

# An empirical validation of the linear- hyperbolic approximation of the I-V characteristic of a solar cell generator

A. A. Penin

**Abstract**—An empirical linearly-hyperbolic approximation of the  $I - V$  characteristic of a solar cell is presented. This approximation is based on hyperbolic dependence of a current of  $p - n$  junctions on voltage for large currents. Such empirical approximation is compared with the early proposed formal linearly-hyperbolic approximation of a solar cell. The expressions defining laws of change of parameters of formal approximation at change of a photo current of family of characteristics are received. It allows simplifying a finding of parameters of approximation on actual curves, to specify their values.

Analytical calculation of load regime for linearly - hyperbolic model leads to quadratic equation. Also, this model allows to define soundly a deviation from the maximum power regime and to compare efficiency of regimes of solar cells with different parameters.

**Keywords**—a solar cell generator,  $I - V$  characteristic,  $p - n$  junction, approximation .

## I. INTRODUCTION

**F**OR the analysis and calculation of power supply systems based on solar cells generators mathematical model of the cell is needed. The traditional model of  $I - V$  characteristic or of  $p - n$  junction [1], requires the iterative numerical calculation methods. Therefore, the use of the simpler models which make it possible to carry out the direct or analytical calculations is useful.

Model in the form of the fractional-quadratic expression leads to the cubic equation of the circuit with the resistance load [2]. It complicates calculation, especially in the real time regime.

More convenient model represents the linear -hyperbolic expression and is based on the similarity with the  $I - V$  characteristic of active two-terminal network with self-limitation of the current [3]. The calculation with the resistive load leads to a quadratic equation. Also, this model makes it possible to soundly determine deviation from the maximum power regime. These obtained relative expressions make it possible to compare the effectiveness of the regime of solar cells with the different parameters [4]. The model parameters are being calculated for the concrete curve (with the appropriate level of insulation) of the set of characteristics from two points near the point of maximum power. But values of the calculated parameters depend on coordinates of these points, which complicate the finding of dependence of parameters on photo-current level. Therefore, there is a problem of a physical

validation of linearly - hyperbolic approximation and obtaining of dependence of the model parameters from photo-current level.

## II. FEATURES OF KNOWN MODELS OF $I - V$ CHARACTERISTIC OF A $p - n$ JUNCTION DIODE.

The known analysis of the idealized diode leads to the exponential expression of  $I - V$  characteristic widely utilized in the theory of semiconductor devices [5]:

$$i = I_0 \left[ \exp \left( \frac{u}{\varphi_T} \right) - 1 \right] \quad (1)$$

where:  $I_0$  is the saturation current or scale current of the diode,  $\varphi_T$  is the thermal voltage,  $u$  is the applied or external voltage. The expression of reverse characteristic takes the form:

$$u = \varphi_T \ln \left( \frac{i}{I_0} + 1 \right) \quad (2)$$

One of the assumptions of idealization is the fact that an external voltage is applied directly to the junction. This condition is being satisfied for the sufficiently low currents and the low resistance of base. We obtain from expression (2) the resistance of diode or of junction to the direct current:

$$R = \frac{u}{i} = \frac{\varphi_T}{i} \ln \left( \frac{i}{I_0} + 1 \right)$$

At the zero point this resistance is equal to:

$$R(0) = \frac{\varphi_T}{I_0} \quad (3)$$

The application of diodes in the power electronics is connected with the large forward currents. Therefore, it is necessary to take into account a voltage drop across the base resistance  $r_B$ . Then, expressions (1), (2) take the form:

$$i = I_0 \left[ \exp \left( \frac{u - ir_B}{\varphi_T} \right) - 1 \right] \quad (4)$$

$$u = \varphi_T \ln \left( \frac{i}{I_0} + 1 \right) + ir_B \quad (5)$$

While the current is increased, the linear component is appeared all in greater degree and exponential characteristic is degenerated, which is demonstrated on Fig. 1. Therefore, this characteristic can be named linear -exponential. Also, for greater current and growing injection level the modulation of the base resistance reveals itself. To take into account this

