

A Shape Optimization Method in Viscous Flow Using Acoustic Velocity and Four-step Explicit Scheme

Yoichi Hikino and Mutsuto Kawahara

Abstract—The purpose of this study is to derive optimal shapes of a body located in viscous flows by the finite element method using the acoustic velocity and the four-step explicit scheme. The formulation is based on an optimal control theory in which a performance function of the fluid force is introduced. The performance function should be minimized satisfying the state equation. This problem can be transformed into the minimization problem without constraint conditions by using the adjoint equation with adjoint variables corresponding to the state equation. The performance function is defined by the drag and lift forces acting on the body. The weighted gradient method is applied as a minimization technique, the Galerkin finite element method is used as a spatial discretization and the four-step explicit scheme is used as a temporal discretization to solve the state equation and the adjoint equation. As the interpolation, the orthogonal basis bubble function for velocity and the linear function for pressure are employed. In case that the orthogonal basis bubble function is used, the mass matrix can be diagonalized without any artificial centralization. The shape optimization is performed by the presented method.

Keywords—Shape Optimization, Optimal Control Theory, Finite Element Method, Weighted Gradient Method, Fluid Force, Orthogonal Basis Bubble Function, Four-step Explicit Scheme, Acoustic Velocity.

I. INTRODUCTION

A Shape design of a body which minimizes the subjected fluid forces has been one of the main theme in the fluid dynamics. Recently, the computational fluid dynamics (CFD) has been developed by the improvement of computer. Until recent years, most of the shape design in engineering field is obtained by experiments and experiences. However, experiments need expensive cost, experiments of large scale and long experimental time. Numerical analysis of the fluid forces by CFD technique makes it possible to reduce the costs and time of experiments.

Most of fluid flows in engineering field are considered as incompressible. However, the fluid existing in the natural world has compressibility at any case. That is to say, the acoustic velocity which passes through the fluid is finite. Considering the acoustic velocity which would be a large number but limited quantity, it is possible to transform the basic equation considering slight compressibility. The equation including the term of time derivative of pressure is obtained by the formulation based on the conservation equations of mass and momentum. In case that the acoustic velocity comes to be

infinite, the resulted equation becomes the conventional continuity equation of the incompressible flow. The equation of continuity, which is more realistic in the actual phenomenon, is obtained because the acoustic velocity can be measured accurately. In this study, the derived equation is used as the continuity equation.

The purpose of this study is to obtain an optimal shape of a body located in the viscous flow based on an optimal control theory. The momentum equation is expressed considering the acoustic velocity in the non-dimensional form. The fluid forces can be derived by integrating the traction acting on the body surface. In this study, the optimal shape is defined to minimize both drag and lift forces. Therefore, the fluid forces are applied as a judgment index of the optimal shape. The performance function is defined by the square sum of the fluid forces and should be minimized satisfying the state equation. Considering that the volume of a body should be kept constant, the volume constraint condition is also introduced.

This problem can be transformed into the minimization problem without constraint by the Lagrange multiplier method. The finite element method is used to solve the state and adjoint equations because this method is suitable for the analysis of arbitrary shapes and computing the fluid forces. As the spatial discretization, the finite element method using the mixed interpolation is applied. As the temporal discretization, the four-step explicit scheme is employed. The orthogonal basis bubble function for the interpolation of the velocity and the linear interpolation for the pressure are used. The orthogonal basis bubble function makes it possible to diagonalize the mass matrix without artificial centralization. Because the orthogonal basis bubble function is used, the mass matrix only includes the diagonal components. Therefore it is possible to solve the state and adjoint equations by the explicit scheme.

