

# Extended Minimal Controller Synthesis for Voltage-Fed Induction Motor Based on the Hyperstability Theory

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**Abstract**—in this work, we present a new strategy of direct adaptive control denoted: Extended minimal controller synthesis (EMCS). This algorithm is designed for an induction motor, which includes both electrical and mechanical dynamics under the assumptions of linear magnetic circuits. The main motivation of the EMCS control is to enhance the robustness of the MRAC algorithms, i.e. the rejection of bounded effects of rapidly varying external disturbances.

**Keywords**—Adaptive Control, Simple model reference adaptive control (SMRAC), Extended Minimal Controller synthesis (EMCS), Induction Motor (IM)

## I. INTRODUCTION

HIGH performance induction motor drives require controllers that yield fast torque response through base frequency and constant power with adequate torque response above base frequency. A vector control is the most successful in meeting these requirements. To date, however, considerable effort is expended in engineering and commissioning the drive, resulting in a complex drive system. Consequently, a vector control drive is complicated and costly and it should be implemented only when the application demands high performance.

Most adaptive approaches are based on self-tuning concepts where in parameters, such as rotor resistance, are identified and used in the field oriented controller, more over the parameter adaptive algorithms usually employ a functional relationship that require priory knowledge of machine inductances and assume a linear machine model.

The main result of this paper is to develop an adaptive version of the controller presented in [2] and [10]. In section II a state space model of an induction motor, which includes both electrical and mechanical dynamics, is given. The section III of the paper is dedicated to a theoretical presentation of the decoupling control. In section IV, the algorithm of a simplified MRAC, is given. It can be mentioned that the papers related to MRAC techniques become more and more numerous (Ohnishi, 1986; Chan, 1990; Fu, 1991). In section V the derivation of the EMCS algorithm is presented. It will

be shown that by adding suitably designed elements to the MRAC algorithms, the resulting EMCS algorithms reject the effects of external disturbances. For the derivation of the EMCS algorithms, the stability proofs will be carried out using the hyperstability theory.

## II. MATHEMATICAL MODEL OF INDUCTION MACHINE

The reader is referred to [9] and [11] for the general theory of electric machines and induction motors, to [12] for related control problems, and to [8] for digital implementation. It is well known that the mathematical model of an induction motor can be obtained using the two-axis theory. By choosing the synchronous reference frame  $(\vec{0}_s, \vec{0}_q)$ , which the supply voltage phases, the dynamics of a squirrel-cage induction motor can be represented by the following non linear differential equations:

$$\dot{X} = A.X + B.U \quad (1)$$

With:

$$A = \begin{bmatrix} -\left(\frac{1}{T_s \sigma} + \frac{1}{T_r} \frac{1-\sigma}{\sigma}\right) & \omega_s & \frac{1-\sigma}{\sigma} \frac{1}{M_{sr} T_r} & \frac{1-\sigma}{\sigma} \frac{1}{M_{sr}} \omega \\ -\omega_s & -\left(\frac{1}{T_s \sigma} + \frac{1}{T_r} \frac{1-\sigma}{\sigma}\right) & -\frac{1-\sigma}{\sigma} \frac{1}{M_{sr}} \omega & \frac{1-\sigma}{\sigma} \frac{1}{M_{sr} T_r} \\ \frac{M_{sr}}{T_r} & 0 & -\frac{1}{T_r} & \omega_{gl} \\ 0 & \frac{M_{sr}}{T_r} & -\omega_{gl} & -\frac{1}{T_r} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{I}_{sd} \\ \dot{I}_{sq} \\ \dot{\phi}_{rd} \\ \dot{\phi}_{rq} \end{bmatrix}, \quad X = \begin{bmatrix} I_{sd} \\ I_{sq} \\ \phi_{rd} \\ \phi_{rq} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad U = \begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix}$$

$$\text{With: } T_r = \frac{L_r}{R_r} \quad \text{and} \quad T_s = \frac{L_s}{R_s}$$

And  $\sigma$ : total leakage coefficient.

