Solution of Fuzzy Maximal Flow Problems Using Fuzzy Linear Programming

Amit Kumar and Manjot Kaur

Abstract—In this paper, the fuzzy linear programming formulation of fuzzy maximal flow problems are proposed and on the basis of the proposed formulation a method is proposed to find the fuzzy optimal solution of fuzzy maximal flow problems. In the proposed method all the parameters are represented by triangular fuzzy numbers. By using the proposed method the fuzzy optimal solution of fuzzy maximal flow problems can be easily obtained. To illustrate the proposed method a numerical example is solved and the obtained results are discussed.

Keywords—Fuzzy linear programming, Fuzzy maximal flow problem, Ranking function, Triangular fuzzy number

I. Introduction

THE maximal flow problem is one of basic problems for combinatorial optimization in weighted directed graphs. It provides very useful models in a number of practical contexts including communication networks, oil pipeline systems and power systems. The maximal flow problem and its variations have wide range of applications and have been studied extensively. The maximal flow problem was proposed by Fulkerson and Dantzig [1] originally and solved by specializing the simplex method for the linear programming and Ford and Fulkerson [2] solved it by augmenting path algorithm. There are efficient algorithms to solve the crisp maximal flow problems [3,4].

In the real life situations there always exist uncertainty about the parameters (e.g.: costs, capacities and demands) of maximal flow problems. To deal with such type of problems, the parameters of maximal flow problems are represented by fuzzy numbers [5] and maximal flow problems with fuzzy parameters are known as fuzzy maximal flow problems. In the literature, the numbers of papers dealing with fuzzy maximal flow problems are less. The paper by Kim and Roush [6] is one of the first on this subject. The authors developed the fuzzy flow theory, presenting the conditions to obtain a optimal flow, by means of definitions on fuzzy matrices. But there were Chanas and Kolodziejczyk [7,8,9] who introduced the main works in the literature involving this subject. They approached this problem using the minimum cuts technique.

In the first paper, Chanas and Kolodziejczyk [7] presented an algorithm for a graph with crisp structure and fuzzy capacities, i.e., the arcs have a membership function associated in their flow. This problem was studied by Chanas and Kolodzijczyk [8] again, in this paper the flow is a real number and the capacities have upper and lower bounds with a satisfaction function. Chanas and Kolodzijczyk [9] had also studied the integer flow and proposed an algorithm. Chanas et al. [10] studied the maximum flow problem when the underlying associated structure is not well defined and must be modeled as a fuzzy graph. Diamond [11] developed interval-valued versions of the max-flow min cut theorem and Karp-Edmonds algorithm and provide robustness estimates for flows in networks in an imprecise or uncertain environment. These results are extended to networks with fuzzy capacities and flows.

Liu and Kao [12] investigated the network flow problems in that the arc lengths of the network are fuzzy numbers. Ji et al. [13] considered a generalized fuzzy version of maximum flow problem, in which arc capacities are fuzzy variables. Hernandes et al. [14] proposed an algorithm, based on the classical algorithm of Ford-Fulkerson. The algorithm uses the technique of the incremental graph and representing all the parameters as fuzzy numbers.

In this paper, the fuzzy linear programming formulation of fuzzy maximal flow problems are proposed and on the basis of the proposed formulation a method is proposed to find the fuzzy optimal solution of fuzzy maximal flow problems. In the proposed method all the parameters are represented by triangular fuzzy numbers. By using the proposed method the fuzzy optimal solution of fuzzy maximal flow problems can be easily obtained. To illustrate the proposed method a numerical example is solved and the obtained results are discussed.

This paper is organized as follows: In section II, some basic definitions, arithmetic operations and the method for conversion of inequality constraints into equality constraints are reviewed. In section III, linear programming formulation of maximal flow problems in crisp and fuzzy environment presented. In section IV, a new method is proposed to find the fuzzy optimal solution of fuzzy maximal flow problems. In section V, to illustrate the proposed method a numerical example is solved. The obtained results are discussed in section VI. The conclusions are discussed in section VII.

II. Preliminaries

In this section some basic definitions, arithmetic operations and conversion of inequality constraint into equality constraint are reviewed.

A. Basic Definitions

In this section some basic definitions are reviewed.