II. STATE EQUATION

In this study, the indicial notation and the summation convention are used. Non-dimensional form of the equation of viscous flow is expressed as follows:

$$\begin{aligned} \dot{u}_i + u_j u_{i,j} + c p_{,i} - \lambda u_{k,ki} - \nu (u_{i,j} + u_{j,i})_{,j} &= 0, \quad (1) \\ \dot{p} + u_j p_{,j} + c u_{i,i} &= 0, \quad (2) \end{aligned}$$

where u_i , p , c , λ and ν are the velocity, pressure, acoustic velocity, coefficient of bulk viscosity ($\lambda = -\frac{2}{3}\nu$) and kinematic viscosity coefficient, respectively. Here, ν is expressed as the inverse of the Reynolds number. A typical problem is

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described in Figure 1, in which a solid body B with a boundary Γ_B is laid in an external flow. Suppose that the boundary and initial conditions for the problem are given as follows:

$$u_i = (U, 0) \quad \text{on } \Gamma_U, \quad (3)$$

$$t_i = 0 \quad \text{on } \Gamma_D, \quad (4)$$

$$t_1 = 0, \quad u_2 = 0 \quad \text{on } \Gamma_S, \quad (5)$$

$$u_i = 0 \quad \text{on } \Gamma_B, \quad (6)$$

$$u_i(t_0) = \hat{u}_{i0}, \quad p(t_0) = \hat{p}_0 \quad \text{in } \Omega, \quad (7)$$

where

$$t_i = \{-p\delta_{ij} + \lambda u_{k,k}\delta_{ij} + \nu(u_{i,j} + u_{j,i})\}n_j. \quad (8)$$

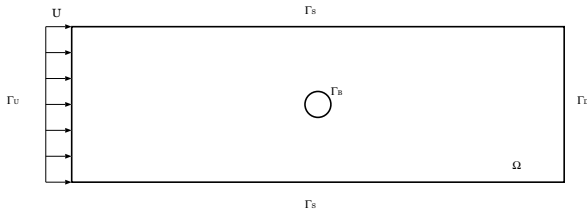


Fig. 1. Computational domain and boundary condition

Here, U , t_i , δ_{ij} and n_j are the constant inflow, traction on the boundary Γ , Kronecker delta and unit outward normal to Γ , respectively. The fluid forces acting on the body B are denoted by F_i , where F_1 and F_2 are the drag and lift forces, respectively. The fluid forces F_i are obtained by integrating the traction t_i on the boundary Γ_B as follows:

$$F_i = - \int_{\Gamma_B} t_i d\Gamma. \quad (9)$$

III. FORMULATION

A. Performance function

In this study, a minimum fluid force problem is treated. The performance function J is defined by the square sum of the fluid forces as follows:

$$J = \frac{1}{2} \int_{t_0}^{t_f} \{q_1(F_1 - \hat{F}_1)^2 + q_2(F_2 - \hat{F}_2)^2\} dt, \quad (10)$$

where q_1 and q_2 are the weighting parameters, \hat{F}_1 and \hat{F}_2 are the target drag and lift forces, respectively. In this study, \hat{F}_1 and \hat{F}_2 are set to zero. In the minimum fluid force theory presented in this paper, at the state that the performance function of the fluid force is minimum, the optimal state is regarded to be obtained. The performance function should be minimized satisfying the constraint conditions which are the state equations (1) and (2). The Lagrange multiplier method is suitable for the minimization problem with constraint conditions. The Lagrange multipliers for the state equations (1) and (2) are defined as the adjoint velocity u_i^* and pressure p^* . The

performance function is extended by adding the inner product between adjoint variables and state equations. The extended performance function J^* is described as follows:

$$\begin{aligned} J^* = & \frac{1}{2} \int_{t_0}^{t_f} (q_1 F_1^2 + q_2 F_2^2) dt \\ & + \int_{t_0}^{t_f} \int_{\Omega} u_i^* \{ \dot{u}_i + u_j u_{i,j} + cp_{,i} - \lambda u_{k,ki} \\ & \quad - \nu(u_{i,j} + u_{j,i})_{,j} \} d\Omega dt \\ & + \int_{t_0}^{t_f} \int_{\Omega} p^* \{ \dot{p} + u_j p_{,j} + cu_{i,i} \} d\Omega dt. \end{aligned} \quad (11)$$

