A Non-linear Damage Model for the Annulus of the Intervertebral Disc Under Cyclic Loading, Including Recovery

Shruti Motiwale, Xianlin Zhou, and Reuben H. Kraft

Abstract—Military and sports personnel are often required to wear heavy helmets for extended periods of time. This leads to excessive cyclic loads on the neck and an increased chance of injury. Computational models offer one approach to understand and predict the time progression of disc degeneration under severe cyclic loading. In this paper, we have applied an analytic non-linear damage evolution model to estimate damage evolution in an intervertebral disc due to cyclic loads over decade-long time periods. We have also proposed a novel strategy for inclusion of recovery in the damage model. Our results show that damage only grows 20% in the initial 75% of the life, growing exponentially in the remaining 25% life. The analysis also shows that it is crucial to include recovery in a damage model.

Keywords—cervical spine, computational biomechanics, damage evolution, intervertebral disc, continuum damage mechanics.

I. Introduction

RECENTLY an increased rate of spinal disc degeneration has been observed in military environments. One hypothesis for this increase is believed to be related to wearing heavier helmets and other head-supported devices such as night vision goggles for extended periods of time while traversing on complex terrains. This leads to complex loads on the cervical spine and is believed to change the head and neck biomechanics. Cohen at al. [1] reported that neck pain is one of the leading causes of medical evacuation out of theaters of combat operations. Heavy helmets and night vision goggles have also been reported as sources of neck pain in military pilots [2]–[5]. Petrén-Mallmin and Linder [6] conducted a magnetic resonance imaging study on military high performance aircraft pilots and found them to be at increased risk of premature development of degenerative lesions of the same type as are seen in an aging population.

In a previous study, we utilized a whole body musculoskeletal model (1) with detailed neck structures (7 cervical vertebral joints and 24 degrees of freedom on head and neck) [7], [8] to simulate the cervical joints during walking and running with a middle sized army combat helmet (1.43kg) [9] as shown in Fig 1(a). The study was aimed to investigate how head supported mass affects neck muscle activation and intervertebral compressive forces during walking and running. It was concluded that head supported mass wear requires increased muscle activation or contraction and consequently increases the intervertebral joint loading. In Fig. 1(b), we show the compressive loading at C4-C5 joint level during one gait cycle of walking. Note that the joint compressive loading is a cumulative result of both inertia effect (due to motion) and muscle contraction induced joint compression. These results form a starting point for this work. We would like to extend the numerical technique to predict the degenerative effects due to fatigue loading on the cervical disc.

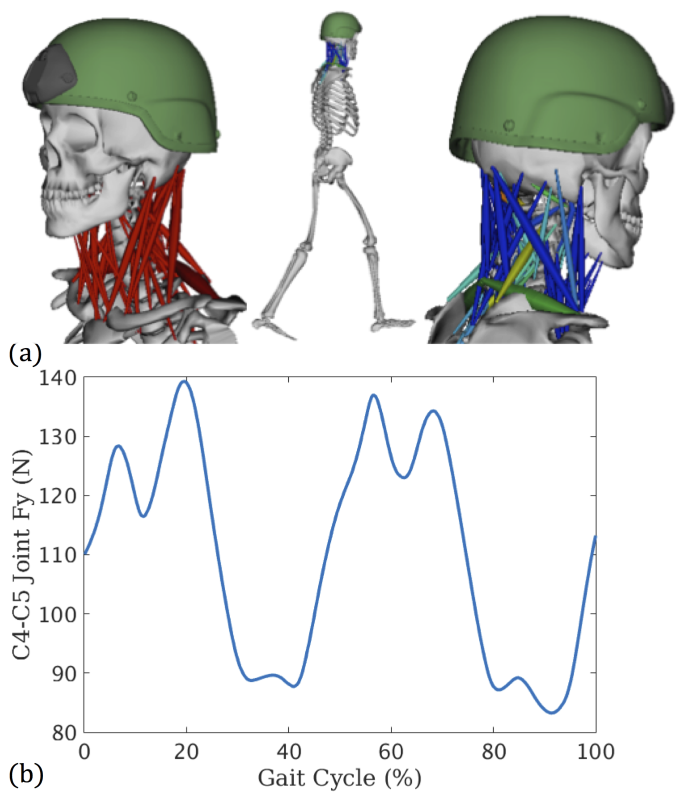

Fig. 1: (a) The detailed neck and whole body musculoskeletal model and predicted muscle contraction activation level (right) at a walking gait point (middle). The predicted muscle contraction forces contribute to disc compressive forces. (b) Loads on a C4 disc during one gait cycle. One gait cycle lasted for 1.234 seconds.

