsilon I}^2 - \frac{1}{2} \tilde{Z}_I^T \Lambda_{|\Psi_I|} \mathbf{Q}_{ZI} \tilde{Z}_I \right]$$

$$\therefore \dot{V} \leq \sum_{I=1}^2 \left[-k_{dI} s_{\epsilon I}^2 - \frac{1}{2} \lambda_{\text{imin}}(\Lambda_{|\Psi_I|} \mathbf{Q}_{ZI}) \tilde{Z}_I^T \tilde{Z}_I \right] \quad (19)$$

From (19), it is clear that $s_{\epsilon 1,2} \in L_2 \cap L_\infty$, and $\tilde{Z}_{1,2}, \tilde{\Theta}_{1,2}, \tilde{\phi}_{1,2} \in L_\infty$. Since $Z_{1,2}, \Theta_{1,2}$, and $\phi_{1,2}$ are bounded and $\tilde{Z}_{1,2}, \tilde{\Theta}_{1,2}, \tilde{\phi}_{1,2} \in L_\infty$, then $\hat{Z}_{1,2}, \hat{\Theta}_{1,2}$, and

$\hat{\phi}_{1,2}$ are also bounded. Then using (18), we can easily conclude that $\dot{s}_{1,2} \in L_\infty$ which implies that $\dot{s}_{\epsilon 1,2} \in L_\infty$. We have $s_{\epsilon 1,2} \in L_2 \cap L_\infty$ and $\dot{s}_{\epsilon 1,2} \in L_\infty$, then $s_{\epsilon 1,2} \rightarrow 0$ as $t \rightarrow \infty$ according to Barbalat's lemma. This would make $\tilde{q}(t)$ and $\tilde{r}(t)$ converging to $\Omega_{\epsilon 1}$ and $\Omega_{\epsilon 2}$ respectively. If $\mathbf{Q}_{Z1,2}$ is chosen to be positive definite matrix, then from (19) we can easily conclude that $\tilde{Z}_{1,2} \in L_2 \cap L_\infty$. Since $\tilde{Z}_{1,2} \in L_\infty$, then from (18), we can see that $\dot{\tilde{Z}}_{1,2} \in L_\infty$. Again invoking to Barbalat's Lemma, then we have $\tilde{Z}_{1,2} \rightarrow 0$ as $t \rightarrow \infty$. ■

V. SIMULATION RESULTS

Simulation results were implemented for a two link robotic manipulator with the following links and friction torques parameters:

Table I The Robot Links and Friction Torques Parameters

| Parameter | Value | Parameter | Value |
|------------|-------|---------------|-------|
| m | 1 | ω_{s1} | 0.14 |
| ML^2 | 3 | σ_2 | 350 |
| σ_1 | 340 | F_{s2} | 15 |
| F_{s1} | 11 | F_{c2} | 1.8 |
| F_{c1} | 1.557 | ω_{s2} | 0.1 |

For the dead zones bounds, we considered the bounds given in the table below:

Table II The Unknown Dead Zones Bounds

| Parameter | Value | Parameter | Value |
|-------------|-------|-------------|-------|
| b_{r1min} | 0.1 | b_{r2min} | 0.2 |
| b_{r1max} | 0.6 | b_{r2max} | 0.75 |
| b_{l1min} | -0.7 | b_{l2min} | -0.3 |
| b_{l1max} | -0.1 | b_{l2max} | 0.1 |
| m_{1min} | 0.85 | m_{2min} | 0.7 |
| m_{1max} | 1.25 | m_{2max} | 1.7 |

As a result of Table 2, we can choose k_1^* and k_2^* to be 2.5 and 3.2 respectively. Other controller constants were chosen as shown in the table below:

Table II Controller Constants

| Parameter | Value | Parameter | Value |
|-------------|-------|-------------|-------|
| k_{d1} | 10 | k_{d2} | 10 |
| λ_1 | 3 | λ_2 | 2 |

In our simulation, we obtained the best results for ε_1 and ε_2 to be 0.001. For the desired positions of link 1 and 2, the following desired trajectories are to be tracked using the suggested controllers and observers:

$$x_{d1}(t) = \sin(0.4\pi t) \quad \text{desired position of link1.}$$

$$x_{d3}(t) = 0.5 \sin(0.4\pi t) \quad \text{desired position of link2.}$$

Using the suggested control actions, unknown parameters update laws, and observers we obtained the simulation results shown in figures 3, 4, and 5.