B. Stationary condition

The minimization problem with a constraint condition results in solving a stationary condition of the extended performance function J^* instead of the performance function J . Adjoint equations and a gradient used to update the shape of a body can be derived by the fact that the first variation of the extended performance function equals to zero. The stationary condition is expressed as follows:

$$\delta J^* = 0. \quad (12)$$

The first variation of the extended performance function can be calculated as follows:

$$\begin{aligned} \delta J^* = & \int_{t_0}^{t_f} \int_{\Omega} \delta u_i^* \{ \dot{u}_i + u_j u_{i,j} + cp_{,i} - \lambda u_{k,ki} \\ & \quad - \nu(u_{i,j} + u_{j,i})_{,j} \} d\Omega dt \\ & + \int_{t_0}^{t_f} \int_{\Omega} \delta p^* \{ \dot{p} + u_j p_{,j} + cu_{i,i} \} d\Omega dt \\ & + \int_{t_0}^{t_f} \int_{\Omega} \delta u_i \{ -\dot{u}_i^* - u_j u_{i,j}^* + u_{j,i} u_j^* + cp_{,i}^* \\ & \quad - \lambda u_{k,ki}^* - \nu(u_{i,j}^* + u_{j,i}^*)_{,j} \} d\Omega dt \\ & + \int_{t_0}^{t_f} \int_{\Omega} \delta p \{ -\dot{p} - u_j p_{,j} + cu_{i,i} \} d\Omega dt \\ & + \int_{t_0}^{t_f} \int_{\Gamma_D} \delta u_i s_i d\Gamma dt + \int_{t_0}^{t_f} \int_{\Gamma_S} \delta u_1 s_1 d\Gamma dt \\ & + \int_{t_0}^{t_f} \int_{\Gamma_B} \delta u_i s_i d\Gamma dt \\ & + \int_{t_0}^{t_f} \int_{\Gamma_U} \delta t_i u_i^* d\Gamma dt + \int_{t_0}^{t_f} \int_{\Gamma_S} \delta t_2 u_2^* d\Gamma dt \\ & + \int_{t_0}^{t_f} \int_{\Gamma_B} \delta t_i (u_i^* - q_j F_j \delta_{ij}) d\Gamma dt \\ & + \int_{\Omega} u_i^*(t_f) \delta u_i(t_f) d\Omega - \int_{\Omega} u_i^*(t_0) \delta u_i(t_0) d\Omega \\ & + \int_{\Omega} p^*(t_f) \delta p(t_f) d\Omega - \int_{\Omega} p^*(t_0) \delta p(t_0) d\Omega, \end{aligned} \quad (13)$$

where

$$s_i = \{u_j u_i^* - p^* \delta_{ij} - \lambda u_{k,k}^* \delta_{ij} + \nu(u_{i,j}^* + u_{j,i}^*)\} n_j \quad (14)$$

Considering $\delta J^* = 0$, each term of eq.(13) must be equal to zero. Therefore, the adjoint equations, the boundary and

terminal conditions are obtained as in the following form.

$$-\dot{u}_i^* - u_j u_{i,j}^* + u_{j,i} u_j^* + c p_{,i}^* - \lambda u_{k,ki}^* - \nu (u_{i,j}^* + u_{j,i}^*),_j = 0 \quad \text{in } \Omega, \quad (15)$$

$$-\dot{p}^* - u_j p_{,j}^* + c u_{i,i}^* = 0 \quad \text{in } \Omega, \quad (16)$$

$$u_i^* = 0 \quad \text{on } \Gamma_U, \quad (17)$$

$$s_i = 0 \quad \text{on } \Gamma_D, \quad (18)$$

$$s_1 = 0, \quad u_2^* = 0 \quad \text{on } \Gamma_S, \quad (19)$$

$$u_1^* - q_1 F_1 = 0, \quad u_2^* - q_2 F_2 = 0 \quad \text{on } \Gamma_B, \quad (20)$$

$$u_i^*(t_f) = 0, \quad p^*(t_f) = 0 \quad \text{in } \Omega. \quad (21)$$

The relation:

$$\delta u_i = u_{i,j} \delta X_j, \quad (22)$$

is used, where X_j means the surface coordinates of the body. Eq.(13) is transformed into the following equation.