The electromechanical torque is given by:

$$T_e = p \frac{M_{sr}}{L_r} \cdot [\vec{\phi}_r \wedge \vec{I}_s] \quad (2)$$

The mechanical equation is given by:

$$J \cdot \frac{d\Omega}{dt} = T_e - T_r - f_r \cdot \Omega \quad (3)$$

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In Figure 3 (a) and 3 (b) flux amplitude  $\Phi_{rd}$  and  $\Phi_{rq}$  are respectively given.

For the phase current (Figure 3 (c)), the first peak is respectively at 22 a while the steady state peak current is at 7A.

The electromechanical torque is represented in figure 3 (d).

The system response and the model reference response at no load ( $T_r = 0$ ) are shown in figure 3 (e).

The speed response is well damped within a rise time of 0.2s and it can be observed a good concordance between system response and model reference response.

The feedforward gain and the feedback gain are shown in figure 3 (f). The error between the system response and the model reference response is reported in Figure 3 (g).

The system response, the model reference response, the adaptive gains and the error when load ( $T_r = 5 N.m$ ) are shown respectively in figure 4 (a), 4 (b) and 4 (c).

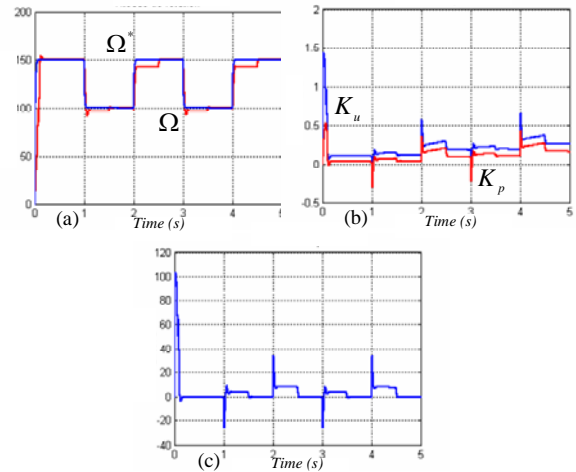


Fig. 4 Simulation results when load ( $T_r = 5 N.m$ )

V. EXTENDED MINIMAL CONTROLLER SYNTHESIS

For the derivation, see appendix A.

The EMCS control input is:

$$U = T_e^* = K_u \cdot \Omega^* + K_p \cdot \Omega + N \cdot \frac{e}{|e| + \xi} \tag{9}$$

The EMCS has been implemented using the configuration figure 5.

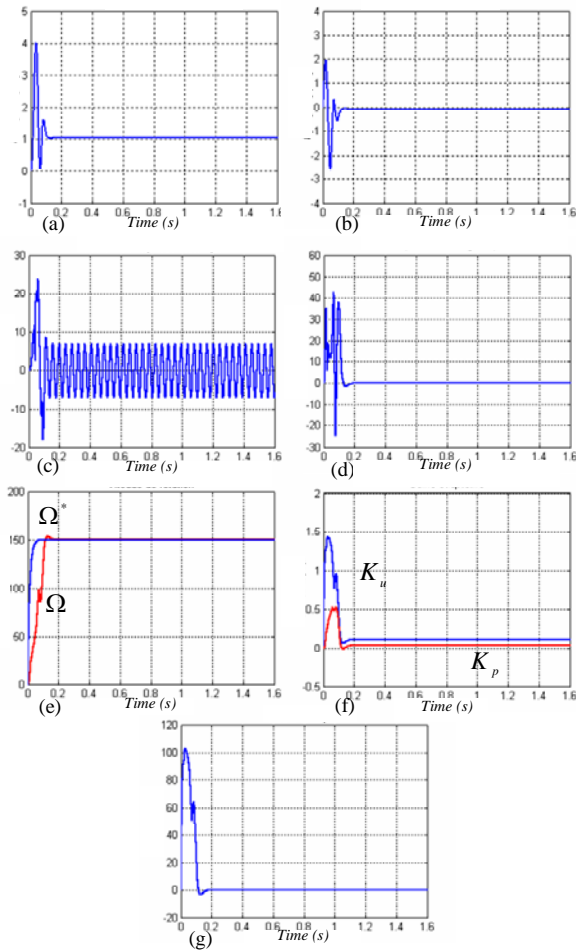


Fig. 3 Simulation results at no load ( $T_r = 0$ )