It is clear that excellent position and velocity tracking performance for both link 1 and 2 is obtained. Moreover, the simulation confirms that all closed loop signals are bounded. It is important to point out that the tuning errors $S_{\varepsilon 1,2}$ will disappear when the filtered errors $S_{1,2}$ is less than ε which is equivalent to creating an adaptation dead band. Moreover the

term $k_i^* s_{\varepsilon i} \text{sat} \left(\frac{s}{s_{\varepsilon i}} \right)$, $i=1,2$, of equation (11) reflect the component for compensating the bounded function $d(v(t))$ that give the robust property to the suggested control laws.

VI. CONCLUSION

The challenge of controlling a two link robotic manipulator with unknown dead zone and friction torques exist at the robot actuations was addressed. The existence of both friction torques and unknown dead zones made the system to contain unknown parameters that was estimated through the suggested parameters update laws. Unmeasured states were resulted from the friction torque and suitable observers were designed. It was proven that using the suggested robust adaptive control, parameters update laws, and observers all closed loop signals are bounded.

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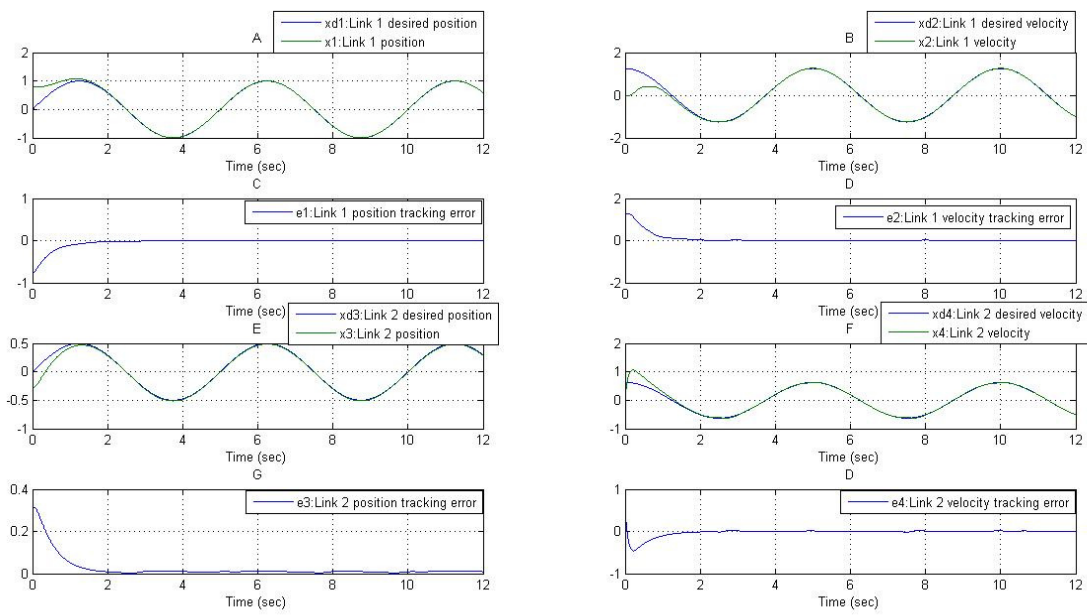


Fig. 3. A. Link1 position tracking performance. B. Link1 velocity tracking performance. C. Link1 position tracking error. D. Link1 velocity tracking error. E. Link2 position tracking performance. F. Link2 velocity tracking performance. G. Link2 position tracking error. H. Link3 velocity tracking error.