A number of researchers have conducted experiments to determine fatigue properties of intervertebral discs [10], [11]. Green, Adams and Dolan [12] presented ultimate tensile strength and fatigue failure data for 19 samples of human inter-

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vertebral discs. They subjected vertical slices from the anterior and posterior margins of annulus part of lumbar intervertebral discs to cyclic loads until the occurrence of failure. These experimental results have been used by researchers to develop computational models. For example, Qasim, Natarajan, An and Andersson [13] have used a linear damage rule to predict damage evolution and proposed a numerical algorithm based on continuum damage methodology to investigate the initiation and progression of damage in the annulus. Makwana *et al.* [14] provide an overview on micromechanics and mechanisms of intervertebral disc degeneration. They have considered a poroelastic finite element model of the disc that captures water loss and disc height change. We seek to develop a non-linear computational model to elucidate the mechanism of disc degeneration in the annulus region that is different from the previous work in three unique ways: a) it captures the non-linearity of damage evolution, b) it takes into consideration recovery mechanisms, and c) it is computationally inexpensive and yet retains the physics of the problem well.

Researchers have approached the topic of damage evolution in many different ways. One of the earliest concepts of linear damage progression and accumulation is the Palmgren-Miner hypothesis, which has been widely used throughout literature due to its simplicity [15]:

$$D = \Sigma n_i/N_{fi} \tag{1}$$

where n_i is the number of cycles at a given stress amplitude $\Delta\sigma_i$, N_{fi} is the number of cycles to failure at the same stress level and D is the damage variable. However, this model does not take into account the effect of the order of application of load. For example, in a two level fatigue stress loading, when a high load is followed by a low load, the summation of cycle ratios is less than 1, whereas when a low load is followed by a high load, the cycle ratio summation is greater than 1 [16]. It also does not consider the cycles ratios below the fatigue limit, as prior loading can result in reduction of fatigue limit [17]. These are major deficiencies of this linear model that may result in a discrepancy of up to an order of magnitude between predicted and experimental life [18]. To remedy the deficiencies associated with the linear damage model, Marko and Starkey [19] proposed the first non-linear load dependent damage theory, which is represented by:

$$D = \Sigma(n_i/N_{fi})^{x_i} \tag{2}$$

where x_i is a variable quantity related to the i^{th} load level. Extending the concept of a non-linear model, Chaboche and Lemaitre [20] proposed an analytical differential equation for a non-linear continuous fatigue damage model, which is of the form:

$$\delta D = f(D, \sigma_{max}, \sigma_{mean})\delta n \tag{3}$$

where D denotes damage, and f is a function of σ_{max}, Others have also used a thermodynamic approach for damage estimation in [21], [22], but it is difficult to find appropriate data to determine parameters in these equations, thus making it hard to implement them.

Very little work has been done on the prediction of long-term damage to an intervertebral disc due to cyclic loads. Linear models have been applied in the past, but the authors have not found any work on application of non-linear models to the disc, which have been found to be more accurate than linear models. In this paper, we propose the application of a non-linear model for estimating evolution of damage in the cervical spine disc subjected to variable loading for extended periods of time. This paper is organized as follows: in Section 2, we describe the non-linear model and its application in detail. In Section 3, we present preliminary work on recovery of damage. In Section 4, we discuss the results. Finally we conclude by summarizing our work and present opportunities and ideas for future development.

II. METHOD

Chaboche and Lemaitre [20] proposed a non-linear model for damage evolution that is given by:

$$\delta D = D^{\alpha(\sigma_{max},\sigma_{mean})}\left[\frac{\sigma_a}{M_0(1-b\sigma_{mean})}\right]^{\beta}\delta n \tag{4}$$

where, σ_a is the alternating stress, σ_{mean} is the mean stress, D is the damage, such that $0 \leq D \leq 1$, n is the number of cycles and M_0, b, β are material parameters.
The damage rate in (4) depends on the present state of damage, leading to a non-linear damage evolution under cyclic loading. The exponent α depends on the applied load and hence results in non-separability of damage and the applied load [20]. This leads to a non-linear damage accumulation and sequence effects, as separate variables are incapable of modelling sequence effects [23]. A number of different forms of α have been proposed in the literature [24]. A simplified form for α, proposed by Chaboche, is given as follows:

$$\alpha = 1 - a\left\langle\frac{\sigma_{max} - (\sigma_f + (1-b\sigma_f)\sigma_{mean}}{\sigma_u - \sigma_{max}}\right\rangle \tag{5}$$

where, σ_u is the ultimate tensile strength (UTS), σ_f is the fatigue limit, i.e. the limiting value of applied stress that can cause failure as the number of cycles to failure becomes very large and a is a material parameter. $\langle \quad \rangle$ is defined as, $\langle x\rangle = 0$ if $x < 0$ and $\langle x\rangle = x$ if $x \geq 0$.
The number of cycles that will be considered in this paper is of the order of 10^8, which is much larger than the number of cycles for which the data is available, 10^4. For such a large number of cycles, it is hard to estimate a stress value below which fatigue damage will not occur for a large number of cycles. Hence, the fatigue limit is assumed to be 0 MPa for the disc as a conservative estimate and we obtain the following form of α:

$$\alpha = 1 - a\left\langle\frac{\sigma_a}{\sigma_u - \sigma_{max}}\right\rangle \tag{6}$$

This is expected to give a more conservative estimate on damage evolution, but as sufficient data on the existence of fatigue limit for the disc is not available, this is a necessary assumption. Integrating (4) from $D = 0$ to $D = 1$ gives the following expression for N_f:

$$N_f = \frac{1}{1-\alpha}\left[\frac{\sigma_a}{M_0(1-b\sigma_{mean})}\right]^{-\beta} \tag{7}$$

where, N_f is the number of cycles for complete failure, i.e. for damage to become 1. Integrating 4 for a generic instant and solving for D gives the following expression for D:

$$D = \left(\frac{n}{N_f}\right)^{\frac{1}{1-\alpha}} \tag{8}$$

A. Variable Loading

The loads on a cervical disc during one gait cycle in real life vary continuously as shown in Fig. 1(b). The equations for damage evolution and the data available are for constant amplitude of load. To address this problem, we calculate the effective number of cycles n_i^{eff} at each amplitude level σ_{ai} that will cause the same damage as the actual load history until iteration i. The damage is then calculated from these effective number of cycles. The effective number of cycles can be given by the following method that is based on the method proposed by Dattoma, Giancane, Nobile and Panella [25]:

$$n_i^{eff} = N_i + n_i \tag{9}$$

where n_i is the number of cycles at amplitude σ_{ai} and N_i is the cumulative effective number of cycles at amplitude σ_{ai}. N_i is calculated by equating damage for the current load level σ_{ai}, and that caused by the previous load level σ_{ai-1} for n_{i-1}^{eff}:

$$D = \left(\frac{n_{i-1}^{eff}}{N_{fi-1}}\right)^{\left(\frac{1}{1-\alpha_{i-1}}\right)} = \left(\frac{N_i}{N_{fi}}\right)^{\left(\frac{1}{1-\alpha_i}\right)} \tag{10}$$

Rearranging this equation, we get:

$$N_i = N_{fi}\left(\frac{n_{i-1}^{eff}}{N_{fi-1}}\right)^{(1-\alpha_i)/(1-\alpha_{i-1})} \tag{11}$$

where N_{fi} is the number of cycles to failure for a stress level σ_{ai}. Combining (9) and (11), we obtain:

$$n_i^{eff} = n_i + N_{fi}\left(\frac{n_{i-1}^{eff}}{N_{fi-1}}\right)^{(1-\alpha_i)/(1-\alpha_{i-1})} \tag{12}$$

where n_i^{eff} signifies the number of cycles of stress level σ_{ai} that should be applied to get the same damage as that due to the damage accumulated by variable loads until iteration i. Thus, the effective damage can be calculated by substituting n in (8) with n_i^{eff} as follows:

$$D = \left(\frac{n_i^{eff}}{N_f}\right)^{\frac{1}{1-\alpha_i}} \tag{13}$$

B. Curve Fitting

To implement these equations for a disc, the parameters a, β, b, M_0 in (6) and (7) need to be found. We used the data from Green *et al.*s' paper [12] to find these parameters. A non-linear curve fitting tool in Matlab (lsqnonlin) was used to find these parameters by fitting (7) and (6) to this data. Since (6) contains ultimate tensile strength as one of the parameters, and the tests conducted by Green *et al.*s' paper were for anterior and posterior parts of the disc, each of which have different ultimate tensile strength values, different parameters were found for both these cases. The plots obtained for curve fitting are shown in Fig. 2.