$$\delta J^* = - \int_{t_0}^{t_f} \int_{\Gamma_B} s_i u_{i,j} \delta X_j d\Gamma dt. \quad (23)$$

Therefore, the gradient to update the shape of the body, $\text{grad}(J^*)_i$, can be derived by eq.(23) as follows:

$$\text{grad}(J^*)_i = -s_j u_{j,i}. \quad (24)$$

C. Volume constraint

The shape of a body is optimized keeping a volume of the initial shape constant at each iteration. The constant volume condition is expressed as follows:

$$\sum_{e=1}^m (a_e(X_i))^{(l)} - A_0 = 0 \quad \text{in } \Omega. \quad (25)$$

where X_i is the surface coordinates of a body, $a_e(X_i)$ is the volume of each element, m is the number of elements and A_0 is the volume of the initial domain.

IV. APPROXIMATION

A. Discretization

As for the discretization, the orthogonal basis bubble function interpolation for the velocity and the linear interpolation for pressure presented by Matsumoto [7] are applied and expressed as follows.

The orthogonal basis bubble function interpolation can be expressed as:

$$u_i = \Phi_1 u_{1i} + \Phi_2 u_{2i} + \Phi_3 u_{3i} + \Phi_4 u_{4i}, \quad (26)$$

$$\Phi_1 = \Psi_1 - \frac{1}{3} \phi_B, \quad \Phi_2 = \Psi_2 - \frac{1}{3} \phi_B,$$

$$\Phi_3 = \Psi_3 - \frac{1}{3} \phi_B, \quad \Phi_4 = \phi_B \quad (27)$$

$$\phi_B = \frac{\alpha_1 \hat{\phi}_B + \alpha_2 \tilde{\phi}_B + \bar{\phi}_B}{\alpha_1 + \alpha_2 + 1}, \quad (28)$$

where α_1 and α_2 are unknown variables, and $\hat{\phi}_B$, $\tilde{\phi}_B$ and $\bar{\phi}_B$

are the bubble functions, which are defined as follows:

$$\hat{\phi}_B = \phi_B^\xi, \quad \tilde{\phi}_B = \tilde{\phi}_B^\xi, \quad \bar{\phi}_B = \bar{\phi}_B^\xi, \quad (29)$$

where ϕ_B^ξ is the ξ th-power bubble function, which is shown in Figure 2, and expressed as follows:

$$\phi_B^\xi = \begin{cases} 3^\xi L_1^\xi & \text{in } w_1 \\ 3^\xi L_2^\xi & \text{in } w_2 \\ 3^\xi L_3^\xi & \text{in } w_3 \end{cases} \quad (30)$$

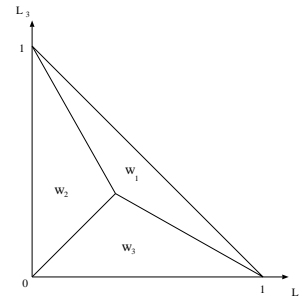


Fig. 2. The ξ th-power bubble function

The number of ξ can be taken arbitrary. Here Φ_α ($\alpha = 1, 2, 3, 4$) is the bubble function interpolation for velocity. The bubble function element is shown in Figure 3. The bubble function can be considered as C_0 continuous.

Eq.(26) can be transformed into the following equation.

$$u_i = \Psi_1 u_{1i} + \Psi_2 u_{2i} + \Psi_3 u_{3i} + \phi_B u'_{Bi}, \quad (31)$$

$$u'_{Bi} = u_{4i} - \frac{1}{3}(u_{1i} + u_{2i} + u_{3i}). \quad (32)$$

The linear interpolation can be expressed as:

$$p = \Psi_1 p_1 + \Psi_2 p_2 + \Psi_3 p_3, \quad (33)$$

$$\Psi_1 = L_1, \quad \Psi_2 = L_2, \quad \Psi_3 = L_3, \quad (34)$$

$$L_1 + L_2 + L_3 = 1 \quad (35)$$

where Ψ_λ ($\lambda = 1, 2, 3$) is the linear interpolation for pressure and L_1, L_2 and L_3 are the area coordinates. The linear element is shown in Figure 4. The criteria for the steady