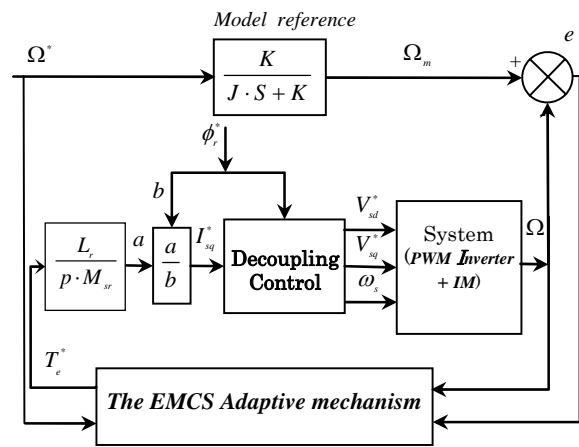


Fig. 5 Configuration of the control system

A final test of the complete drive when using EMCS control law has been performed starting-up towards 1500 rpm at ( $T_r = 5 N.m$ ) Figure 6. The response system (Speed) and the response of model reference are plotted in (rd/s) against time in Figure 6 (a). The adaptive gains (feedforward and feedback gains) and the error between response system (speed) and response of model reference are shown in figure 6 (b) and figure 6 (c).

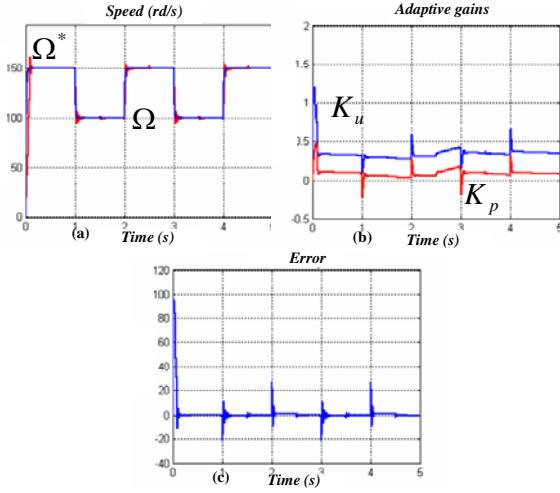


Fig. 6 The EMCS simulation results nonlinear

A test of robustness has been also performed by tuning the rotor resistance parameter with both over-estimation and under-estimation.

To compare the tracking performances of EMCS algorithm with those of MRAC algorithms in the presence of external disturbances, the tracking performance is improved in the case of EMCS algorithm. As we can see, when the EMCS algorithm is used, the effect of rapidly varying external disturbances is rejected and the error is asymptotically stable.

## VI. CONCLUSION

In this paper we propose, an adaptive decoupling control which has some advantages over classical algorithm control of MRAC. First a MRAC scheme for vector-controlled induction machine has been proposed starting from the very beginning of MRAC theory.

The use of the EMCS algorithm yields complete rejection of external disturbances. Therefore, the robustness properties are enhanced in comparison with those of the MRAC algorithms.

The improvement in drive performances can be evaluated in term of robustness in front of machine parameter changes without having to implement identification procedure.

## APPENDIX A

Consider the plant described by [1] and [3]:

$$\dot{X}(t) = A(t) \cdot X(t) + B(t) \cdot U(t) + d(X, t) \quad (\text{A.1})$$

$$A(t) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -a_1 & -a_2 & \dots & \dots & -a_n \end{bmatrix}, \quad (\text{A.2})$$

$$B(t) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_1 \end{bmatrix}, \quad d(X, t) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ d_1 \end{bmatrix}$$

Where:  $A(t) \in \mathfrak{R}^{n \times n}$ ,  $B(t) \in \mathfrak{R}^{n \times 1}$ ,  $d(X, t) \in \mathfrak{R}^{n \times 1}$ .

The elements of  $A(t)$  and  $B(t)$  are unknown and assumed to be slowly varying, i.e. constant during the adaptation process. The term  $d(X, t)$  represents the effects of rapidly varying external disturbances.

