Two-Dimensional Symmetric Half-Plane Recursive Doubly Complementary Digital Lattice Filters

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OUTLINE

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INTRODUCTION

- Using DC Filters $\rightarrow$ The goals of \textit{allpass-complementary} & \textit{power-complementary} for digital video systems.

- Using LDAF to construct DC Filters $\rightarrow$ The designed DC Filters with approximately linear phase response without magnitude distortion, low pass-band sensitivity and robustness to filter coefficient quantization error.
LDAF BASED 2-D DC FILTERS

\[ A(z_1, z_2) : 2\text{-D DAF with Lattice Structure} \]

\[
\frac{Y(z_1, z_2)}{X(z_1, z_2)} = z_1^{-M} \frac{R_N(z_1, z_2)}{U(z_1, z_2)} \quad \frac{U(z_1, z_2)}{Q_N(z_1, z_2)} = z_1^{-M} z_2^{-N} L_N(z_1^{-1}, z_2^{-1}) = A(z_1, z_2)
\]

**G(z_1,z_2), H(z_1,z_2): Frequency Responses**

\[
G(e^{j\omega_1}, e^{j\omega_2}) = \frac{e^{-jM\omega_1} e^{-jN\omega_2} + A(e^{j\omega_1}, e^{j\omega_2})}{2}
\]

\[
H(e^{j\omega_1}, e^{j\omega_2}) = \frac{e^{-jM\omega_1} e^{-jN\omega_2} - A(e^{j\omega_1}, e^{j\omega_2})}{2}
\]

**all-pass-complementary property:**

\[
|G(e^{j\omega_1}, e^{j\omega_2}) + H(e^{j\omega_1}, e^{j\omega_2})| = 1, \text{ for } \forall (\omega_1, \omega_2)
\]

**power-complementary property:**

\[
|G(e^{j\omega_1}, e^{j\omega_2})|^2 + |H(e^{j\omega_1}, e^{j\omega_2})|^2 = 1, \text{ for } \forall (\omega_1, \omega_2)
\]

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STABILITY CONSTRAINTS

- **Stable LDAF → Stable DC Filters**
- **Stability Constraints of LDAF:**

  (I) $\arg\{A(e^{j\omega_1}, e^{j\omega_2})\}$ is monotonically decreasing along $\omega_1$ axis and $\arg\{A(e^{j\pi}, e^{j\omega_2})\} = \arg\{A(e^{j0}, e^{j\omega_2})\} - M\pi$ for $-\pi \leq \omega_2 \leq \pi$;

  (II) $\arg\{A(e^{j\omega_1}, e^{j\omega_2})\}$ is monotonically decreasing along $\omega_2$ axis and $\arg\{A(e^{j\omega_1}, e^{j\pi})\} = \arg\{A(e^{j\omega_1}, e^{j0})\} - N\pi$ for $-\pi \leq \omega_1 \leq \pi$.  

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PROPOSED DESIGN METHOD

The Frequency response of $A(z_1,z_2)$:

$A(e^{j\omega_1},e^{j\omega_2}) = e^{j(-M_1\omega_1-N_1\omega_2-2\phi(\omega_1,\omega_2))}$

$G(e^{j\omega_1},e^{j\omega_2}) = \frac{e^{-jM_1\omega_1} e^{-jN_1\omega_2} + e^{j(-M_1\omega_1-N_1\omega_2-2\phi(\omega_1,\omega_2))}}{2}$

$= \cos\{\theta_m(\omega_1,\omega_2)\} \exp\{j\theta_p(\omega_1,\omega_2)\}$

and

$H(e^{j\omega_1},e^{j\omega_2}) = \frac{e^{-jM_1\omega_1} e^{-jN_1\omega_2} - e^{j(-M_1\omega_1-N_1\omega_2-2\phi(\omega_1,\omega_2))}}{2}$

$= \sin\{\theta_m(\omega_1,\omega_2)\} \exp\{j(\theta_p(\omega_1,\omega_2) + \frac{\pi}{2})\}$

→ Least Squares design problem:

Finding the phase $\phi(\omega_1,\omega_2)$ to approximate the desired phase response $\phi_d(\omega_1,\omega_2)$ of $L_N(z_1,z_2)$

Minimize $\|W(\omega_1,\omega_2)\left\{\arg\{L_N(e^{j\omega_1},e^{j\omega_2})\} - \phi_d(\omega_1,\omega_2)\right\}\|^2$

$\|x\|^2$ is the squared norm of $x$, $W(\omega_1,\omega_2)$ is the preset frequency weighting function.

→ Easily solved by using the trust-region optimization method.

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SIMULATION RESULTS

Peak Stopband Attenuation (PSA)

\[
PSA = -\max_{(\omega_1, \omega_2) \in \Omega_s} 20 \log_{10} \left( \left| G \left( e^{j\omega_1}, e^{j\omega_2} \right) \right| \right) \text{ (dB)},
\]

Passband Magnitude Mean-Squared Errors (PMSE)

\[
PMSE = \frac{\sum \sum \left( \left| G \left( e^{j\omega_1}, e^{j\omega_2} \right) \right| - \left| G_d \left( \omega_1, \omega_2 \right) \right| \right)^2}{\text{number of grid points in the passband}},
\]

Stopband Magnitude Mean-Squared Errors (SMSE)

\[
SMSE = \frac{\sum \sum \left( \left| G \left( e^{j\omega_1}, e^{j\omega_2} \right) \right| - \left| G_d \left( \omega_1, \omega_2 \right) \right| \right)^2}{\text{number of grid points in the stopband}},
\]

Passband Phase Mean-Squared Error (PPMSE)

\[
PPMSE = \frac{\sum \sum \left[ \arg \left\{ L_{N_2} \left( e^{j\omega_1}, e^{j\omega_2} \right) \right\} - \phi_d \left( \omega_1, \omega_2 \right) \right]^2}{\text{number of grid points in the passband}} \text{ (radian}^2\text{)}
\]

Peak Passband Phase Error (PPPE)

\[
PPPE = \max_{(\omega_1, \omega_2) \in \Omega_p} \left| \arg \left\{ L_{N_2} \left( e^{j\omega_1}, e^{j\omega_2} \right) \right\} - \phi_d \left( \omega_1, \omega_2 \right) \right| \text{ (radian)}
\]

\[ M_1 = M_2 = 4, \text{ and } N_1+1 = N_2 = 7. \]

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## Significant Results for the Design Example

<table>
<thead>
<tr>
<th></th>
<th>Conventional Direct-Form Design [17]</th>
<th>Proposed Lattice-Form Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMSE</td>
<td>$5.1715 \times 10^{-10}$</td>
<td>$8.1617 \times 10^{-11}$</td>
</tr>
<tr>
<td>SMSE</td>
<td>$1.3422 \times 10^{-6}$</td>
<td>$8.5503 \times 10^{-7}$</td>
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<td>PSA</td>
<td>34.9551</td>
<td>39.8172</td>
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<tr>
<td>PPMSE</td>
<td>$4.2567 \times 10^{-6}$</td>
<td>$3.1589 \times 10^{-6}$</td>
</tr>
<tr>
<td>PPPE</td>
<td>$4.9311 \times 10^{-2}$</td>
<td>$3.4069 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
CONCLUSION

- New Lattice Structure for DAF-Based DC Filters.
- New Stability Constraints for LDAF-Based DC Filters.
- New Formulation of the WLS Design.
- Proposed Method Provides More Satisfactory 2-D DC Filters’ Frequency Response.

Thank you very much!