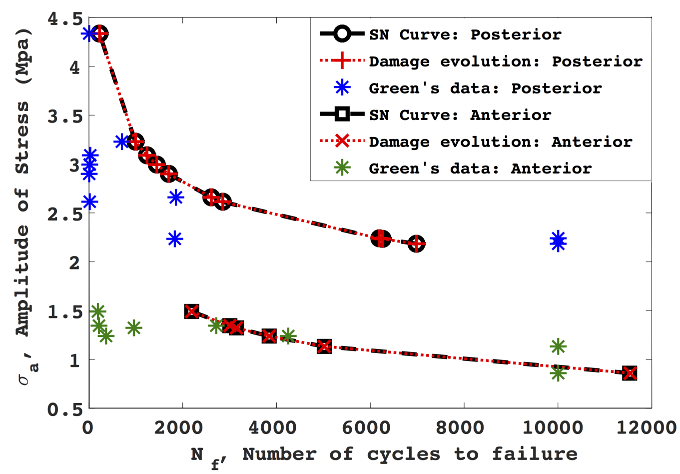


Fig. 2: Green's data for fatigue of anterior and posterior part of the disc fitted to a standard SN curve. We have fitted a standard Stress-Life (SN) curve for reference along with the stress-life curve for the non-linear model in (7). The two curves were found to overlap exactly.

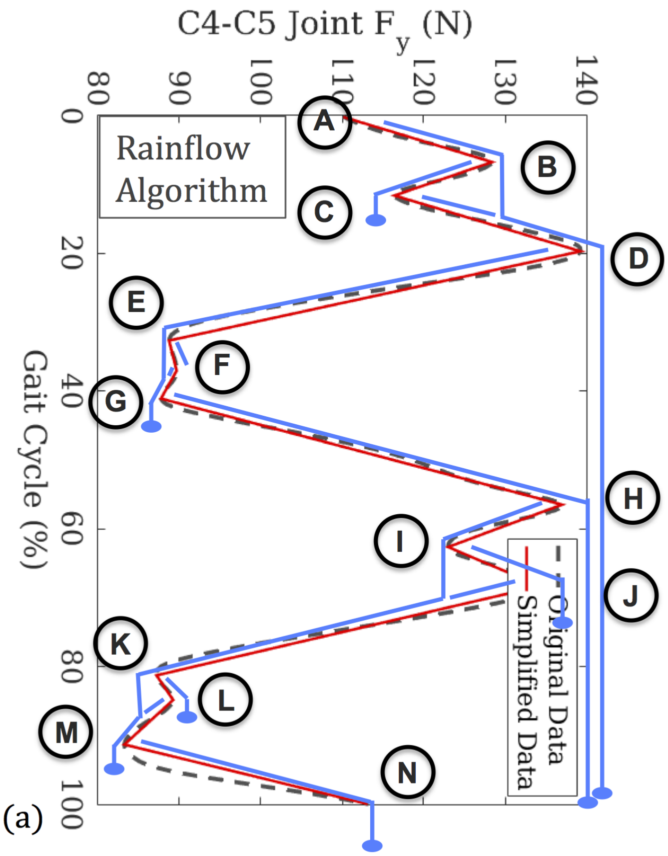
A standard stress-life (SN) curve was fitted to the same data for reference and (7), the equation that relates stresses and total life for the non-linear model was found to fit the data exactly like the standard SN curve. From Fig. 2 (a) and (b), we see that the anterior part of the disc fails at almost half of the stress level than posterior part of the disc. We have used the properties of the anterior part of the disc to plot damage evolution curves in this paper.

C. Rainflow Cycle Counting

The Rainflow cycle counting method is widely used for counting cycles for various damage evolution strategies. Further details of this method can be found in [26]–[29]. We used this method to extract cycles from the complicated loading history in Fig. 1(b) to obtain simplified loading history as shown in Fig. 3(a). A table was formed using this method, based on Fig. 3(a) in terms of mean and alternating forces as shown in Fig. 3(b)). From such a table, we can extract F_{mean} and F_{amp}, which can then be given as inputs to the code.The area of the disc was measured to be $312mm^2$ and the stresses were calculated using this area.