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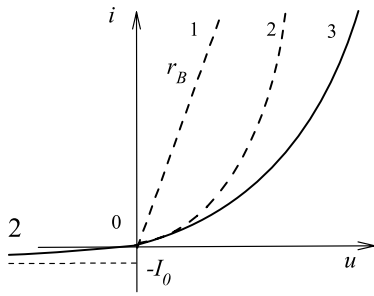


Fig. 1. The influence of the base resistance on forward characteristic of the diode: 1 -linear component, 2 - exponential component, 3 -resulting characteristic

effect, parameter  $m = 1.5 - 2$  is introduced into expressions (4), (5):

$$i = I_0 \left[ \exp \left( \frac{u - ir_B}{m\varphi_T} \right) - 1 \right] \quad (6)$$

$$u = m\varphi_T \ln \left( \frac{i}{I_0} + 1 \right) + ir_B \quad (7)$$

But more strict analysis shows that for the greater currents and high levels of injection, the elementary exponential dependence of current on voltage is not valid essentially already, which is a consequence of Maxwell - Boltzmann distribution. Such qualitative changes appear when the voltage barrier is decreased approximately to  $\varphi_T$ . In this case, the  $I - V$  characteristic ceases to be exponential and gradually approaches to power-mode dependency. The quadratic dependence  $i = au^2$  is taken with some assumptions where the factor  $a$  depends on parameters of the base material and is inversely proportional to its thickness in degree  $2/3$  [5].

Let us consider the variants of the approximations of the forward characteristics of the row of industrial power diodes [6], [7] according to expression (7) on Fig. 2, 3, 4, 5.

It is possible to notice that comprehensible accuracy of approximation obtained with parameters which are considerably different from the recommended and actual values. In particular, for diode *KD243* the best variants of the approximation correspond to curves 1 and 4. The curve 3 describes the average area of the current better, and the curve 2 describes the area of small current. For diode *KD247* the curve 1 describes the small and greater current better, and the curve 2 describes the small and average current. Moreover, for these curves the base resistance is not taken into account. The characteristics of the diode *HFA70NH60* are well described by exponential approximation, but for different ranges of current the parameters of approximations are differ. Certainly, this is explained by imprecision of the simple theoretical expression (7). So, for practical calculation such an expression carries the empirical nature in greater degree when it is necessary to define (or to adjust) parameters of the approximations on actual curve. This implies that various convenient models and approximations of

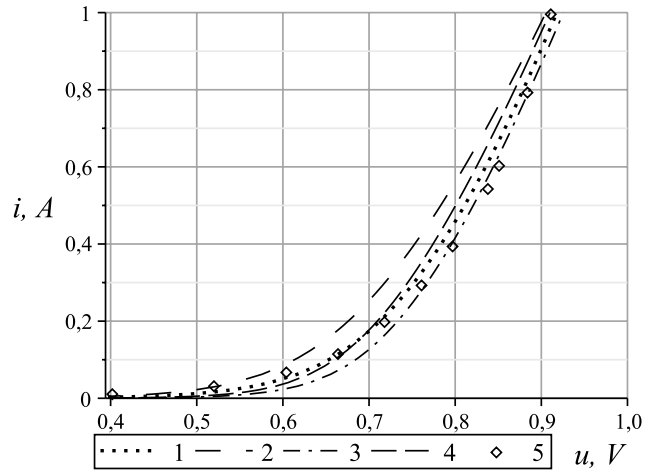


Fig. 2. The variants of exponential approximations of the forward  $I - V$  characteristic of the *KD243* diode : 1-  $m\varphi_T = 0.068V, r_B = 0.12\Omega, I_0 = 8\mu A$ , 2-  $m\varphi_T = 0.068V, r_B = 0.15\Omega, I_0 = 15\mu A$ , 3-  $m\varphi_T = 0.05V, r_B = 0.14\Omega, I_0 = 0.15\mu A$ , 4-  $m\varphi_T = 0.05V, r_B = 0.15\Omega, I_0 = 0.25\mu A$ , 5- actual date