Definition 2.1 [15] The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in

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X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0, 1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 [15] A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ \frac{(x-c)}{(b-c)}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.3 [15] A triangular fuzzy number (a, b, c) is said to be non-negative fuzzy number iff $a \geq 0$

Definition 2.4 [16] A ranking function is a function $\Re : F(R) \rightarrow R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number then $\Re(\tilde{A}) = \frac{a+2b+c}{4}$

B. Arithmetic Operations

In this subsection, arithmetic operations between two triangular fuzzy numbers, defined on universal set of real numbers R, are reviewed [15] .

Let $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$ be two triangular fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

(ii) $\tilde{A}_1 = \tilde{A}_2$ iff $a_1 = a_2, b_1 = b_2, c_1 = c_2$

C. Conversion of Inequality Constraint into Equality Constraint

In this subsection the method for conversion of inequality constraints into equality constraints are reviewed [17].

Let $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ be two triangular fuzzy numbers, then

$\tilde{A} \preceq \tilde{B}$ iff $a_1 \leq a_2, b_1 - a_1 \leq b_2 - a_2, c_1 - b_1 \leq c_2 - b_2$.

If $\tilde{A} \preceq \tilde{B}$, then it can be converted into $\tilde{A} \oplus \tilde{C} = \tilde{B}$.

where $\tilde{C}$ is non-negative triangular fuzzy number.

III. Linear Programming Formulation of Maximal Flow Problems in Crisp and Fuzzy Environment

In this section linear programming formulation of maximal flow problems in crisp and fuzzy environment are presented.

A. Linear Programming Formulation of Maximal Flow Problems in Crisp Environment

In this section the linear programming formulation of maximal flow problems in crisp environment is presented [14].

Let us consider a directed graph $G = (V, E)$ with capacities upper bound u_{ij}, and associating to each arc (i, j) a value x_{ij} corresponding to the flow in this arc (i, j). Let f represent the amount of maximal flow in the network from source node (say s) to destination node (say t). The maximal flow problems in crisp environment may be formulated as follow:

Maximize f

Subject to

$$\begin{aligned} &\sum_j x_{ij} = \sum_k x_{ki} + f \quad ; i = s \\ &\sum_j x_{ij} = \sum_k x_{ki} \quad ; \forall i \neq s, t \\ &\sum_j x_{ij} + f = \sum_k x_{ki} \quad ; i = t \\ &0 \leq x_{ij} \leq u_{ij} \quad \forall (i, j) \in E \end{aligned}$$

B. Linear Programming Formulation of Maximal Flow Problems in Fuzzy Environment

In this section the linear programming formulation of maximal flow problems in fuzzy environment is proposed.

Let us consider a directed graph $G = (V, E)$ with fuzzy capacities upper bound $\tilde{u}_{ij}$, and associating to each arc (i, j) a value $\tilde{x}_{ij}$ corresponding to the fuzzy flow in this arc (i, j). Let $\tilde{f}$ represent the amount of fuzzy maximal flow in the network from source node s to destination node t. The fuzzy maximal flow problems in fuzzy environment may be formulated as follow:

Maximize $\tilde{f}$

Subject to

$$\begin{aligned} &\sum_j \tilde{x}_{ij} = \sum_k \tilde{x}_{ki} \oplus \tilde{f} \quad ; i = s \\ &\sum_j \tilde{x}_{ij} = \sum_k \tilde{x}_{ki} \quad ; \forall i \neq s, t \\ &\sum_j \tilde{x}_{ij} \oplus \tilde{f} = \sum_k \tilde{x}_{ki} \quad ; i = t \\ &\tilde{x}_{ij} \preceq \tilde{u}_{ij} \quad \forall (i, j) \in E \\ &\tilde{x}_{ij} \text{ is a non-negative fuzzy number} \end{aligned}$$

Remark 3.1 In this paper, at all the places $\sum_i^m x_i$ and $\sum_i^m \tilde{x}_i$ represents the crisp and fuzzy addition respectively i.e., $\sum_i^m x_i = x_1 + x_2 + \ldots + x_m$ and $\sum_i^m \tilde{x}_i = \tilde{x}_1 \oplus \tilde{x}_2 \oplus \ldots \oplus \tilde{x}_m$, where x_i and $\tilde{x}_i$ are real number and fuzzy number respectively.