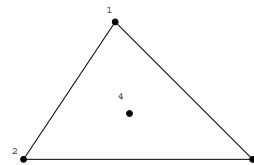


Fig. 3. Element for the interpolation of orthogonal basis bubble function

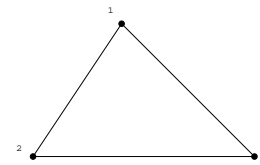


Fig. 4. Element for linear interpolation

problem is used, in which the discretized form of the bubble function element is equivalent to those of SUPG (Streamline-Upwind/Petrov-Galerkin) method. Therefore, in the bubble

function element for the steady problem, the stabilized parameter τ_{eB} which determines the magnitude of the streamline stabilized term. The stabilized parameter τ_{eB} is expressed as follows:

$$\tau_{eB} = \frac{\langle \phi_B, 1 \rangle_{\Omega_e}^2}{(\nu + \nu') \langle \phi_{B,j} u'_{Bi} + \frac{1}{3} \phi_{B,i} u'_{Bj}, \phi_{B,j} \rangle_{\Omega_e} A_e} \delta_{ij} u'_{Bi}, \quad (36)$$

where Ω_e is an element domain and

$$\langle u, v \rangle_{\Omega_e} = \int_{\Omega_e} uv d\Omega, \quad \|u\|_{\Omega_e}^2 = \int_{\Omega_e} uu d\Omega, \\ A_e = \int_{\Omega_e} d\Omega.$$

The integral of bubble function is expressed as follows:

$$\langle \phi_B, 1 \rangle_{\Omega_e} = \|\phi_B\|_{\Omega_e}^2 = \frac{3}{4} A_e, \quad (37)$$

$$\langle \phi_{B,i}, \phi_{B,j} \rangle_{\Omega_e} = \frac{27}{4} A_e \Psi_{\lambda,i} \Psi_{\lambda,j}. \quad (38)$$

From the criteria for the stabilized parameter in SUPG method, an parameter τ_{eR} can be given as follows:

$$\tau_{eR} = \left[\left(\frac{2|u_i|}{h_e} \right)^2 + \left(\frac{4\nu}{h_e^2} \right)^2 \right]^{-\frac{1}{2}}, \quad (39)$$

where h_e is an element size. Generally, the stabilized parameter eq.(36) is not equal to the optimal parameter eq.(39). In this study, the bubble function which gives an optimal viscosity is assumed to satisfy the following equation.

$$\tau_{eR} = \tau_{eB}. \quad (40)$$

From eq.(40), stabilized viscosity ν' can be determined using the following equation.

$$(\nu + \nu') \langle \phi_{B,j} u'_{Bi} + \frac{1}{3} \phi_{B,i} u'_{Bj}, \phi_{B,j} \rangle_{\Omega_e} \\ = \frac{\langle \phi_B, 1 \rangle_{\Omega_e}^2}{A_e} \tau_{eR}^{-1} \delta_{ij} u'_{Bi}. \quad (41)$$

Based on ν' , the stable computation can be carried out.

B. Finite element equation

The finite element equation of the state and adjoint equations are expressed as follows:

For the state equation;

$$M_{\alpha\beta} \dot{u}_{\beta i} + K_{\alpha\beta\gamma j} u_{\beta j} u_{\gamma i} - cH_{\lambda\alpha i} p_{\lambda} + S_{\alpha i\beta j} u_{\beta j} = F_{\alpha i}, \quad (42)$$

$$N_{\mu\lambda} \dot{p}_{\lambda} + B_{\mu\beta\lambda j} u_{\beta j} p_{\lambda} + cH_{\mu\beta i} u_{\beta i} = 0, \quad (43)$$

and for the adjoint equation;

$$-M_{\alpha\beta} \dot{u}_{\beta i}^* - K_{\alpha\beta\gamma j} u_{\beta j} u_{\gamma i}^* + K_{\alpha\gamma\beta i} u_{\beta j} u_{\gamma j}^* \\ - cH_{\lambda\alpha i} p_{\lambda}^* + S_{\alpha i\beta j} u_{\beta j}^* = 0, \quad (44)$$