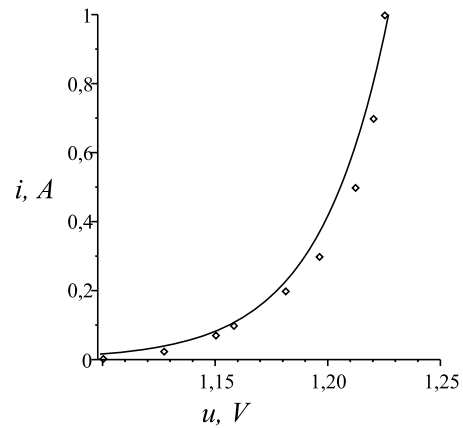


Fig. 3. The variants of exponential approximations of the forward  $I - V$  characteristic of the *KD247* diode for  $r_B = 0$  : 1-  $m\varphi_T = 0.0308V, I_0 = 0.5 \cdot 10^{-17} A$ , 2-  $m\varphi_T = 0.032V, I_0 = 0.2 \cdot 10^{-16} A$ , 3-actual values

characteristics of the diode are quite admissible even from the physical point of view.

### III. CONNECTION OF PARAMETERS OF LINEARLY-HYPERBOLIC APPROXIMATION WITH THE PARAMETERS OF EXPONENTIAL MODEL $I - V$ CHARACTERISTIC OF DIODE

According to the stated representations, for exponential model of the idealized diode, at approach of forward bias voltage to the built-in potential  $V_{bi}$  the forward current sharply grows, and the saturation current, in turn, does not depend on reverse voltage. Such behaviour of currents allows applying the hyperbolic function presented on Fig. 6. In this case, asymptotes are the voltage  $V_{bi}$  and a current  $-aI_0$ , where  $a$  is the scale factor. Such hyperbolic function is described by

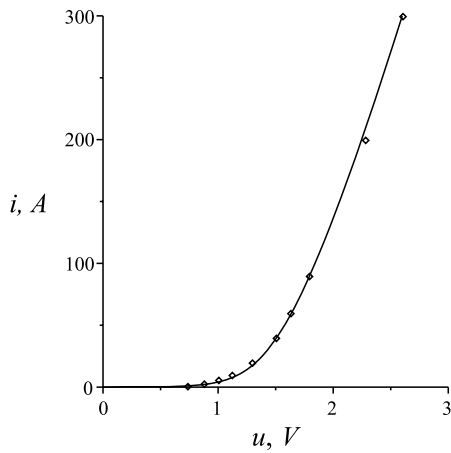


Fig. 4. The exponential approximations of the forward  $I - V$  characteristic of the  $HFA70NH60$  diode for range  $300A$  : line-  $m\varphi_T = 0.18V, r_B = 0.0028\Omega, I_0 = 0.017A$ , boxes-actual values

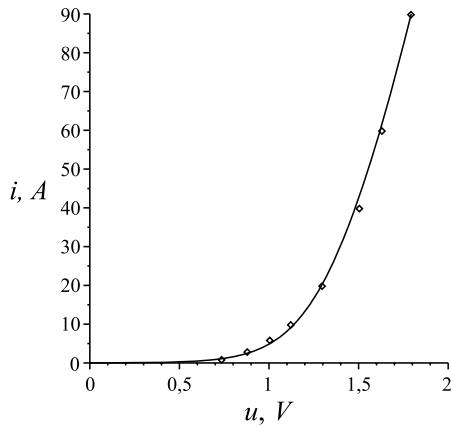


Fig. 5. The exponential approximations of the forward  $I - V$  characteristic of the  $HFA70NH60$  diode for range  $90A$  : line-  $m\varphi_T = 0.17V, r_B = 0.0035\Omega, I_0 = 0.017A$ , boxes - actual values