D. Approximation Over a Day

Equations (4) to (8) presented in this paper are valid for a constant load level. In Section A, we present the equations for variable load to calculate the damage for varying load. As our loads vary for every cycle, the damage increment needs to be calculated at every step. This led to a damage increment of the order of 10^{-21} or lesser in each step, which is less than the machine precision (10^{-16}). This can result in accumulation of error possibly greater than the actual value itself. Thus an approximation strategy was developed for calculation of damage by rearrangement of the load history



(a)

Path	Cycles	$F_{max}(N)$	$F_{min}(N)$
A-D	0.5	128.4010	110.3100
B-C	1	128.4010	116.4170
D-G	0.5	139.2750	87.7287
E-F	1	89.6508	88.7240
G-H	0.5	137.0030	87.7287
H-M	0.5	137.0030	83.2289
I-J	1	134.3080	123.0020
K-L	1	89.2255	87.1773
M-N	0.5	112.9650	83.2289

(b)

Fig. 3: (a) Rainflow cycle counting method is used to extract cycles from a variable profile of cyclic loading by rotating the loading history by 90 degrees and extracting each cycle by assuming that water droplets are falling down a roof. [26] (b) A table that shows the systematic cycle data that was extracted extracted from (a) using rainflow cycle counting. F_{max} and F_{min} can be used to find mean and amplitude of stresses.

over one complete day. One day was signified by with 9000 steps on the basis of results from a meta-analysis by Bohannan [30]. All the cycles were arranged in the descending order of σ_{max} and each load was applied for 9000 cycles at once. This was done in order to avoid very small increments of damage, which may result in accumulation of a large machine error over time. This leads to a more conservative estimate on damage, as we are applying the higher loads at the beginning, but it also yields a more trustworthy value for damage, as the machine error is only 0.01% of the smallest damage increment (10^{-12}). We compared the plots for the case without approximation and with approximation for 5 years (Fig. 4) and did not find any observable difference in the results. The approximation also resulted in reducing the computational time to 0.1% of the original time.

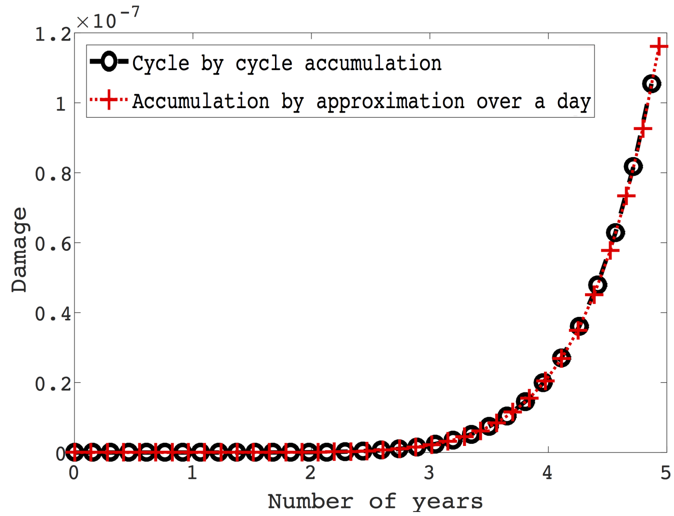


Fig. 4: Comparison of damage evolution for the original method, where damage is calculated for every new cycle of load, vs. the method of approximation, where all the cycles of same amplitude over one day are clubbed together. No difference was observed in the value of damage over a period of 5 years.

E. Recovery

It has been shown that intervertebral discs usually recover considerably from the damage caused due to cyclic loads during a recovery or rest period, i.e., when the disc is not loaded [31]–[33]. We have proposed a novel method to try to capture this effect in our damage model. At the end of each day, we assume that a resting period occurs during sleep, when the disc recovers from some of the damage caused during the day. We have introduced a recovery parameter, r, that proportionately reduces the damage caused over the period of one day as shown in (14).

$$D_i^r = D_{i-1}^r + (D_i - D_{i-1}^r) * (1 - r) \tag{14}$$

where, D_i is the damage in the i^{th} iteration without recovery, and D_i^r is the damage in the i^{th} iteration with recovery. The recovery parameter is a measure of the percentage of damage accumulated during the day that is recovered during the rest period. To incorporate the effect of this change in the next iteration, the value of N_f needs to be modified.

$$N_{fi} = \frac{n_i^{eff}}{D_i^{1/(1-\alpha_i)}} \tag{15}$$

III. Results

In Fig. 5, we show a plot of damage evolution for a C4 cervical disc without considering any recovery. The disc reaches a stage of complete failure in 36 years, which is a relatively small number as compared to lifetimes of healthy individuals. This result emphasizes the need of inclusion of recovery in the model.