IV. Proposed Method

In this section, a new method is proposed to find the fuzzy optimal solution of fuzzy maximal flow problems. The steps of the proposed method are as follows:

Step 1 Formulate the given fuzzy maximal flow problem into the following fuzzy linear programming problem:

Maximize $\tilde{f}$

Subject to

$$\begin{aligned} &\sum_j \tilde{x}_{ij} = \sum_k \tilde{x}_{ki} \oplus \tilde{f} \quad ; i = s \\ &\sum_j \tilde{x}_{ij} = \sum_k \tilde{x}_{ki} \quad ; \forall i \neq s, t \\ &\sum_j \tilde{x}_{ij} \oplus \tilde{f} = \sum_k \tilde{x}_{ki} \quad ; i = t \\ &\tilde{x}_{ij} \preceq \tilde{u}_{ij} \quad \forall (i, j) \in E \\ &\tilde{x}_{ij} \text{ is a non-negative fuzzy number} \end{aligned}$$

Step 2 Let all the parameters $\tilde{x}_{ij}$, $\tilde{f}$ and $\tilde{u}_{ij}$ are represented by non-negative triangular fuzzy numbers (a_{ij}, b_{ij}, c_{ij}), (f_1, f_2, f_3) and (u_{ij}, v_{ij}, w_{ij}) respectively then the fuzzy linear programming formulation of fuzzy maximal flow problem, obtained in step 1, may be written as:
Maximize (f_1, f_2, f_3)
Subject to

$$\sum_j (a_{ij}, b_{ij}, c_{ij}) = \sum_k (a_{ki}, b_{ki}, c_{ki}) \oplus (f_1, f_2, f_3) \quad ; i = s$$
$$\sum_j (a_{ij}, b_{ij}, c_{ij}) = \sum_k (a_{ki}, b_{ki}, c_{ki}) \quad ; \forall i \neq s, t$$
$$\sum_j (a_{ij}, b_{ij}, c_{ij}) \oplus (f_1, f_2, f_3) = \sum_k (a_{ki}, b_{ki}, c_{ki}) \quad ; i = t$$
$$(a_{ij}, b_{ij}, c_{ij}) \preceq (u_{ij}, v_{ij}, w_{ij}) \quad \forall (i,j) \in E$$

Step 3 Converting the inequality constraints into equality constraint by introducing non-negative variable $\tilde{S}_{ij} = (s'_{ij}, s''_{ij}, s'''_{ij})\ \forall (i,j) \in E$ the fuzzy linear programming problem, obtained in step 2, may be written as:
Maximize (f_1, f_2, f_3)
Subject to

$$\sum_j (a_{ij}, b_{ij}, c_{ij}) = \sum_k (a_{ki}, b_{ki}, c_{ki}) \oplus (f_1, f_2, f_3) \quad ; i = s$$
$$\sum_j (a_{ij}, b_{ij}, c_{ij}) = \sum_k (a_{ki}, b_{ki}, c_{ki}) \quad ; \forall i \neq s, t$$
$$\sum_j (a_{ij}, b_{ij}, c_{ij}) \oplus (f_1, f_2, f_3) = \sum_k (a_{ki}, b_{ki}, c_{ki}) \quad ; i = t$$
$$(a_{ij}, b_{ij}, c_{ij}) \oplus (s'_{ij}, s''_{ij}, s'''_{ij}) = (u_{ij}, v_{ij}, w_{ij}) \quad \forall (i,j) \in E$$

Step 4 Using ranking formula and arithmetic operations, presented in subsection A and B of section II respectively, the fuzzy linear programming problem, obtained in step 3, is converted into the following crisp linear programming problem:
Maximize $(\frac{f_1 + 2f_2 + f_3}{4})$
Subject to

$$\begin{aligned} \sum_j a_{ij} &= \sum_k a_{ki} + f_1 \quad ; i = s \\ \sum_j a_{ij} &= \sum_k a_{ki} \quad ; \forall i \neq s, t \\ \sum_j a_{ij} + f_1 &= \sum_k a_{ki} \quad ; i = t \end{aligned} \tag{1}$$