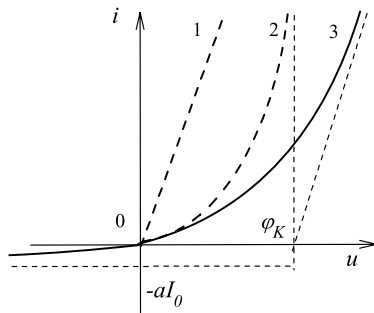


Fig. 6. The proposed approximation of the forward  $I - V$  characteristic of diode : 1 - line component, 2 - hyper component, 3 - resulting line- hyper characteristic

the following expression:

$$i = aI_0 \frac{u}{V_{bi} - u} \tag{8}$$

This implies that for the voltage  $u \rightarrow V_{bi}$ , the current  $i \rightarrow \infty$  and for the voltage  $u \rightarrow -\infty$ , the current  $i \rightarrow -aI_0$ . Reverse expression looks like:

$$u = V_{bi} \frac{i}{aI_0 + i} \tag{9}$$

Then the non linear component of resistance to a direct current takes the form:

$$R(i) = \frac{u}{i} = \frac{V_{bi}}{aI_0 + i}$$

For zero values of a current and voltage the value of resistance is equal:

$$R(0) = \frac{V_{bi}}{aI_0} \tag{10}$$

Equating expression (10) with similar expression (3) we receive the parameter  $a$ :

$$a = \frac{V_{bi}}{\varphi_T}$$

Definitively, we receive expressions of the hyperbolic characteristic correct for the forward branch:

$$i = aI_0 \frac{V_{bi} - u}{\varphi_T V_{bi} - u} \tag{11}$$

$$u = \frac{V_{bi}}{\varphi_T} \frac{i}{V_{bi}I_0 + i\varphi_T} \tag{12}$$

The received expressions also represent the idealized model of the diode. It is possible to accept that for  $u \rightarrow V_{bi}$  the forward current is limited by a linear part of resistance of junction from base  $r_{B1}$  and by resistance of the base neutral region  $r_{B2}$ . Therefore, expression (12) is naturally supplemented by a linear component as well as expression (7):

$$u = \frac{V_{bi}}{\varphi_T} \frac{i}{V_{bi}I_0 + i\varphi_T} + ir_B, \tag{13}$$

where  $r_B = r_{B1} + r_{B2}$ . The influence of resistance of base is shown on Fig. 6. Thus, empirical expression of the characteristic is received. This implies the equivalent scheme of the diode presented on Fig. 7. The diode is replaced with consecutive connection of nonlinear and linear resistance (or of conductance). From expression (13) we receive the direct

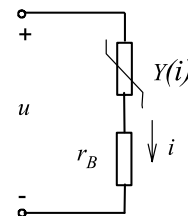


Fig. 7. The equivalent scheme of the diode in linearly-hyperbolic approximation of the I-V characteristic

expression of characteristic:

$$i^2 r_B + i(V_{bi} - u - r_B I_0 V_{bi} / \varphi_T) - u I_0 V_{bi} / \varphi_T = 0$$

The current is calculated as the solution of quadratic equation for set voltage. It essentially differs from the transcendental exponential expressions (6). Approximations of same diodes corresponding to expression (13) are presented on Fig. 8, 9,10, 11 .

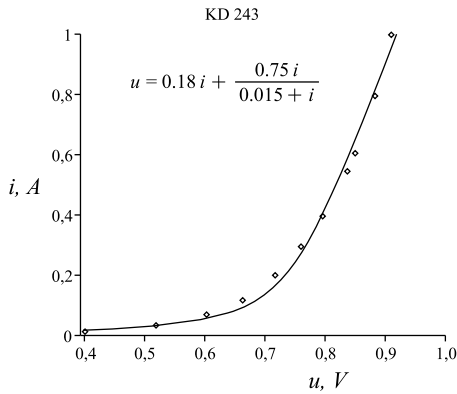


Fig. 8. The linearly-hyperbolic approximations of the forward  $I - V$  characteristic of the  $KD243$  diode

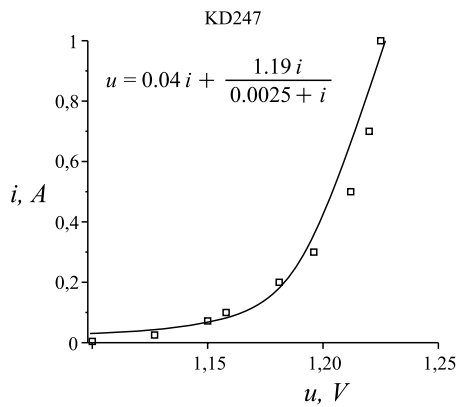


Fig. 9. The linearly-hyperbolic approximations of the forward  $I - V$  characteristic of the  $KD247$  diode