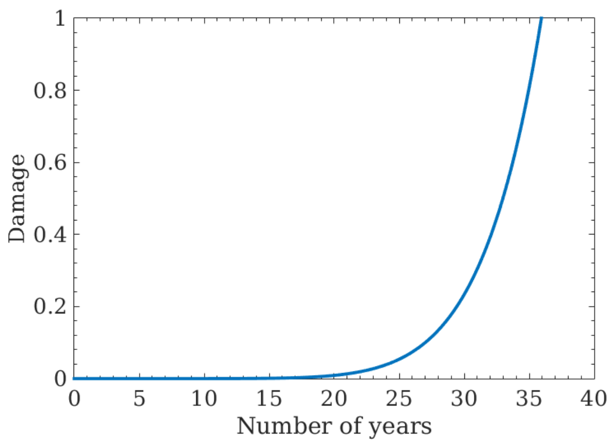


Fig. 5: Complete damage of the disc without considering recovery. The disc reaches complete failure, i.e. the value of damage reaches 1 within 36 years.

A. Recovery

The plots of damage evolution for various values of the recovery parameter are shown in Fig. 6. Addition of a recovery parameter makes a considerable difference in the results - for $r = 0.4$, the damage in the disc at the end of 75 years was found to be 0.4, as opposed to occurrence of complete failure in 36 years without recovery.

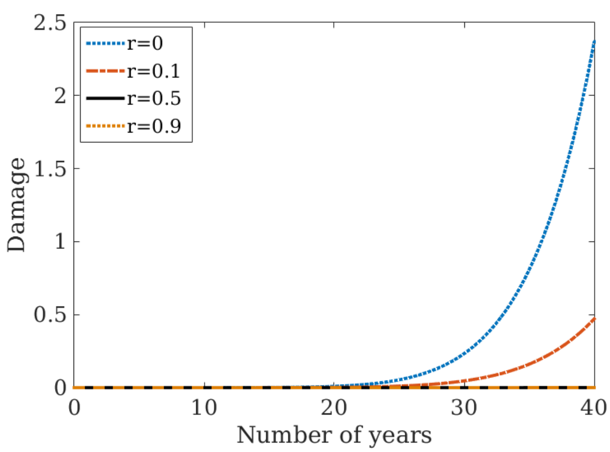


Fig. 6: Effect of recovery parameter, r. The addition of a recovery parameter was found to make a considerable difference in the results. For $r = 0.4$, at the end of 75 years, the damage in the disc was found to be 0.4, whereas without recovery, the disc was found to fail in 36 years.

B. Comparison of damage accumulation for different values of b

Since the data available for model calibration was only for fully reversed loading, the parameter b, that is associated with the mean stress, σ_{mean} in (7), could not be determined. The parameter b controls how much effect the mean stress has on evolution of damage. Here, we present plots that compare damage evolution for different values of b. Upon application of same loading cycle, it was observed that the the rate of damage evolution decreased with increasing values of b for compressive mean stress and increased with increasing value of b for a tensile mean stress. Moreover, damage was found to occur much earlier for a tensile mean stress.