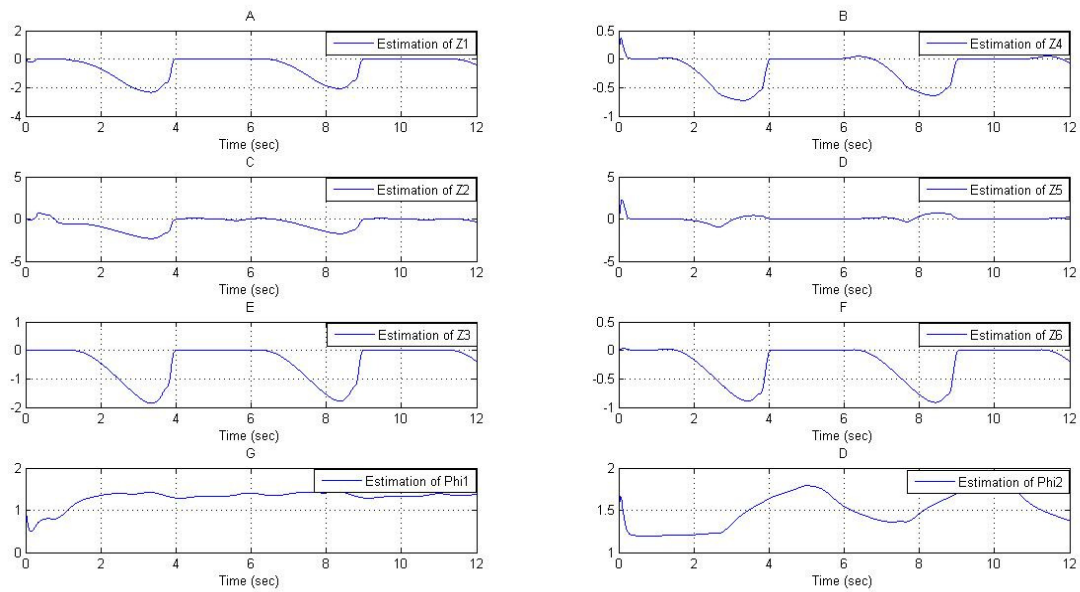


Fig. 4. A. Estimation of Z_1 . C. Estimation of Z_2 . E. Estimation of Z_3 . B. Estimation of Z_4 . D. Estimation of Z_5 . F. Estimation of Z_6 . G. Estimation of Φ_1 . H. Estimation of Φ_2 .

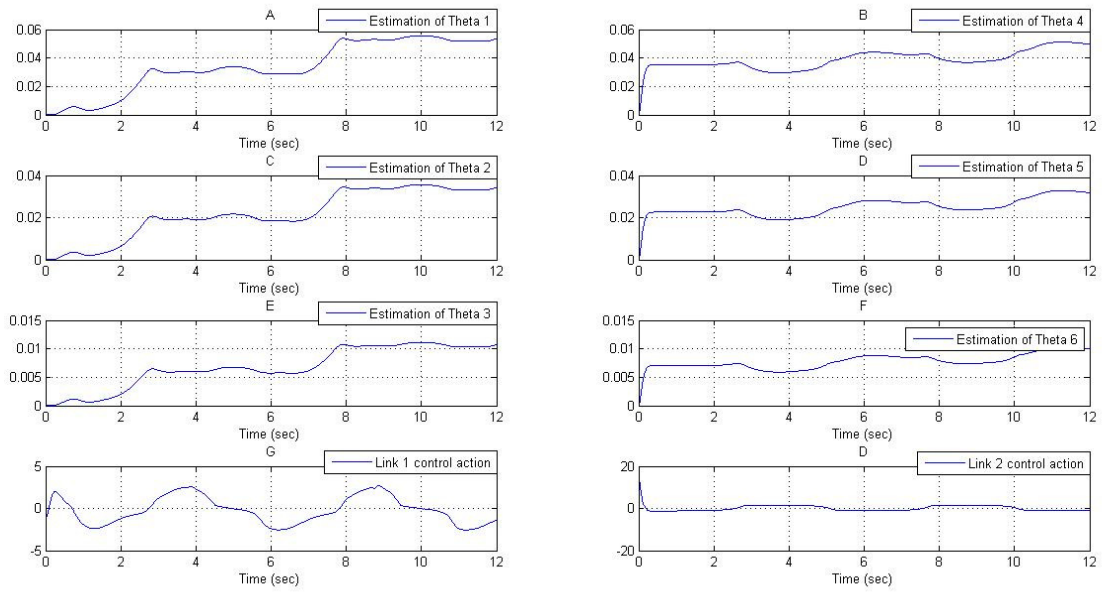


Fig. 5. A. Estimation of Ψ_1 . C. Estimation of Ψ_2 . E. Estimation of Ψ_3 . B. Estimation of Ψ_4 . D. Estimation of Ψ_5 . F. Estimation of Ψ_6 . G. Link1 control action. H. Link2 control action.