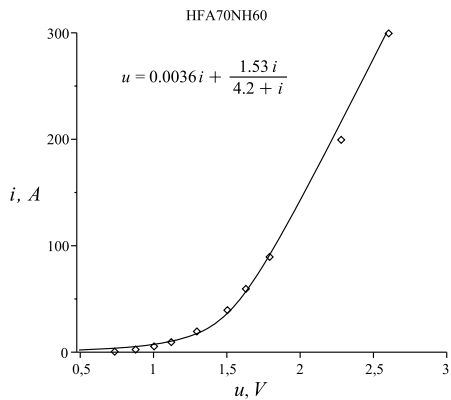


Fig. 10. The linearly-hyperbolic approximations of the forward  $I - V$  characteristic of the  $HFA70NH60$  diode for range 300A

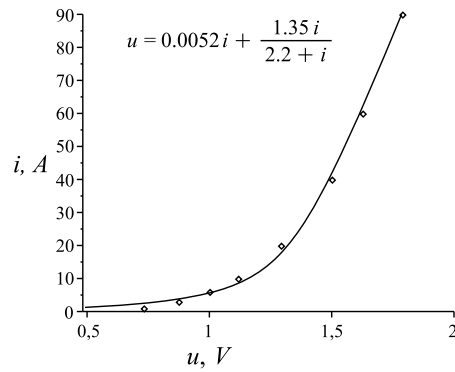


Fig. 11. The linearly-hyperbolic approximations of the forward  $I - V$  characteristic of the  $HFA70NH60$  diode for range 90A

It is obviously that the offered linearly-hyperbolic approximation quite satisfies to practical application.

#### IV. THE CONNECTION OF PARAMETERS OF EMPIRICAL AND FORMAL LINEARLY - HYPERBOLIC APPROXIMATION OF THE SOLAR CELL CHARACTERISTIC

Let us consider the equivalent scheme of a solar cell on Fig. 12. We give the expression of its characteristic:

$$u = \varphi_T \ln \left( \frac{I_{ph} - i}{I_0} + 1 \right) - ir_S \quad (14)$$

where  $r_S$  is a resistance of losses,  $I_{ph}$  is a photo-current.

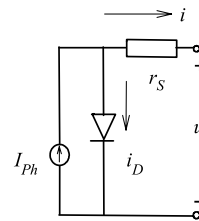


Fig. 12. The equivalent scheme of the solar cell

The light characteristic is presented on Fig. 13. Current  $I_{ph} = 0$  for the dark characteristic. Therefore, the dark characteristic represents simply the diode characteristic. This implies that the working region of a solar cell characteristic corresponds to the forward characteristic of the diode displaced on an axis of currents on value  $I_{ph}$ . Therefore, it is possible to use the offered linearly-hyperbolic approximation of a forward characteristic of the diode for approximation of a working region of a solar cell characteristic.

On this basis, it is possible to present the equivalent linearly-hyperbolic scheme of a solar cell on Fig. 14. Taking into account the specified directions of currents and polarity of voltage of the scheme elements, we receive the following relations:

$$i_D = I_{ph} - i, u = u_D + u_{B1} - u_{B2}.$$

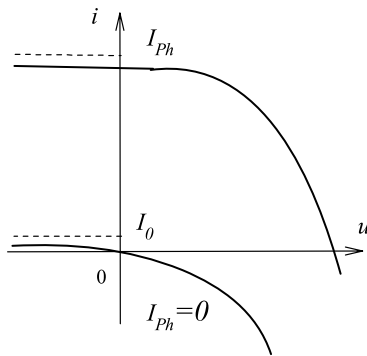


Fig. 13. The light characteristic of a solar cell

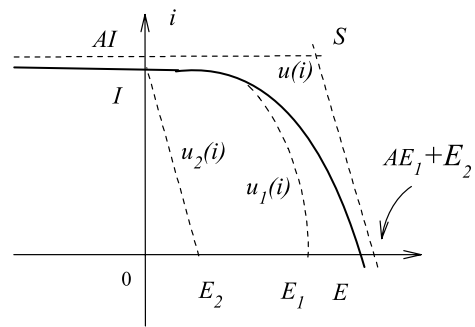


Fig. 15. The formal approximation of a solar cell characteristic

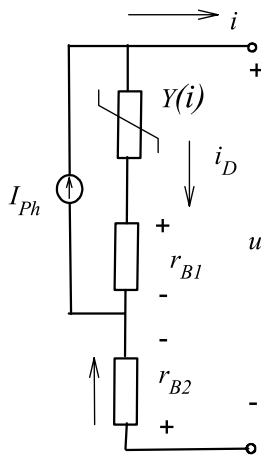


Fig. 14. The equivalent linearly-hyperbolic scheme of a solar cell

Using expression of the characteristic of the diode (13), we find the load voltage:

$$u = \frac{V_{bi} I_{ph}}{I_{ph} + I_0 V_{bi} / \varphi_T} \cdot \frac{1 - i / I_{ph}}{1 - i / [(1 + I_0 V_{bi} / I_{ph} \varphi_T) I_{ph}]} + I_{ph} r_{B1} [1 - i(r_{B1} + r_{B2}) / I_{ph} r_{B1}] \quad (15)$$

Let us present the obtained early expression (expression (4) in [3]) of formal approximation of a solar cell characteristic:

$$u(i) = u_1(i) + u_2(i) = E_1 \frac{1 - i/I}{1 - i/AI} + E_2(1 - i/I)$$

The plot of this expression is presented on Fig. 15 where a hyperbolic  $u_1(i)$ , linear  $u_2(i)$  component of voltage and short circuit current  $I$  are selected.

The parameter  $A$  sets the degree of curvature of a hyperbole. If we compare this expression with the offered empirical

expression (15), it is possible to determine the following:

$$E_2 = I_{ph} r_{B1}, I = I_{ph} r_{B1} / (r_{B1} + r_{B2}) \approx I_{ph},$$

$$E_1 = \frac{V_{bi} I_{ph}}{I_{ph} + I_0 V_{bi} / \varphi_T}, \quad (16)$$

$$A = 1 + I_0 V_{bi} / I_{ph} \varphi_T \quad (17)$$

From expression (16) follows that the voltage  $E_1 \rightarrow V_{bi}$  while the current  $I_{ph}$  is increasing. And voltage  $E_1$  is proportional to a current  $I_{ph}$  for small values of this current, which corresponds to physical sense. From expression (17) it also follows that parameter  $A$  does not depend significantly on current  $I_{ph}$  for enough great values of this current and parameter  $A$  is decreased while current  $I_{ph}$  is increased. From expressions (16), (17) it turns out that  $E_1 A = V_{bi}$ . This implies that at processing of actual curves and calculation of parameters of formal approximation the relation  $E_1 A \approx const$  should be carried out. It is confirmed also by the date [4]. In a practical example of solar cell *MSX120* [8], the constant value of parameter  $A = 1.004$  for different insulation levels is quite soundly accepted. As it has already been specified, the results of parameter calculations of formal approximation strongly depend on a choice of points near to the maximum power point. Therefore, relations (16), (17) allow reducing this uncertainty.

## V. CONCLUSION

1. Results of exponential approximation of actual characteristics of some power diodes show that values of physical parameters (as approximation parameters) considerably differ from recommended theoretical values. Therefore, it is possible to consider such models qualitatively approached. This implies that other more exact and practical models are possible also.

2. The offered empirical linearly-hyperbolic approximation is based on corresponding interpretation of influence of a potential barrier on a current of junction. Results of approximation of actual characteristics of some power diodes show a good coordination of actual and predicted values.

3. We received the expressions defining the rules of change of parameters of formal approximation when parameters of

family of curves of a solar cell characteristic are changed. It allows simplifying a method of finding of parameters of approximation on actual curves, to specify their values.

4. The suggested approach can be applied to the description of  $I - V$  characteristics of bipolar transistors.

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