$$-N_{\mu\lambda} \dot{p}_{\lambda}^* - B_{\mu\beta\lambda j} u_{\beta j} p_{\lambda}^* + cH_{\mu\beta i} u_{\beta i}^* = 0, \quad (45)$$

where

$$M_{\alpha\beta} = \int_{\Omega_e} \Phi_{\alpha} \Phi_{\beta} d\Omega, \quad K_{\alpha\beta\gamma j} = \int_{\Omega_e} \Phi_{\alpha} \Phi_{\beta} \Phi_{\gamma,j} d\Omega,$$

$$H_{\lambda\alpha i} = \int_{\Omega_e} \Psi_{\lambda} \Phi_{\alpha,i} d\Omega, \quad F_{\alpha i} = \int_{\Gamma} \Phi_{\alpha} t_i d\Gamma,$$

$$S_{\alpha i\beta j} = \lambda \int_{\Omega_e} \Phi_{\alpha,i} \Phi_{\beta,j} d\Omega + \nu \int_{\Omega_e} \Phi_{\alpha,j} \Phi_{\beta,i} d\Omega +$$

$$\nu \int_{\Omega_e} \Phi_{\alpha,k} \Phi_{\beta,k} \delta_{ij} d\Omega,$$

$$N_{\mu\lambda} = \int_{\Omega_e} \Psi_{\mu} \Psi_{\lambda} d\Omega, \quad B_{\mu\beta\lambda j} = \int_{\Omega_e} \Psi_{\mu} \Phi_{\beta} \Psi_{\lambda,j} d\Omega.$$

C. Finite element method in time

As the temporal discretization, the four-step explicit scheme is applied. The state equation is transformed as follows:

$$M_{\alpha\beta} u_{\beta i}^{n+\frac{1}{4}} = M_{\alpha\beta} u_{\beta i}^n - \frac{\Delta t}{4} (K_{\alpha\beta\gamma j} u_{\beta j}^n u_{\gamma i}^n \\ - cH_{\lambda\alpha i} p_{\lambda}^n + S_{\alpha i\beta j} u_{\beta j}^n - F_{\alpha i}^n), \\ \bar{N}_{\mu\lambda} p_{\lambda}^{n+\frac{1}{4}} = \bar{N}_{\mu\lambda} p_{\lambda}^n - \frac{\Delta t}{4} (B_{\mu\beta\lambda j} u_{\beta j}^n p_{\lambda}^n \\ + cH_{\mu\beta i} u_{\beta i}^n),$$

$$M_{\alpha\beta} u_{\beta i}^{n+\frac{2}{4}} = M_{\alpha\beta} u_{\beta i}^n - \frac{\Delta t}{3} (K_{\alpha\beta\gamma j} u_{\beta j}^{n+\frac{1}{4}} u_{\gamma i}^{n+\frac{1}{4}} \\ - cH_{\lambda\alpha i} p_{\lambda}^{n+\frac{1}{4}} + S_{\alpha i\beta j} u_{\beta j}^{n+\frac{1}{4}} - F_{\alpha i}^{n+\frac{1}{4}}), \\ \bar{N}_{\mu\lambda} p_{\lambda}^{n+\frac{2}{4}} = \bar{N}_{\mu\lambda} p_{\lambda}^n - \frac{\Delta t}{3} (B_{\mu\beta\lambda j} u_{\beta j}^{n+\frac{1}{4}} p_{\lambda}^{n+\frac{1}{4}} \\ + cH_{\mu\beta i} u_{\beta i}^{n+\frac{1}{4}}),$$