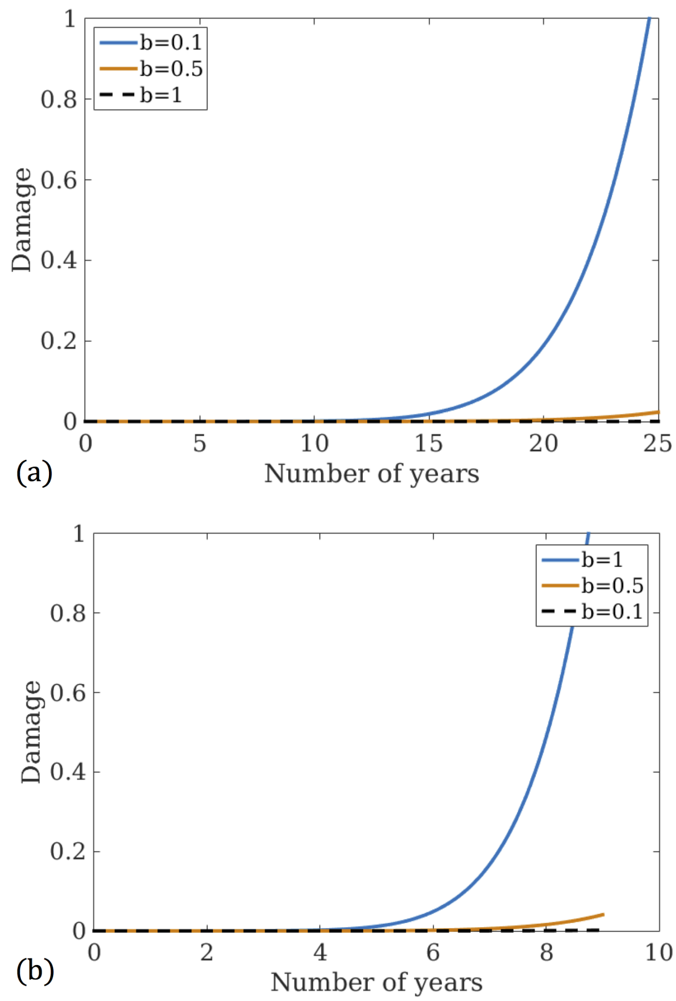


Fig. 7: (a) Effect of mean stress parameter $'b'$ on compressive mean stress. (b) Effect of mean stress parameter $'b'$ on tensile mean stress.

IV. Discussion

We have applied a non-linear model to study disc degeneration of a cervical spine due to long-term cyclic loads. We also proposed a preliminary recovery model to account for the recovery of damage experienced by the disc during a resting period.

Fig. 5 shows the damage evolution without considering recovery. Only 20% of the damage was found to occur until 75% of the total life. Moreover, negligible damage happens in the first 50% of life, after which, the damage is found to grow exponentially. We found that the disc reached complete damage within 36 years. One day, in the context of this study, is a collection of steps equal to the average number of human steps in a day. Thus, a damage model without recovery may mean that the disc is continuously being loaded for 1.2×10^8 steps (number of steps an average person is expected to walk in 36 years). This is equivalent to walking 6.2×10^4 miles, considering 1900 steps is a mile [34].

For this reason, we added recovery to our model. We found that for a recovery parameter $r = 0.5$, the disc achieves complete damage by 70 years. The recovery parameter can vary for people based on health status and age. Youngsters are expected to have a higher recovery parameter than older people. It can also be considered a function of damage - when the disc is already damaged, the recovery capacity of the disc can be assumed to be lower than when the disc is healthy. One aspect that the current recovery model does not consider is lower and upper bounds on the recovery possible during a recovery period. In this paper, the recovery is considered a percentage of damage incurred during the day. This may not be true for certain cases when the damage is too less and the entire damage is recovered during the rest period, or when the damage is too high, and only a small percentage of damage can be recovered.

Further, we estimated the effect of the mean stress parameter b on our model by plotting three different damage evolution plots for $b = 0.1, b = 0.5, b = 1$. The effect of increasing b on the rate of damage evolution was found to be opposite for tensile and compressive mean stress. Compressive mean stress is found to decrease the rate of damage growth; as b is directly related to σ_{mean}, this behaviour due to changing its value is expected. We found that the results depend heavily on the value of b - when b was increased from 0.1 to 0.5 for compressive mean stress, the final value of damage was found to decrease from 1.1 to 0.1. Further experiments are necessary to determine the value of b accurately.

The fatigue data used in this paper was from experiments on a lumbar spine, as the authors could not find any similar experimental data for a cervical spine. It is necessary to conduct experiments for the cervical spine to be able to predict damage more accurately for this region. This model does not account for disc damage due to any disease, which might accelerate damage growth. Also, this model only considers damage in the annulus part of the disc.

Despite a few limitations, this model provides very good insights into damage evolution of the intervertebral disc. The three important advancements that this model brings to the area of modeling intervertebral disc degeneration are:

- Inclusion of recovery into the damage model that makes the model resemble the biological phenomenon more accurately than any other pre-existing models.
- Consideration of non-linearity of damage evolution.
- Efficient approximation strategy that makes the model computationally inexpensive and fast while retaining the physics of the process.

The model can be further developed to aid in evaluation of head supported devices and yield recommendations towards better helmet designs and usage scenarios.

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