Similarly, the stable reference model is defined as:

$$\dot{X}_m(t) = A_m X_m(t) + B_m U_m(t) \quad (\text{A.3})$$

$$A_m = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -a_{m1} & -a_{m2} & \dots & \dots & -a_{mn} \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ b_m \end{bmatrix} \quad (\text{A.4})$$

Where:  $A_m \in \mathfrak{R}^{n \times n}$  and  $B_m \in \mathfrak{R}^{n \times 1}$ .

According to [1], The EMCS control law is synthesised by adding a suitably designed extra term to the existing MRAC algorithms. Therefore, we define an EMCS control law as follow:

$$U = \left[ \theta^T + \frac{1}{b_1} \gamma^T \right] W + N \frac{y_n}{|y_n| + \xi} \quad (\text{A.5})$$

Where  $\xi$  is a small positive constant. Note that if:  $\xi \rightarrow 0$ , then:  $\frac{|y_n| + \xi}{|y_n|} \rightarrow \text{sign}(y_n)$ , with:

$$\text{sign}(y_n) = \begin{cases} 1 & \text{if } y_n > 0 \\ 0 & \text{if } y_n = 0 \\ -1 & \text{if } y_n < 0 \end{cases} \quad (\text{A.6})$$

$$\theta^T = \left[ \int_0^t \alpha y_n X^T d\tau \quad \int_0^t \alpha y_n U_m d\tau \right] \quad (\text{A.7})$$

$$\gamma = b_1 \beta y_n W = b_1 \beta b^T P X_e W \quad (\text{A.8})$$

$$W = \begin{bmatrix} X^T \\ U_m \end{bmatrix} \quad (\text{A.9})$$

The error vector  $X_e$  is defined as:

$$X_e = X_m - X \quad (\text{A.10})$$

Then the error dynamics are given by:

$$\dot{X}_e = A_m X_e - b W^T \Phi - b W^T \gamma - b \left[ d_1(X, t) + b_1 N \frac{y_n}{|y_n| + \xi} \right] \quad (\text{A.11})$$

$$\Phi = \begin{bmatrix} b_1 \int_0^t \alpha y_n X_1 d\tau + a_{m1} - a_1 \\ b_1 \int_0^t \alpha y_n X_1 d\tau + a_{m2} - a_2 \\ \vdots \\ b_1 \int_0^t \alpha y_n X_1 d\tau + a_{mn} - a_n \\ b_1 \int_0^t \alpha y_n U_m d\tau - b_m \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad (\text{A.12})$$

$b \in \mathfrak{R}^{n \times 1}$

When applying Popov's criterion to the system defined by (A.11), we obtain: for all  $t \geq 0$ :

$$\int_0^T y_n \left[ W^T \Phi + W^T \gamma + d_1(X, t) + b_1 N \frac{y_n}{|y_n| + \xi} \right] dt \geq -C_1^2 + \int_0^T |y_n| \left[ d_1(X, t) \text{sign}(y_n) + b_1 N \frac{y_n}{|y_n| + \xi} \right] dt \geq -C_1^2 \quad (\text{A.13})$$

If the following condition is satisfied for all  $t \geq 0$ :

$$d_1(X, t) \text{sign}(y_n) + b_1 N \frac{y_n}{|y_n| + \xi} \geq 0 \quad (\text{A.14})$$

The error  $X_e$  is therefore globally asymptotically stable if condition (A.14) is satisfied. If the coefficient  $N$  is chosen so that  $b_1 N$  is positive, then a lower limit of  $N$  is obtained from (A.14) as follow:

$$b_1 N \geq \max \left\{ \left| d_1(X, t) \right| \cdot \frac{|y_n| + \xi}{y_n} \right\} \quad (\text{A.15})$$

And:  $b_1 N > 0$  for all  $t \geq 0$ .

If the parameter  $\xi$  is sufficient small so that the term  $\frac{|y_n| + \xi}{|y_n|}$  is approximated to 1.

We conclude therefore that if the parameters  $N$  and  $\xi$  are chosen appropriately, the use of the EMCS control law (A.5) yields a globally asymptotically stable error  $X_e$ .

REMARK:

The satisfaction of the condition (A.15) is not related to the speed of variation of  $d_1$  and therefore no matter how rapidly varying  $d_1$  is, suitable values of  $N$  and  $\xi$  must ensure the global asymptotic stability of the error  $X_e$ .

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