$$\begin{aligned} \sum_j b_{ij} &= \sum_k b_{ki} + f_2 \quad ; i = s \\ \sum_j b_{ij} &= \sum_k b_{ki} \quad ; \forall i \neq s, t \\ \sum_j b_{ij} + f_2 &= \sum_k b_{ki} \quad ; i = t \end{aligned} \tag{2}$$

$$\begin{aligned} \sum_j c_{ij} &= \sum_k c_{ki} + f_3 \quad ; i = s \\ \sum_j c_{ij} &= \sum_k c_{ki} \quad ; \forall i \neq s, t \\ \sum_j c_{ij} + f_3 &= \sum_k c_{ki} \quad ; i = t \end{aligned} \tag{3}$$

$$a_{ij} + s'_{ij} = u_{ij}$$
$$b_{ij} + s''_{ij} = v_{ij}$$
$$c_{ij} + s'''_{ij} = w_{ij}$$
$$b_{ij} - a_{ij} \geq 0,\ c_{ij} - b_{ij} \geq 0,\ a_{ij} \geq 0,\ b_{ij} \geq 0,\ c_{ij} \geq 0$$
$$s''_{ij} - s'_{ij} \geq 0,\ s'''_{ij} - s''_{ij} \geq 0,\ s'_{ij} \geq 0,\ s''_{ij} \geq 0,\ s'''_{ij} \geq 0$$
$$f_2 - f_1 \geq 0,\ f_3 - f_2 \geq 0,\ f_1 \geq 0,\ f_2 \geq 0,\ f_3 \geq 0$$
$$\forall (i,j) \in E$$

Step 5 Find the optimal flow f_1, f_2 and f_3 by solving the crisp linear programming problem obtained in step 4.
Step 6 Find the fuzzy maximal flow by putting the values of f_1, f_2 and f_3 in $\tilde{f} = (f_1, f_2, f_3)$.

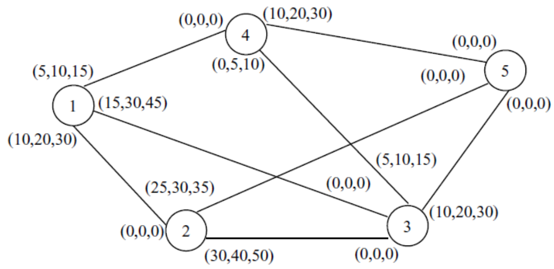

Fig. 1 Fuzzy value of flow along each arc

V. Illustrative Example

In this section, the proposed method is illustrated by solving a numerical example.
Example 5.1 The problem is to find out the fuzzy maximal flow between node 1 (say source node) and node 5 (say destination node) on the network shown above in Fig. 1.
Solution: The fuzzy maximal flow between node 1 and node 5 can be obtained by using the following steps:
Step 1 The given fuzzy maximal flow problem may be formulated into the following fuzzy linear programming problem:
Maximize $\tilde{f}$
Subject to

$$\tilde{x}_{12} \oplus \tilde{x}_{13} \oplus \tilde{x}_{13} = \tilde{f}$$
$$\tilde{x}_{23} \oplus \tilde{x}_{25} = \tilde{x}_{12}$$
$$\tilde{x}_{34} \oplus \tilde{x}_{35} = \tilde{x}_{13} \oplus \tilde{x}_{23}$$
$$\tilde{x}_{45} = \tilde{x}_{14} \oplus \tilde{x}_{34}$$
$$\tilde{x}_{25} \oplus \tilde{x}_{35} \oplus \tilde{x}_{45} = \tilde{f}$$
$$\tilde{x}_{ij} \preceq \tilde{u}_{ij} \quad \forall (i,j) \in E$$

$\tilde{x}_{ij}$ is a non-negative fuzzy number.
Step 2 Let all the parameters $\tilde{x}_{ij}$, $\tilde{f}$ and $\tilde{u}_{ij}$ are represented by non-negative triangular fuzzy numbers (a_{ij}, b_{ij}, c_{ij}), (f_1, f_2, f_3) and (u_{ij}, v_{ij}, w_{ij}) respectively, then the fuzzy linear programming formulation of fuzzy maximal flow problem, obtained in step 1, may be written as:
Maximize (f_1, f_2, f_3)
Subject to