$$M_{\alpha\beta} u_{\beta i}^{n+\frac{3}{4}} = M_{\alpha\beta} u_{\beta i}^n - \frac{\Delta t}{2} (K_{\alpha\beta\gamma j} u_{\beta j}^{n+\frac{2}{4}} u_{\gamma i}^{n+\frac{2}{4}} \\ - cH_{\lambda\alpha i} p_{\lambda}^{n+\frac{2}{4}} + S_{\alpha i\beta j} u_{\beta j}^{n+\frac{2}{4}} - F_{\alpha i}^{n+\frac{2}{4}}), \\ \bar{N}_{\mu\lambda} p_{\lambda}^{n+\frac{3}{4}} = \bar{N}_{\mu\lambda} p_{\lambda}^n - \frac{\Delta t}{2} (B_{\mu\beta\lambda j} u_{\beta j}^{n+\frac{2}{4}} p_{\lambda}^{n+\frac{2}{4}} \\ + cH_{\mu\beta i} u_{\beta i}^{n+\frac{2}{4}}),$$

$$M_{\alpha\beta} u_{\beta i}^{n+1} = M_{\alpha\beta} u_{\beta i}^n - \Delta t (K_{\alpha\beta\gamma j} u_{\beta j}^{n+\frac{3}{4}} u_{\gamma i}^{n+\frac{3}{4}} \\ - cH_{\lambda\alpha i} p_{\lambda}^{n+\frac{3}{4}} + S_{\alpha i\beta j} u_{\beta j}^{n+\frac{3}{4}} - F_{\alpha i}^{n+\frac{3}{4}}), \\ \bar{N}_{\mu\lambda} p_{\lambda}^{n+1} = \bar{N}_{\mu\lambda} p_{\lambda}^n - \Delta t (B_{\mu\beta\lambda j} u_{\beta j}^{n+\frac{3}{4}} p_{\lambda}^{n+\frac{3}{4}} \\ + cH_{\mu\beta i} u_{\beta i}^{n+\frac{3}{4}}),$$

where $\bar{N}_{\mu\lambda}$ is the lumped mass matrix of $N_{\mu\lambda}$, and $\tilde{N}_{\mu\lambda}$ is expressed as following equation.

$$\tilde{N}_{\mu\lambda} = e\bar{N}_{\mu\lambda} + (1 - e)N_{\mu\lambda}, \quad (47)$$

where e is the lumping parameter. The fluid forces can be evaluated as:

$$\hat{\Gamma}_{\alpha i} = \int_{\Gamma_B} \Phi_{\alpha} t_i d\Gamma. \quad (48)$$

V. MINIMIZATION

A. The weighted gradient method

The weighted gradient method is applied for minimizing the performance function. In this method a modified performance function, which can be obtained by adding a penalty term to the performance function, should be introduced. The modified performance function K is expressed as follows:

$$K^{(l)} = J^{*(l)} + \frac{1}{2} \int_{\Gamma_B} \left(X_{\alpha}^{(l+1)} - X_{\alpha}^{(l)} \right) W_{\alpha\beta}^{(l)} \left(X_{\beta}^{(l+1)} - X_{\beta}^{(l)} \right) d\Gamma, \quad (49)$$

where l is the iteration number for the minimization, $W_{\alpha\beta}^{(l)}$ is the weighting diagonal matrix, which will be updated during iteration. Applying the stationary condition $\delta K^{(l)} = 0$, the updated surface coordinates of the body are calculated at each iteration by the following equation:

$$W_{\alpha\beta}^{(l)} X_{\beta}^{(l+1)} = W_{\alpha\beta}^{(l)} X_{\beta}^{(l)} - \text{grad}(J^{*(l)})_{\alpha}. \quad (50)$$

B. Algorithm

The following algorithm is employed for the computation.

- 1) Select initial surface coordinates $X_{\alpha}^{(0)}$ on Γ_B .
- 2) Solve $u_i^{(0)}, p^{(0)}$ by the state equations (42) and (43) from start time to final time.
- 3) Solve $u_i^{*(0)}, p^{*(0)}$ by the adjoint equations (44) and (45) from final time to start time.
- 4) Compute $X_{\alpha}^{(1)}$ by eq.(50).
- 5) Move the surface coordinate keeping the volume constant as eq.(25).
- 6) Remeshing.
- 7) Solve $u_i^{(1)}, p^{(1)}$ by the state equations (42) and (43).
- 8) IF $|X_{\alpha}^{(l+1)} - X_{\alpha}^{(l)}| < \epsilon$ THEN stop
ELSE solve $u_i^{*(l)}, p^{*(l)}$ by the adjoint equation (44) and (45), $l = l + 1$ and go to 4.

VI. NUMERICAL STUDY

The minimization problem of drag and lift forces subjected to a body located in a viscous flow is studied. The weighting parameters q_1 and q_2 in eq.(10) are both 1.0. The computational domain and boundary conditions are shown in Figure 5. The finite element mesh is represented in Figure 6. The total number of nodes and element are 2474 and 4800, respectively. The Reynolds numbers are set to 1.0 and 40.0 in case 1 and 2, respectively.

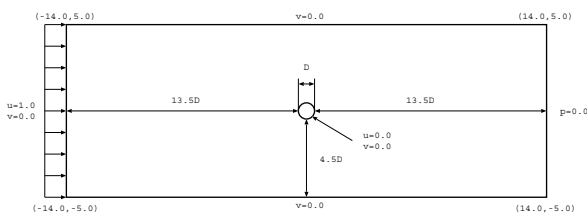


Fig. 5. Computational domain and boundary condition

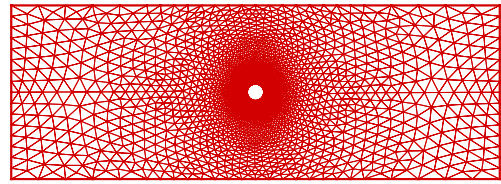


Fig. 6. Finite element mesh

A. Computational result

1) Case 1 ($Re=1.0$): The variation of the performance function is plotted in Figure 7. Figures 8 and 9 show the initial and final shapes in case 1.

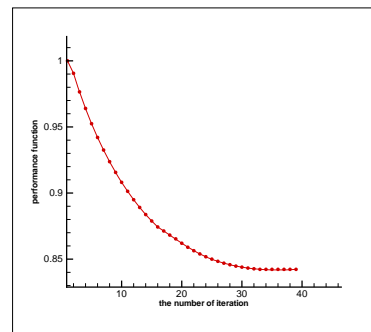


Fig. 7. performance function ($Re=1.0$)

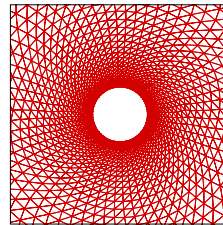


Fig. 8. initial shape ($Re=1.0$)

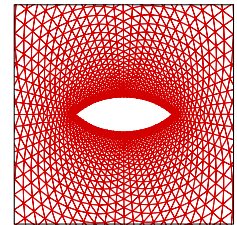


Fig. 9. final shape ($Re=1.0$)

2) Case 2 ($Re=40.0$): The variation of the performance function is plotted in Figure 10. Figures 11 and 12 show the initial and final shapes in case 2.

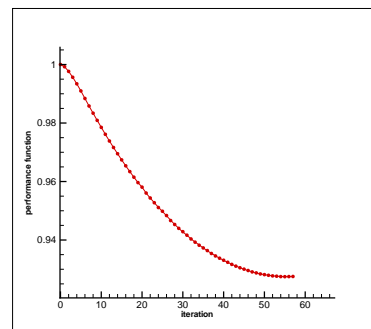


Fig. 10. performance function ($Re=40.0$)

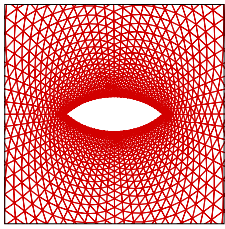


Fig. 11. initial shape (Re=40.0)

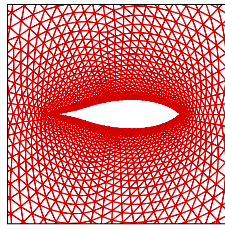


Fig. 12. final shape (Re=40.0)

VII. CONCLUSION

In this study, a shape optimization method in viscous flow using the acoustic velocity is presented. The optimal shapes of the body in the viscous flow has been obtained by the finite element method using the acoustic velocity and orthogonal basis bubble function.

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