$$(a_{12}, b_{12}, c_{12}) \oplus (a_{13}, b_{13}, c_{13}) \oplus (a_{14}, b_{14}, c_{14}) = (f_1, f_2, f_3)$$
$$(a_{23}, b_{23}, c_{23}) \oplus (a_{25}, b_{25}, c_{25}) = (a_{12}, b_{12}, c_{12})$$
$$(a_{34}, b_{34}, c_{34}) \oplus (a_{35}, b_{35}, c_{35}) = (a_{13}, b_{13}, c_{13}) \oplus (a_{23}, b_{23}, c_{23})$$
$$(a_{45}, b_{45}, c_{45}) = (a_{14}, b_{14}, c_{14}) \oplus (a_{34}, b_{34}, c_{34})$$
$$(a_{25}, b_{25}, c_{25}) \oplus (a_{35}, b_{35}, c_{35}) \oplus (a_{45}, b_{45}, c_{45}) = (f_1, f_2, f_3)$$
$$\tilde{x}_{12} \preceq (10, 20, 30),\ \tilde{x}_{13} \preceq (15, 30, 45),\ \tilde{x}_{14} \preceq (5, 10, 15),$$
$$\tilde{x}_{23} \preceq (30, 40, 50),\ \tilde{x}_{25} \preceq (25, 30, 35),\ \tilde{x}_{34} \preceq (5, 10, 15),$$
$$\tilde{x}_{35} \preceq (10, 20, 30),\ \tilde{x}_{45} \preceq (10, 20, 30)$$

Step 3 Converting the inequality constraints into equality constraint by introducing non-negative variable $\tilde{S}_{ij} = (s'_{ij}, s''_{ij}, s'''_{ij})\ \forall (i,j) \in E$ the fuzzy linear programming problem, obtained in step 2, may be written as:
Maximize (f_1, f_2, f_3)
Subject to

$$(a_{12}, b_{12}, c_{12}) \oplus (a_{13}, b_{13}, c_{13}) \oplus (a_{14}, b_{14}, c_{14}) = (f_1, f_2, f_3)$$
$$(a_{23}, b_{23}, c_{23}) \oplus (a_{25}, b_{25}, c_{25}) = (a_{12}, b_{12}, c_{12})$$
$$(a_{34}, b_{34}, c_{34}) \oplus (a_{35}, b_{35}, c_{35}) = (a_{13}, b_{13}, c_{13}) \oplus (a_{23}, b_{23}, c_{23})$$

$(a_{45}, b_{45}, c_{45}) = (a_{14}, b_{14}, c_{14}) \oplus (a_{34}, b_{34}, c_{34})$
$(a_{25}, b_{25}, c_{25}) \oplus (a_{35}, b_{35}, c_{35}) \oplus (a_{45}, b_{45}, c_{45}) = (f_1, f_2, f_3)$
$(a_{12}, b_{12}, c_{12}) \oplus (s'_{12}, s''_{12}, s'''_{12}) = (10, 20, 30)$
$(a_{13}, b_{13}, c_{13}) \oplus (s'_{13}, s''_{13}, s'''_{13}) = (15, 30, 45)$
$(a_{14}, b_{14}, c_{14}) \oplus (s'_{14}, s''_{14}, s'''_{14}) = (5, 10, 15)$
$(a_{23}, b_{23}, c_{23}) \oplus (s'_{23}, s''_{23}, s'''_{23}) = (30, 40, 50)$
$(a_{25}, b_{25}, c_{25}) \oplus (s'_{25}, s''_{25}, s'''_{25}) = (25, 30, 35)$
$(a_{34}, b_{34}, c_{34}) \oplus (s'_{34}, s''_{34}, s'''_{34}) = (5, 10, 15)$
$(a_{35}, b_{35}, c_{35}) \oplus (s'_{35}, s''_{35}, s'''_{35}) = (10, 20, 30)$
$(a_{45}, b_{45}, c_{45}) \oplus (s'_{45}, s''_{45}, s'''_{45}) = (10, 20, 30)$

Step 4 Using ranking formula and arithmetic operations, presented in subsection A and B of section II respectively, the fuzzy linear programming problem, obtained in step 3, is converted into the following crisp linear programming problem:

Maximize $(\frac{f_1+2f_2+f_3}{4})$

Subject to

$$
\begin{aligned}
& a_{12} + a_{13} + a_{14} - f_1 = 0,\ b_{12} + b_{13} + b_{14} - f_2 = 0 \\
& c_{12} + c_{13} + c_{14} - f_3 = 0,\ a_{23} + a_{25} - a_{12} = 0 \\
& b_{23} + b_{25} - b_{12} = 0,\ c_{23} + c_{25} - c_{12} = 0 \\
& a_{34} + a_{35} - a_{13} - a_{23} = 0,\ b_{34} + b_{35} - b_{13} - b_{23} = 0 \\
& c_{34} + c_{35} - c_{13} - c_{23} = 0,\ a_{14} + a_{34} - a_{45} = 0 \\
& b_{14} + b_{34} - b_{45} = 0,\ c_{14} + c_{34} - c_{45} = 0 \\
& a_{25} + a_{35} + a_{45} - f_1 = 0,\ b_{25} + b_{35} + b_{45} - f_2 = 0 \\
& c_{25} + c_{35} + c_{45} - f_3 = 0 \\
& a_{12} + s'_{12} = 10,\ b_{12} + s''_{12} = 20,\ c_{12} + s'''_{12} = 30 \\
& a_{13} + s'_{13} = 15,\ b_{13} + s''_{13} = 30,\ c_{13} + s'''_{13} = 45 \\
& a_{14} + s'_{14} = 5,\ b_{14} + s''_{14} = 10,\ c_{14} + s'''_{14} = 15 \\
& a_{23} + s'_{23} = 30,\ b_{23} + s''_{23} = 40,\ c_{23} + s'''_{23} = 50 \\
& a_{25} + s'_{25} = 25,\ b_{25} + s''_{25} = 30,\ c_{25} + s'''_{25} = 35 \\
& a_{34} + s'_{34} = 5,\ b_{34} + s''_{34} = 10,\ c_{34} + s'''_{34} = 15 \\
& a_{35} + s'_{35} = 10,\ b_{35} + s''_{35} = 20,\ c_{35} + s'''_{35} = 30 \\
& a_{45} + s'_{45} = 10,\ b_{45} + s''_{45} = 20,\ c_{45} + s'''_{45} = 30 \\
& b_{ij} - a_{ij} \geq 0,\ c_{ij} - b_{ij} \geq 0,\ a_{ij} \geq 0,\ b_{ij} \geq 0,\ c_{ij} \geq 0 \\
& s''_{ij} - s'_{ij} \geq 0,\ s'''_{ij} - s''_{ij} \geq 0,\ s'_{ij} \geq 0,\ s''_{ij} \geq 0,\ s'''_{ij} \geq 0 \\
& f_2 - f_1 \geq 0,\ f_3 - f_2 \geq 0,\ f_1 \geq 0,\ f_2 \geq 0,\ f_3 \geq 0 \\
& \forall (i,j) \in E
\end{aligned}
$$

Step 5 Solving the crisp linear programming problem obtained in step 4, the optimal flow is $f_1 = 30$, $f_2 = 55$ and $f_3 = 85$.

Step 6 Putting the values of $f_1 = 30$, $f_2 = 55$ and $f_3 = 85$ in $\tilde{f} = (f_1, f_2, f_3)$, the fuzzy maximal flow is $\tilde{f} = (30, 55, 85)$.

VI. Results And Discussion

The obtained result can be explained as follows:

(i) The amount of flow between source and sink is greater than 30 and less than 85 units.

(ii) Maximum number of persons are in favour that amount of flow will be 55 units.

(iii) The percentage of favourness for remaining flow can be obtained as follow:

Let x represents the amount of flow, then the percentage of the favourness for $x = \mu_{\tilde{f}}(x) \times 100$

$$
\text{where, } \mu_{\tilde{f}}(x) = \begin{cases} \frac{(x-30)}{25}, & 30 \leq x \leq 55 \\ \frac{(x-85)}{-30}, & 55 \leq x \leq 85 \\ 0 \quad, & \text{otherwise} \end{cases}
$$

VII. Conclusion

A new method, based on fuzzy linear programming formulation of maximal flow problems, is proposed. To illustrate the proposed method a numerical example is solved and the obtained results are discussed. Using the proposed method the fuzzy optimal solution of maximal flow problems occurring in real life situations can be easily obtained.

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